

The journey towards differentiable hadronization models

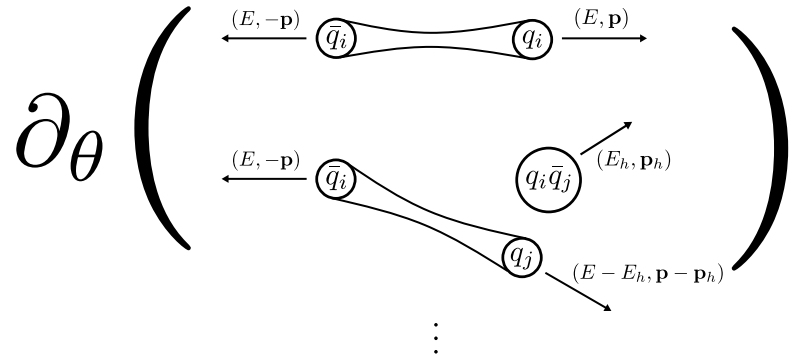
University of Alabama

HEP Seminar

October 17th, 2025

Tony Menzo

Postdoc @ University of Alabama & Fermilab



Based upon works in [2203.04983](#), [2308.13459](#), [2311.09296](#), [2410.06342](#), [2411.02194](#), [2505.00142](#),
[2503.05667](#), [2509.03592](#), [2511.xxxxx](#)

MLHAD



Jure Zupan



Phil Ilten



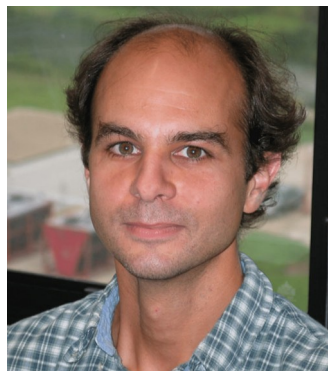
Manuel Szwec



Rikab Gambhir



Christian Bierlich



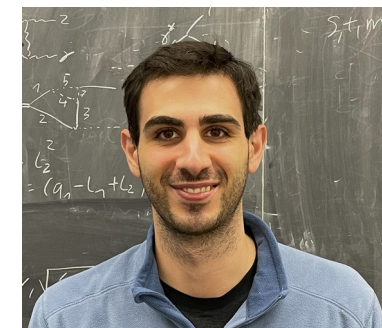
Steve Mrenna



Ahmed Youssef

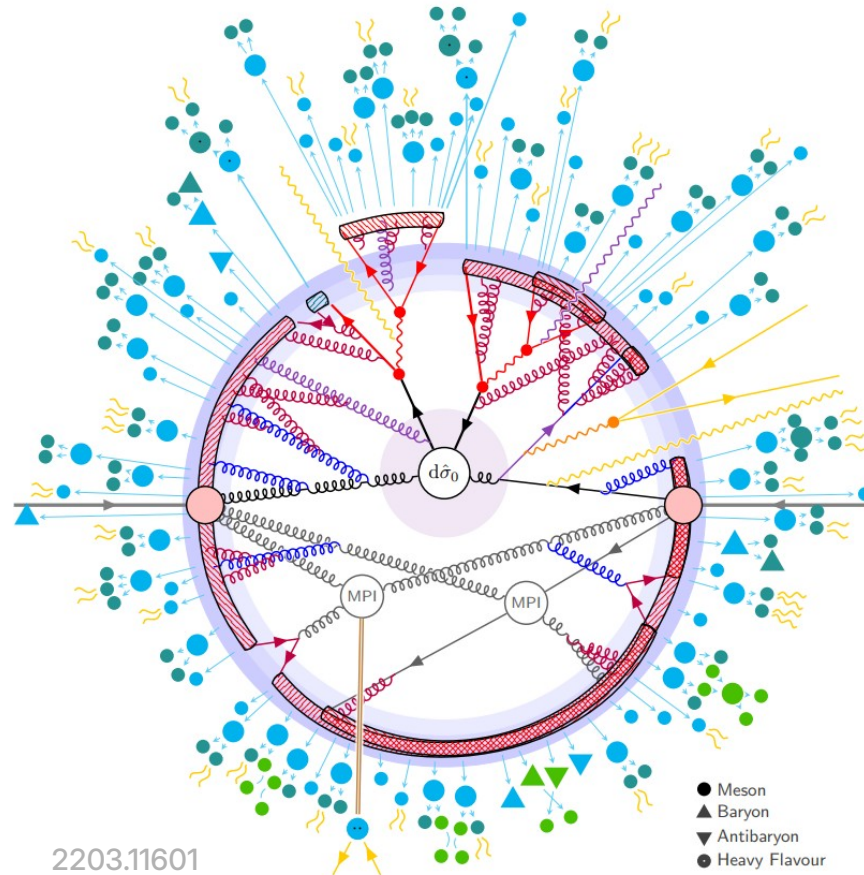


Michael Wilkinson

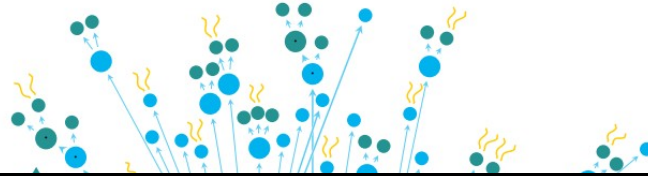


Ben Assi

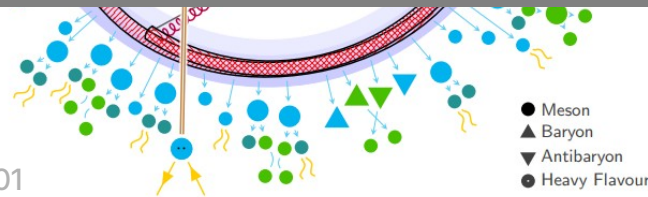
Monte Carlo Event generators



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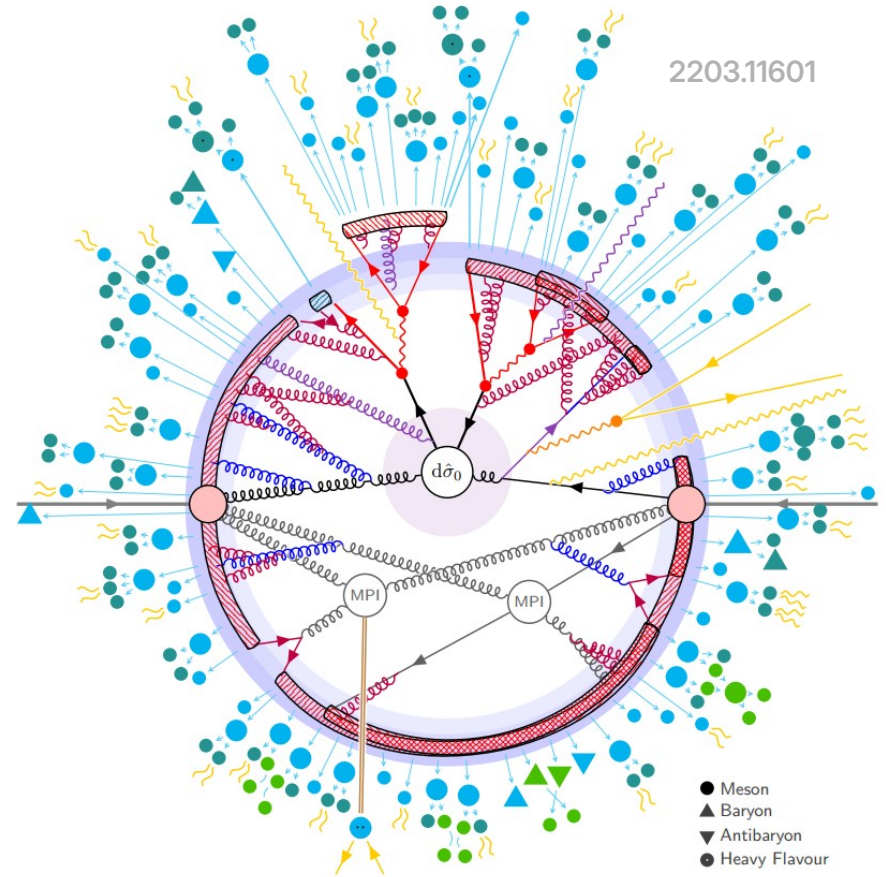
$$\mathcal{G} : \underbrace{\mathcal{S}(\mathcal{D}(\mathcal{H}(\mathcal{P}(\mathcal{M}))))}_{\text{Simulation}} = \mathcal{E} \simeq \underbrace{\begin{pmatrix} \{\text{id}, E, p_x, p_y, p_z, \dots\}_1 \\ \{\text{id}, E, p_x, p_y, p_z, \dots\}_2 \\ \vdots \\ \{\text{id}, E, p_x, p_y, p_z, \dots\}_N \end{pmatrix}}_{\text{Event record}} .$$



2203.11601

Hadronization

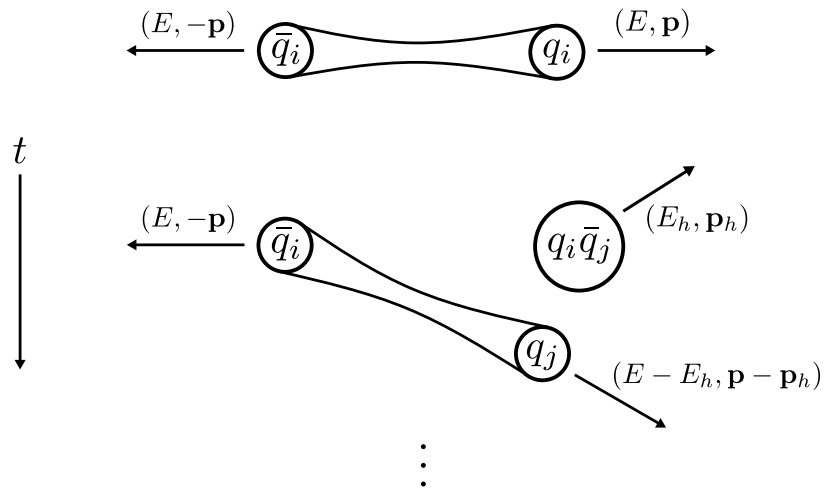
The conversion of a partonic system consisting of quarks, anti-quarks, and gluons ($q\bar{q}$, $qg\bar{q}$, gg , qqq , $qq\bar{q}\bar{q}$, $qqqgg$, ...) to a final state consisting of hadrons (h_1, h_2, \dots, h_n).



Phenomenological models

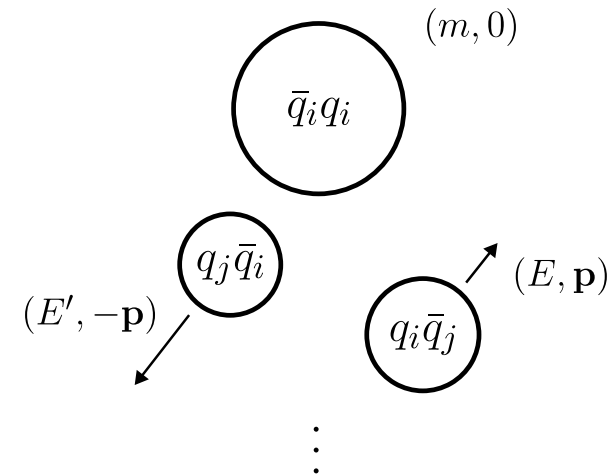
Lund string model

(used in Pythia)



Cluster model

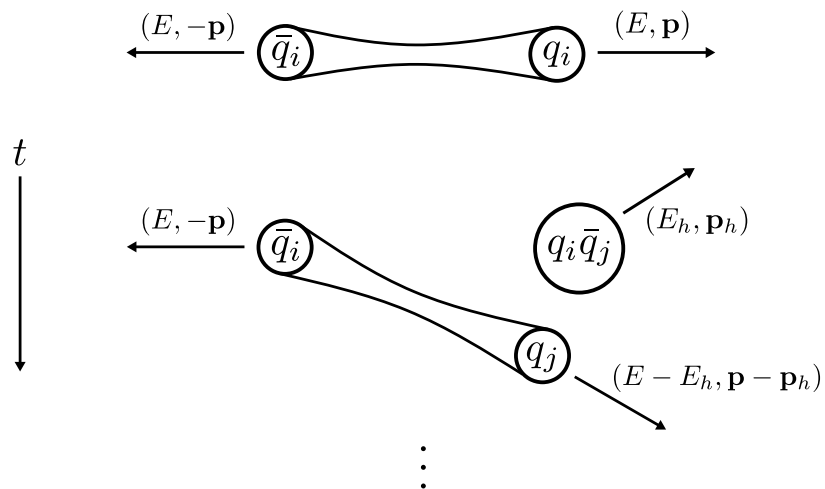
(used in Herwig, Sherpa)



Phenomenological models

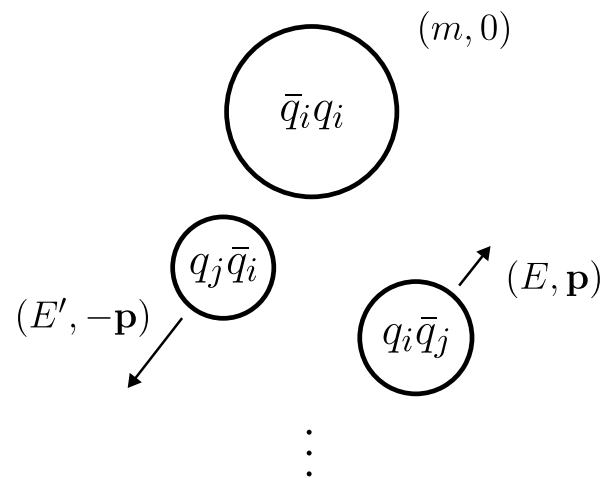
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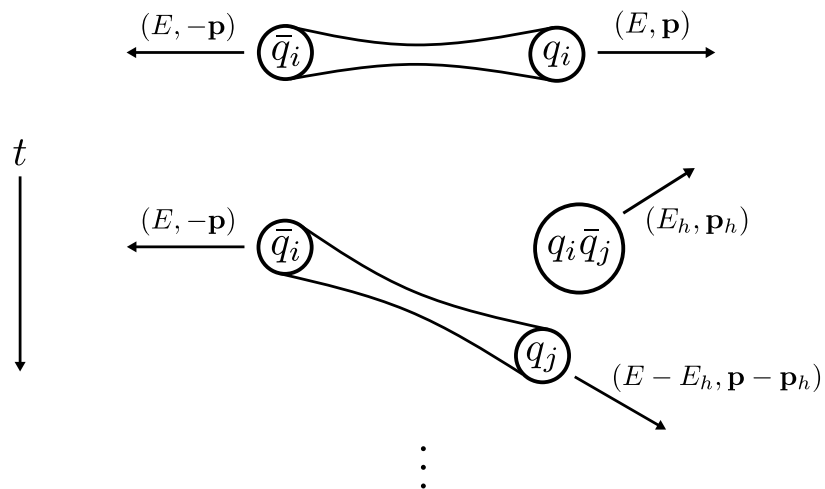
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Phenomenological models

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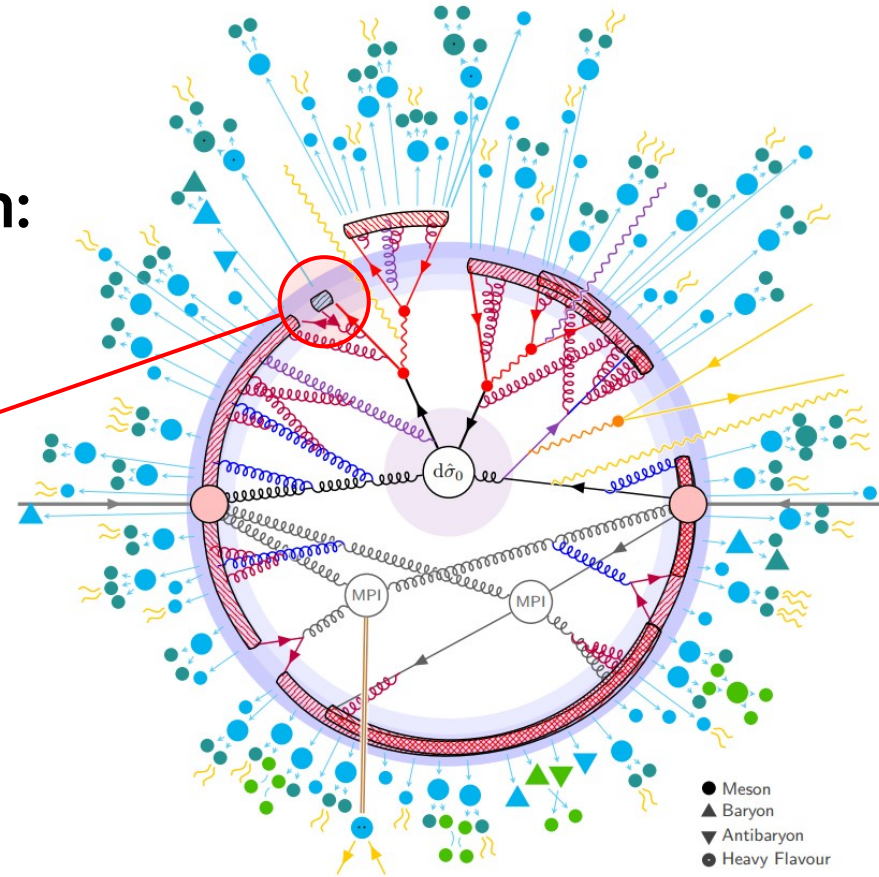
[HadML collaboration](#)

For ML methods applied to
the cluster model:

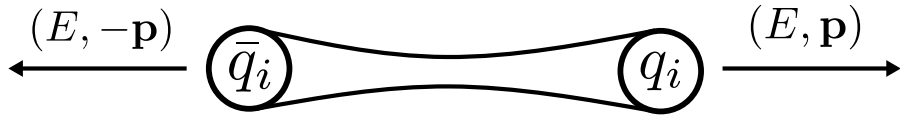
2203.12660, 2305.17169,
2312.08453

⋮

The "atomic" hadronizing system: Quark-antiquark ($q\bar{q}$)



The algorithm ($q\bar{q}$)



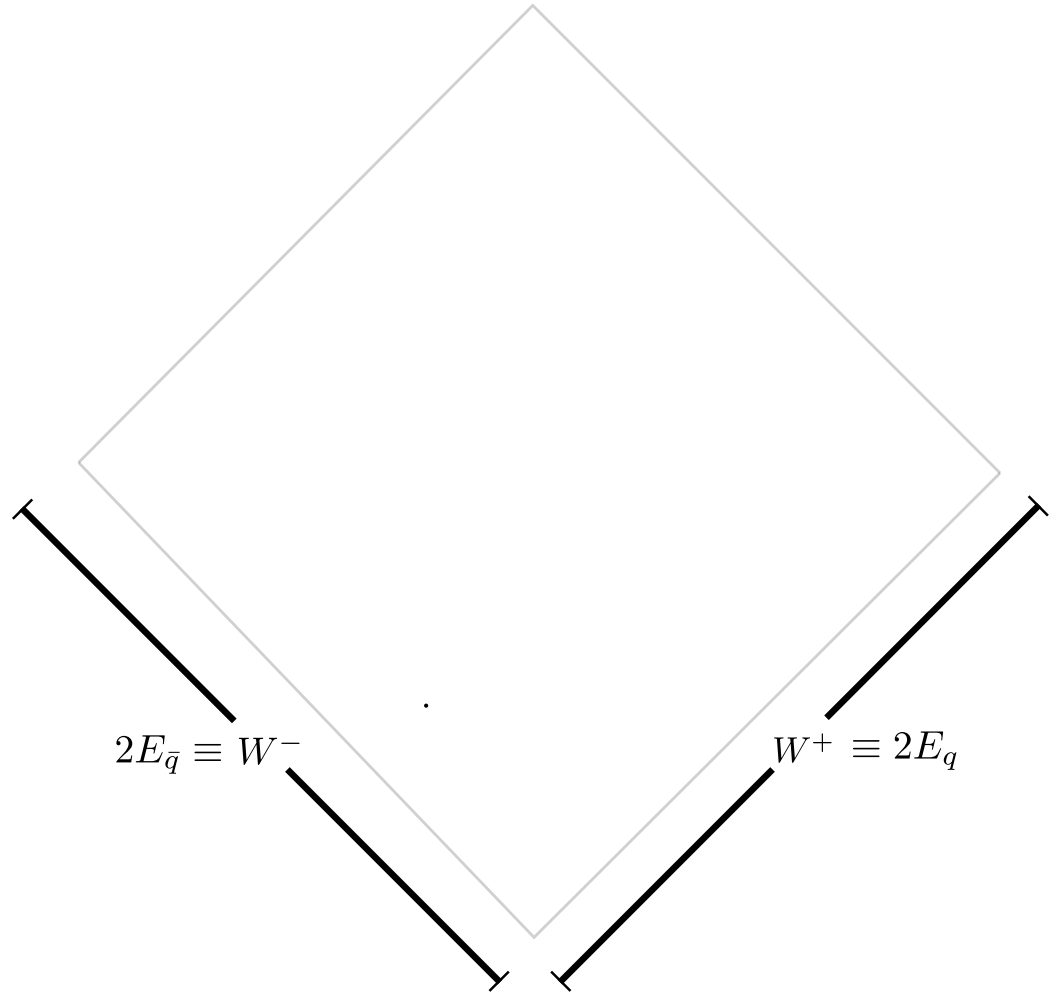
- 1) Randomly select one of the string ends
- 2) Sample string break flavor
- 3) Sample transverse momentum of new quarks

$$\mathcal{P}(p_x, p_y; \sigma_{p_T}) = \frac{1}{\pi\sigma_{p_T}^2} \exp\left(-\frac{p_x^2 + p_y^2}{\sigma_{p_T}^2}\right)$$

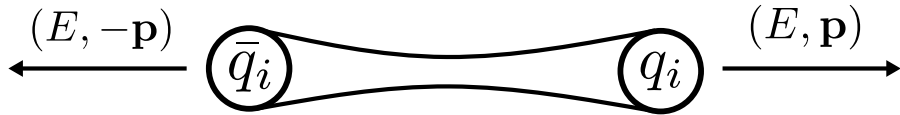
- 4) Sample longitudinal momentum fraction of new hadron

$$f(z) \propto \frac{(1-z)^a}{z} \exp\left(-\frac{bm_T^2}{z}\right), \quad z = \frac{p_z + E_h}{2E}$$

- 5) Repeat steps 1-4



The algorithm ($q\bar{q}$)



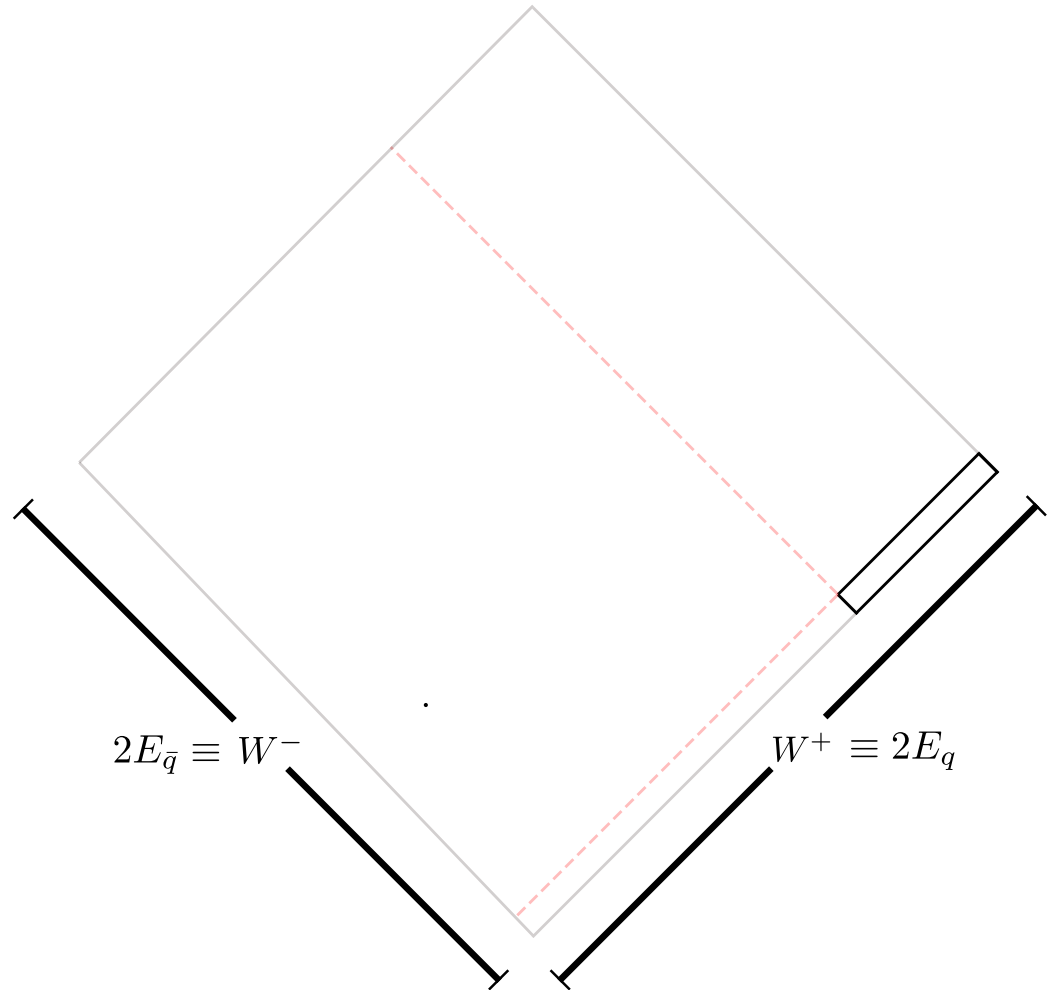
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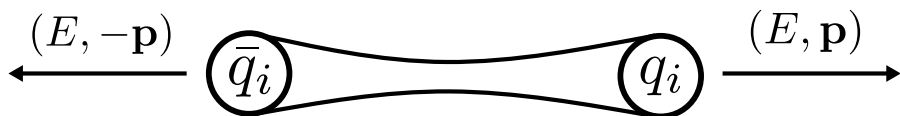
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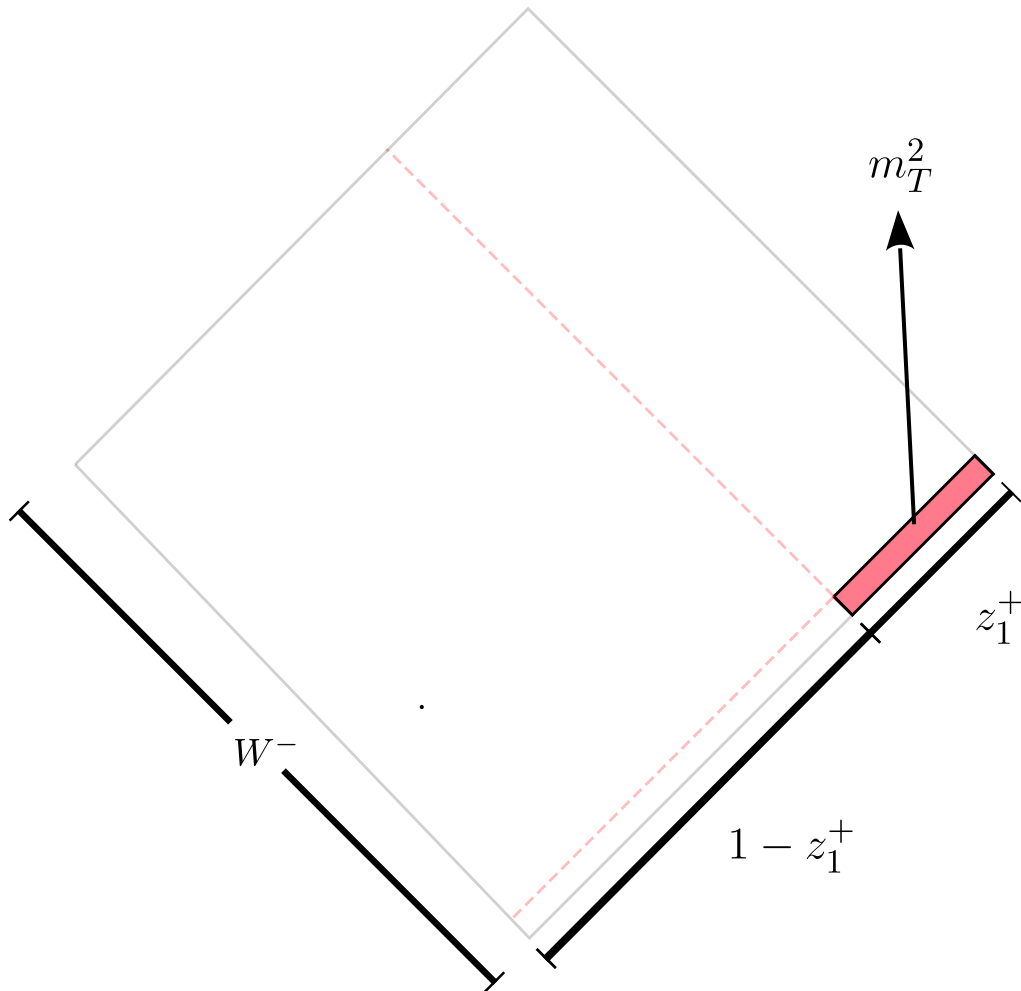
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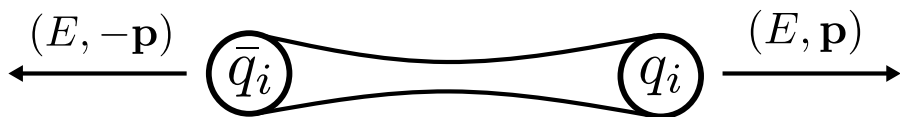
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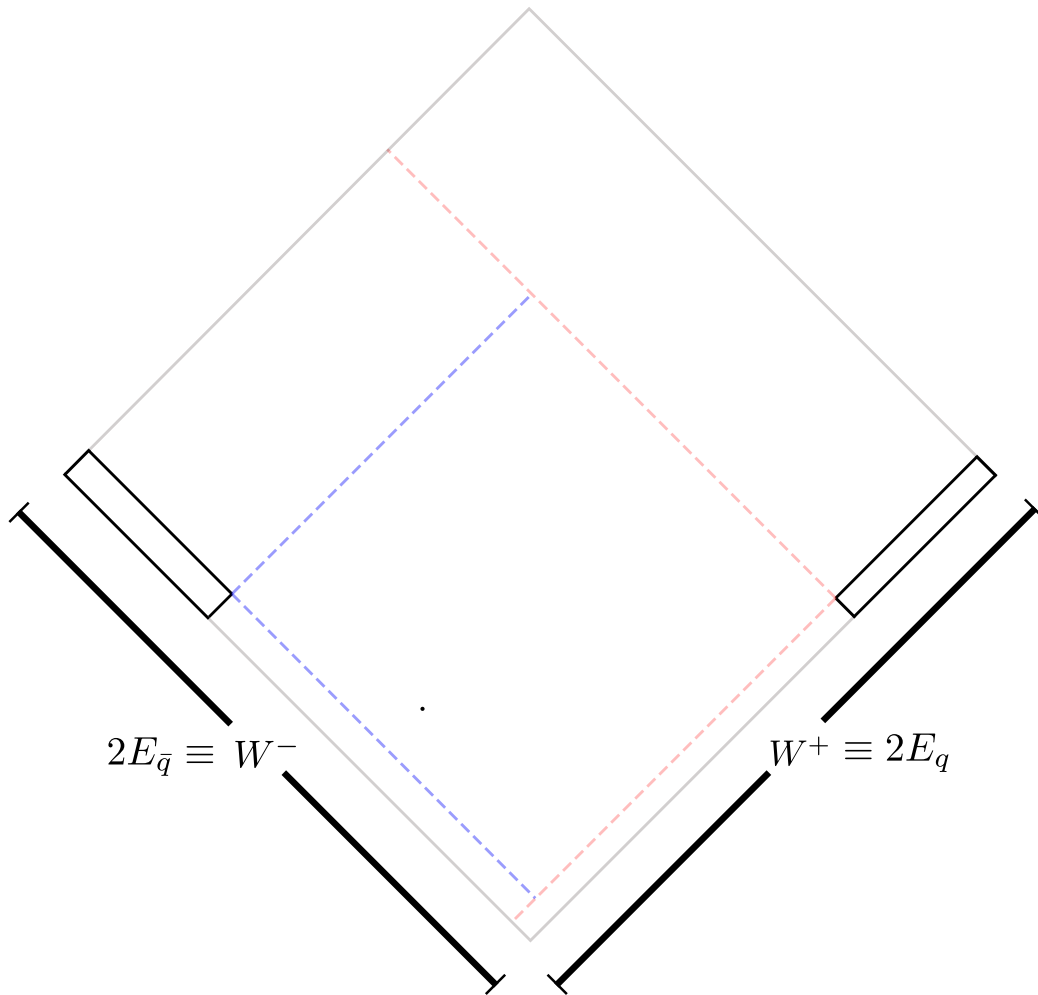
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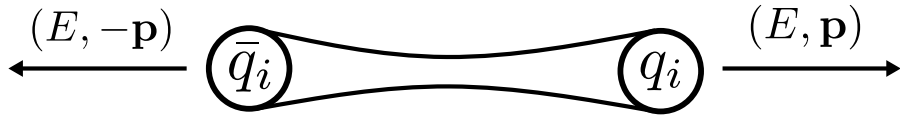
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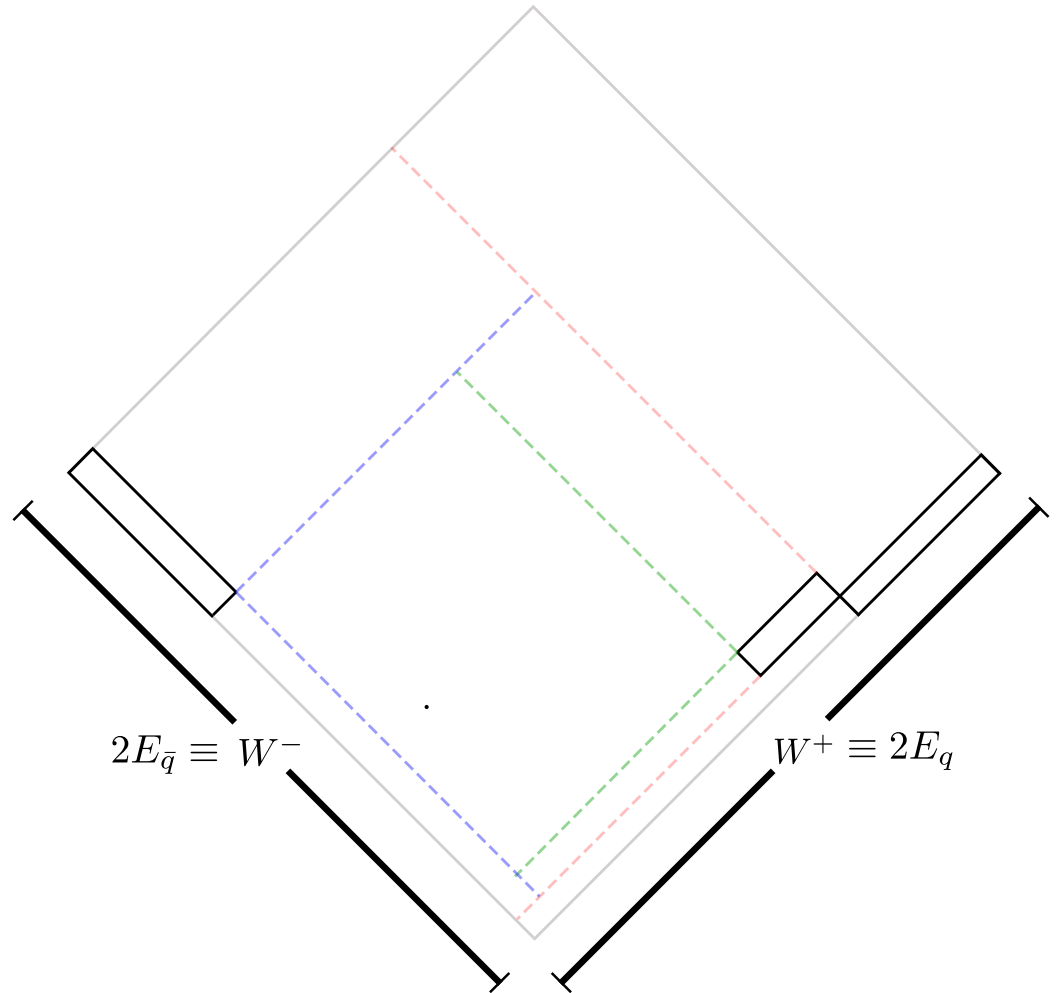
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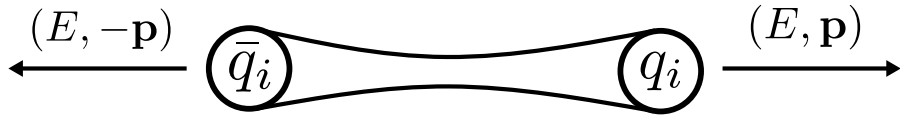
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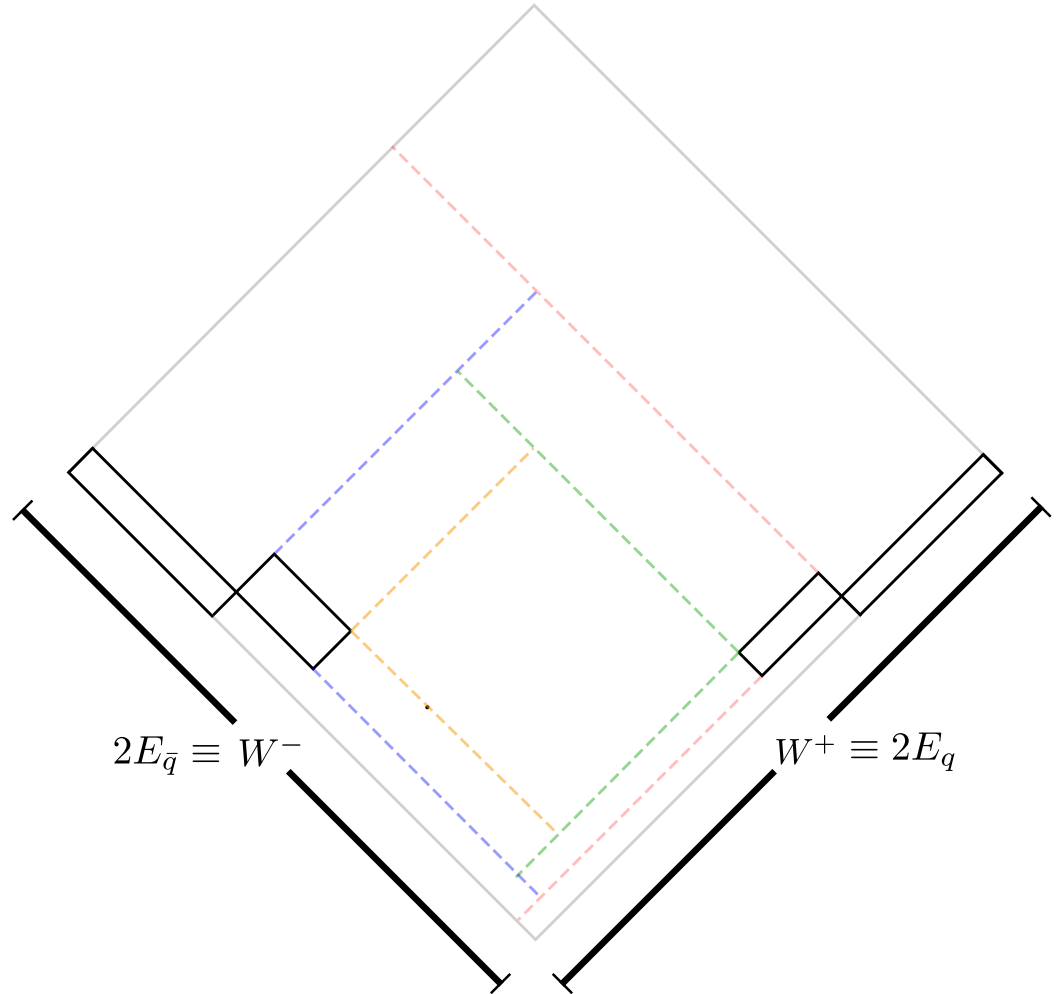
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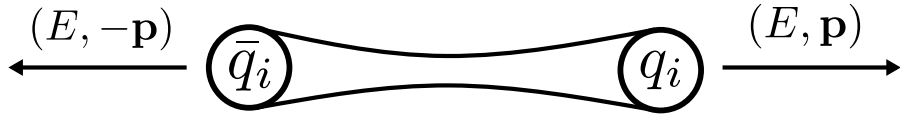
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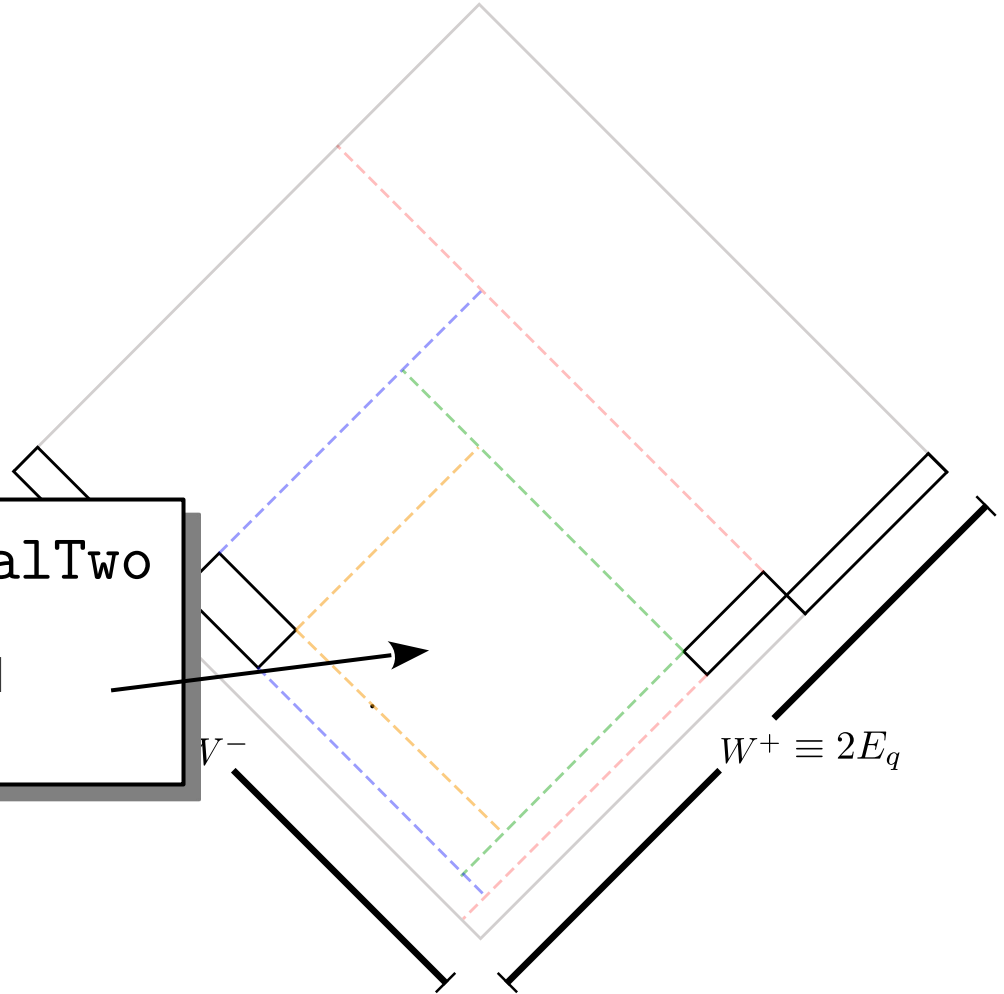
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$\mathcal{P}(z)$
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When E_{CM} goes below $\sim 2 \text{ GeV}$, finalTwo is called. This can fail, when it does, the full string system is re-simulated from the beginning.

$$f(z) \propto \frac{1}{z} \exp\left(-\frac{1}{z}\right), \quad z = \frac{E_q}{2E}$$

- 5) Repeat steps 1-4



The algorithm ($q\bar{q}$)



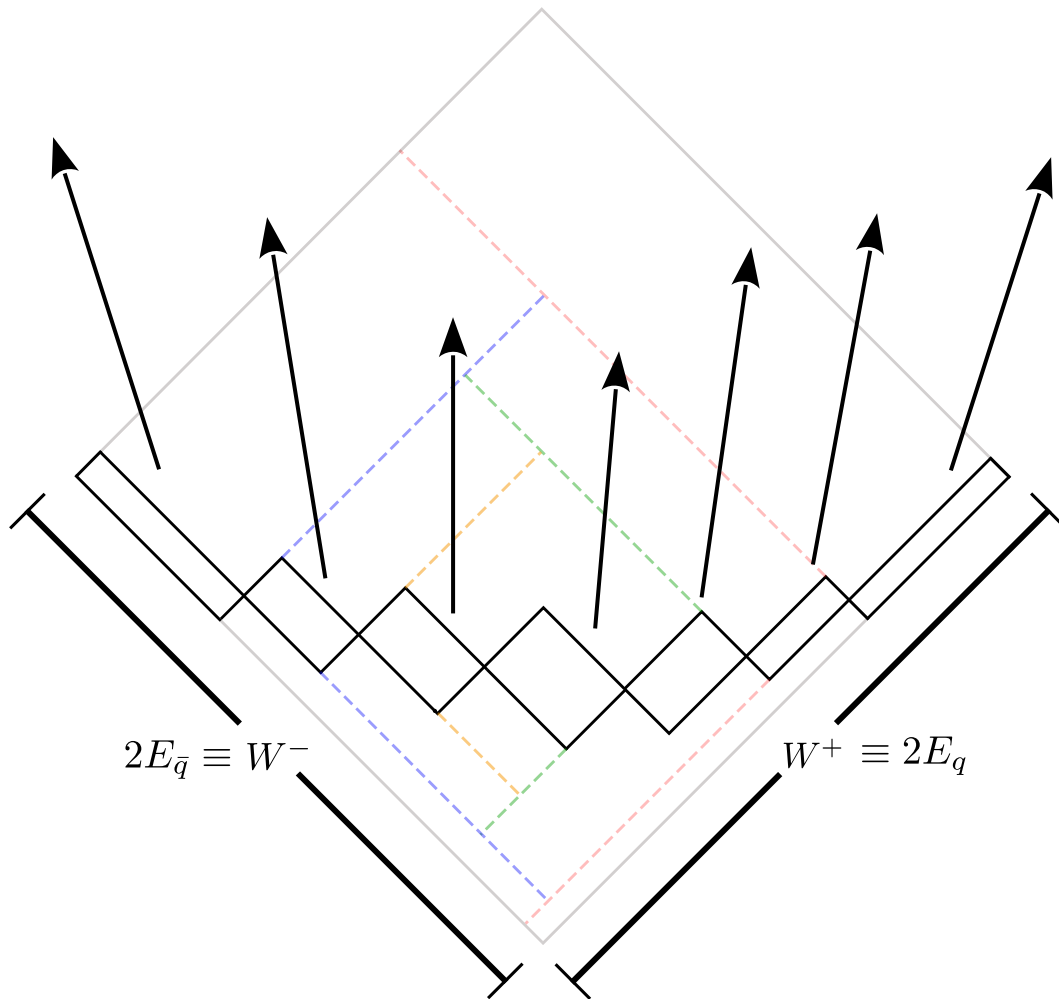
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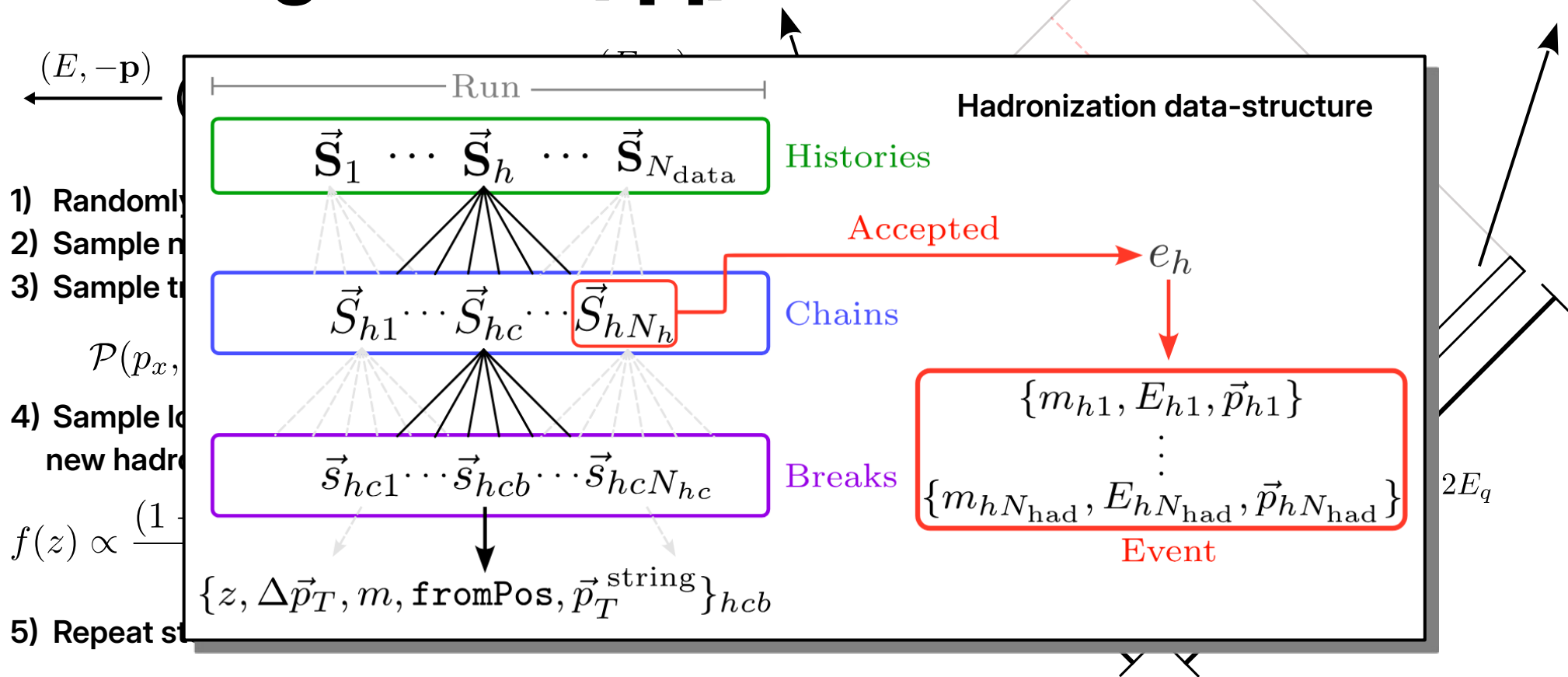
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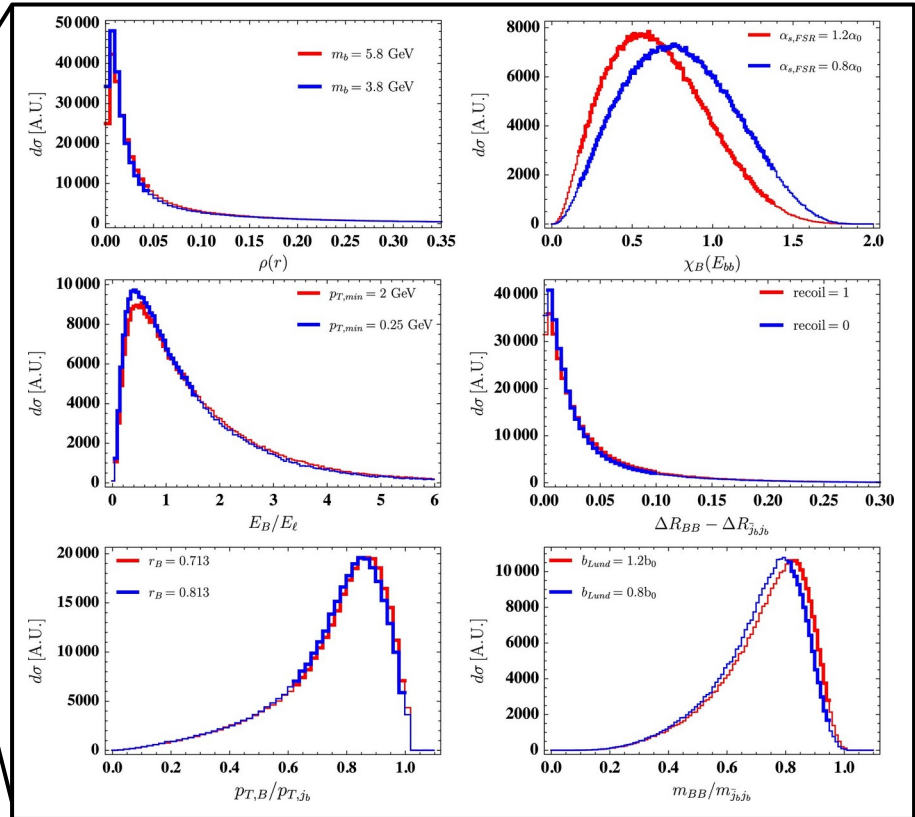
The algorithm (q \bar{q})



Why?

Precision (exclusive) measurements are sensitive to hadronization!

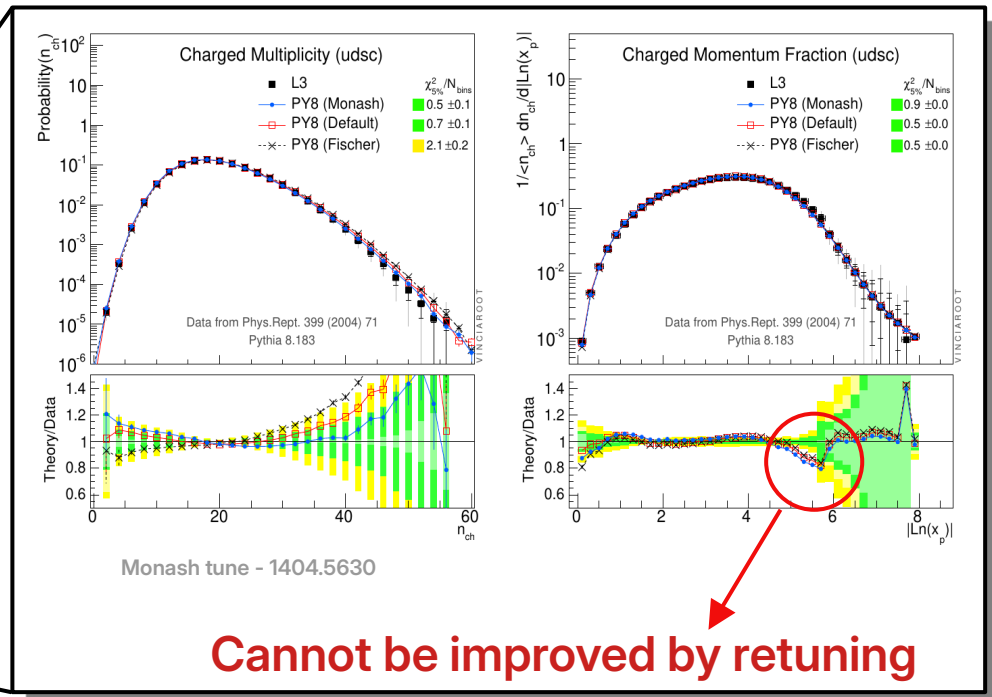
- Uncertainty reduction
 - Top quark mass measurement (r_b)
 - e^+e^- determination of α_s
- Mis-modeling bias
 - High-multiplicity events
 - Tuning discrepancies



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Three roads to improvement

- Improve model

- MPIs, rope hadronization, transverse mass suppression, flavor asymmetries, hadronic rescattering, multiscale models (string → hydrodynamical), flavor selector, etc.

Hard to come up with mathematically precise model without established calculational techniques

- Tend towards model independence

- Sample directly from global distributions

Non-universal and extremely difficult to convert into **representative** particle flow data (uninterpretable)

- Improve observables

- Come up with better, hadronization sensitive (IR unsafe), observables

New observables must be measured by experiments = **time + \$\$\$**

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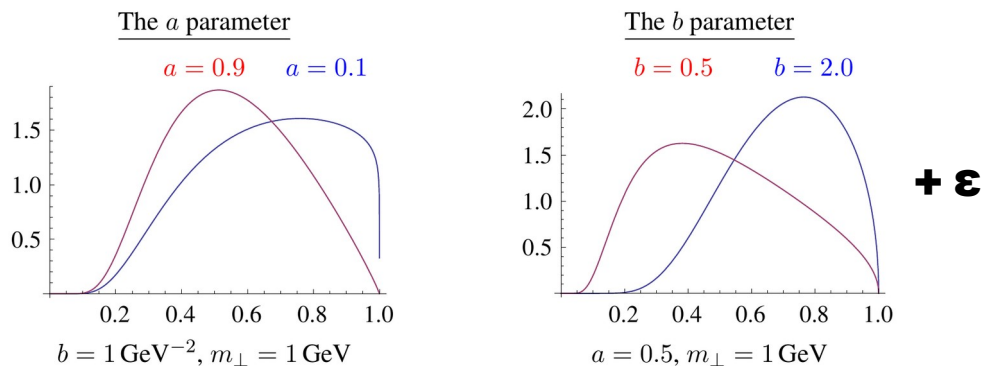
New observables must be measured by experiments = **time + \$\$\$**

***or some combination of all three**

Data-driven fine tuning

The phenomenological models of hadronization already give an acceptable description of a large amount of data.

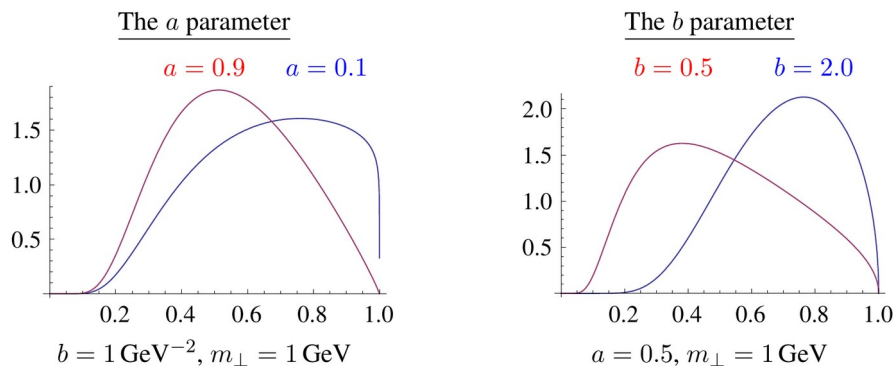
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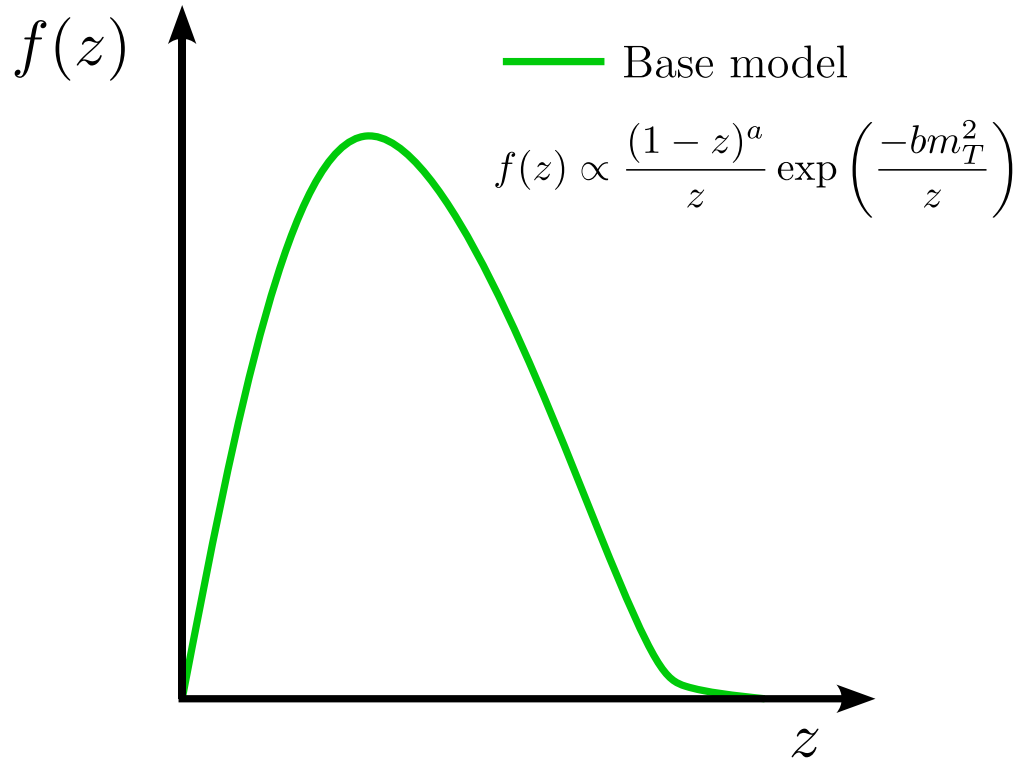


+ ϵ

Machine learning offers a nice framework to tackle this problem

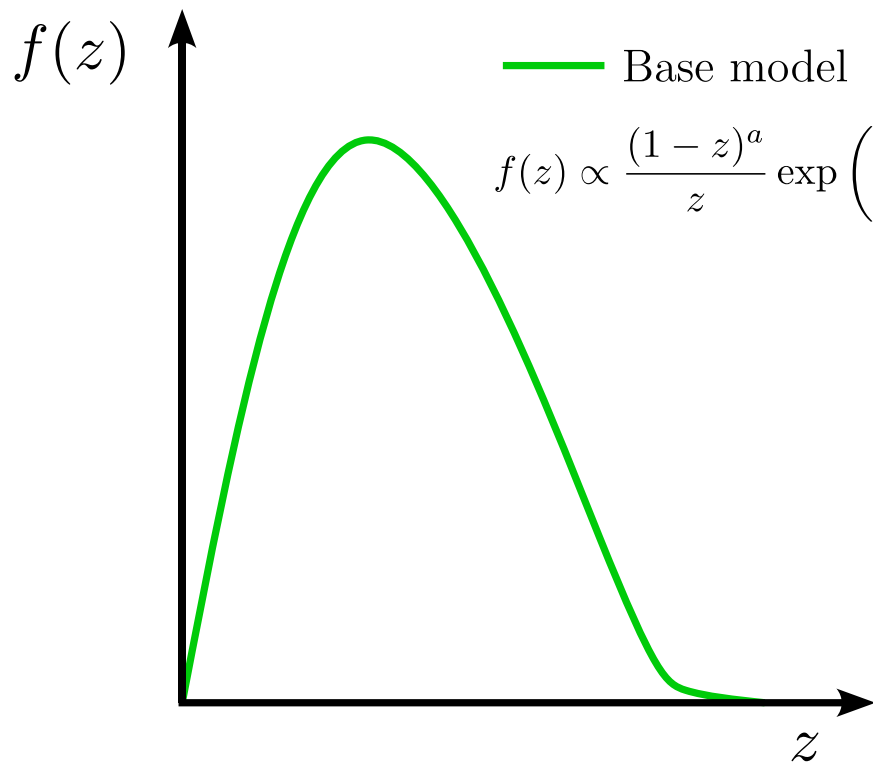
MLHAD **efforts: big picture**

Solving the “inverse problem of hadronization”



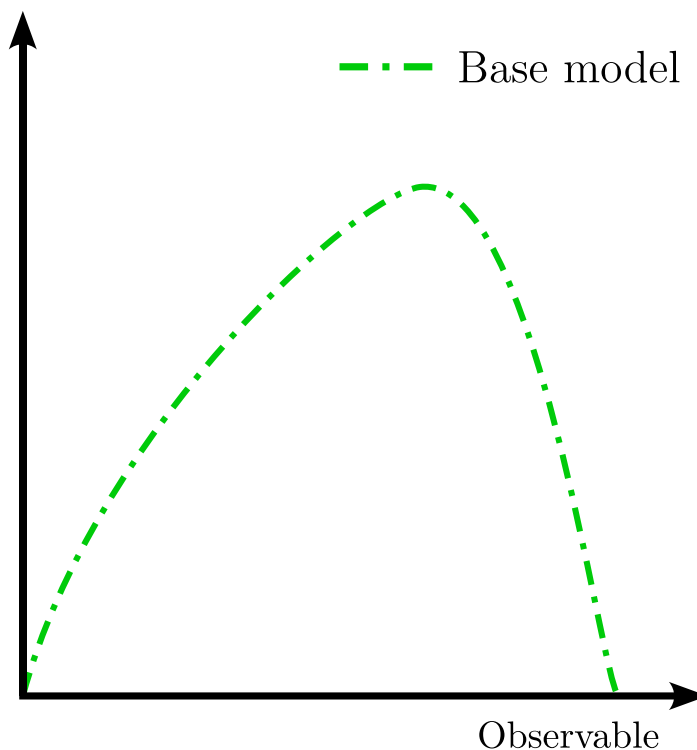
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— Base model

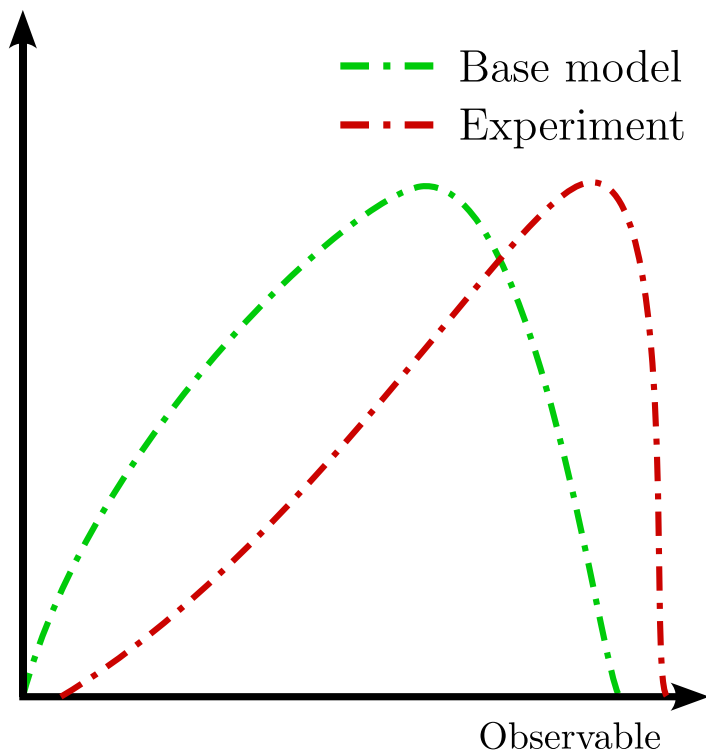
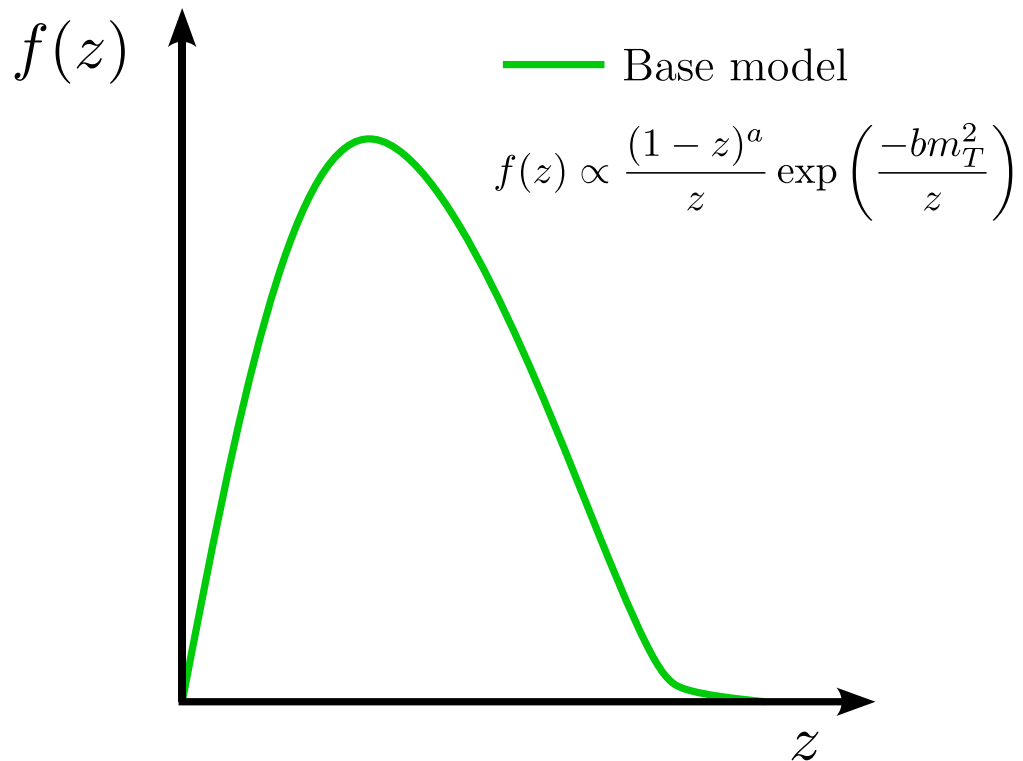
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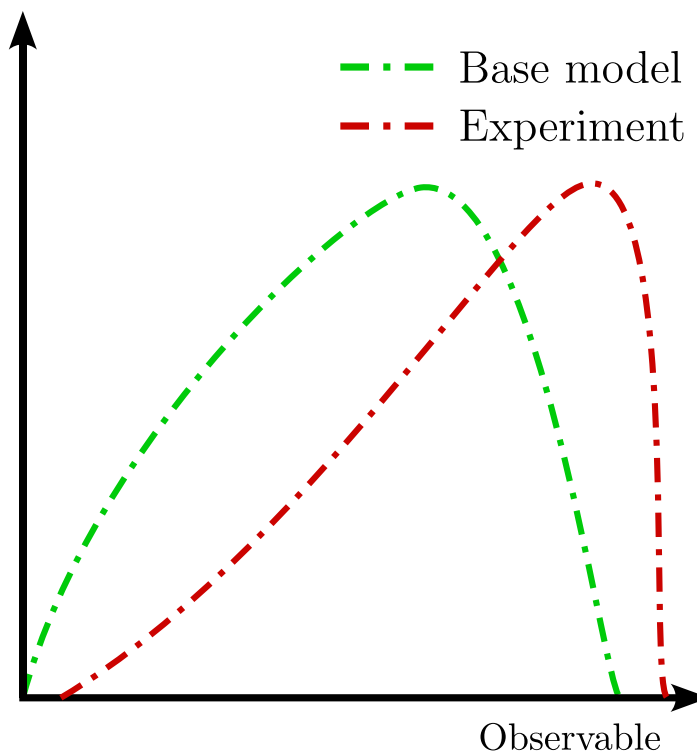
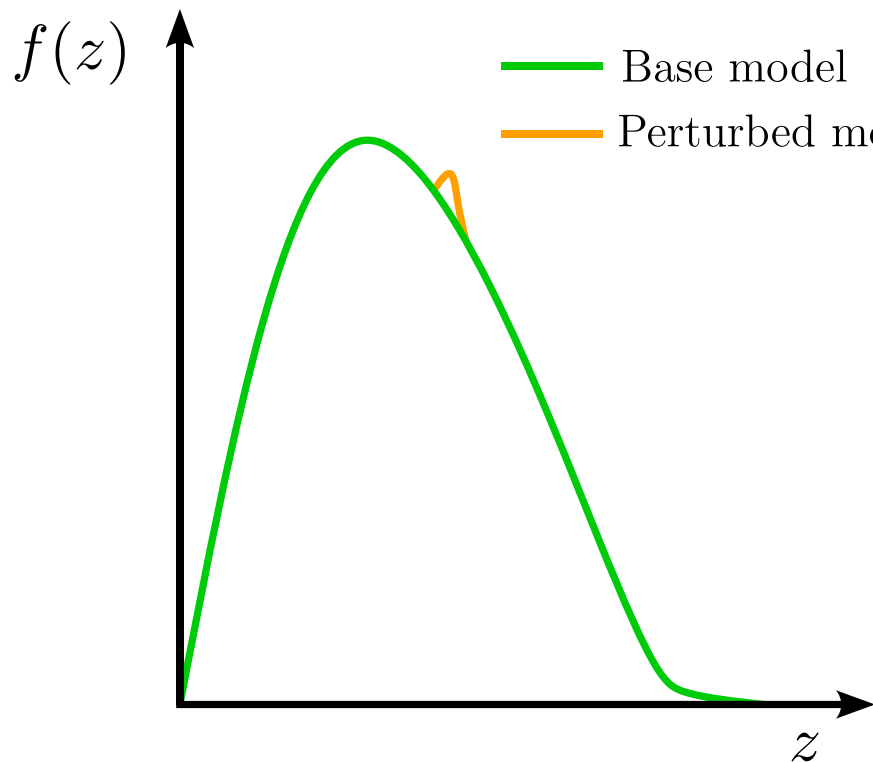
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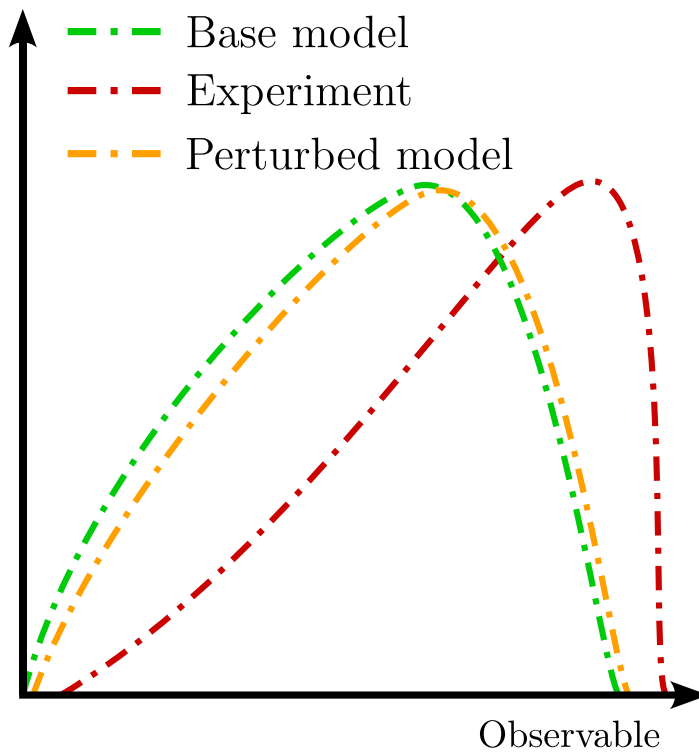
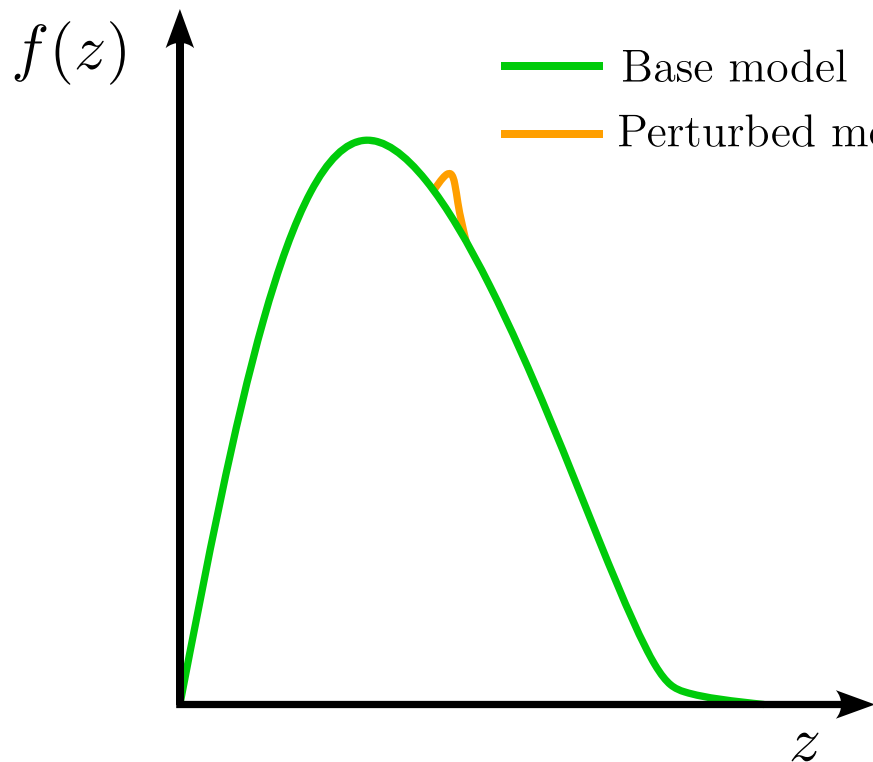
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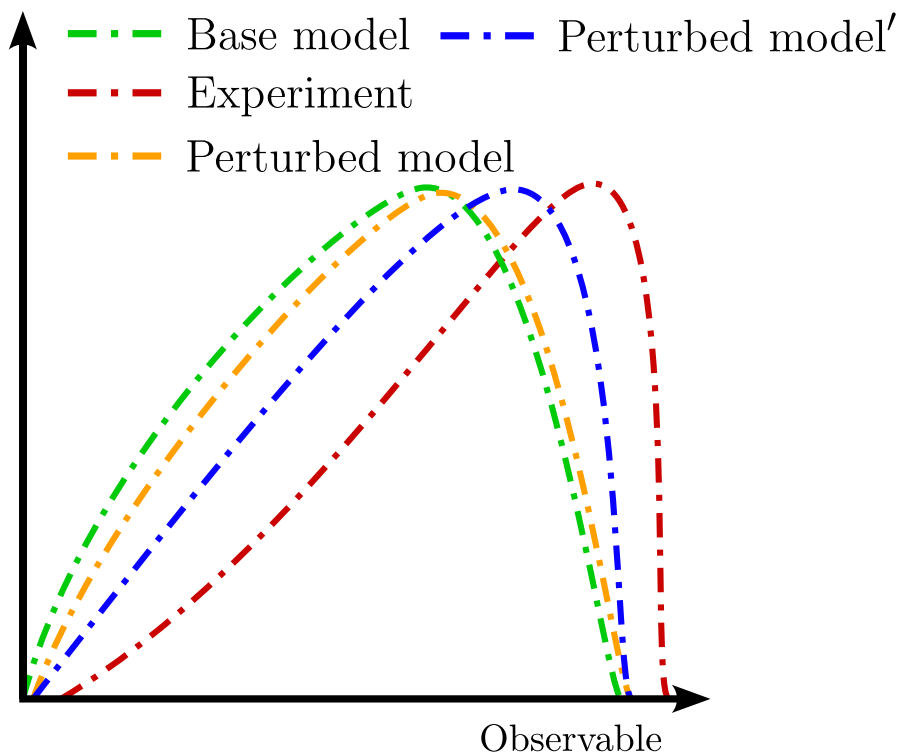
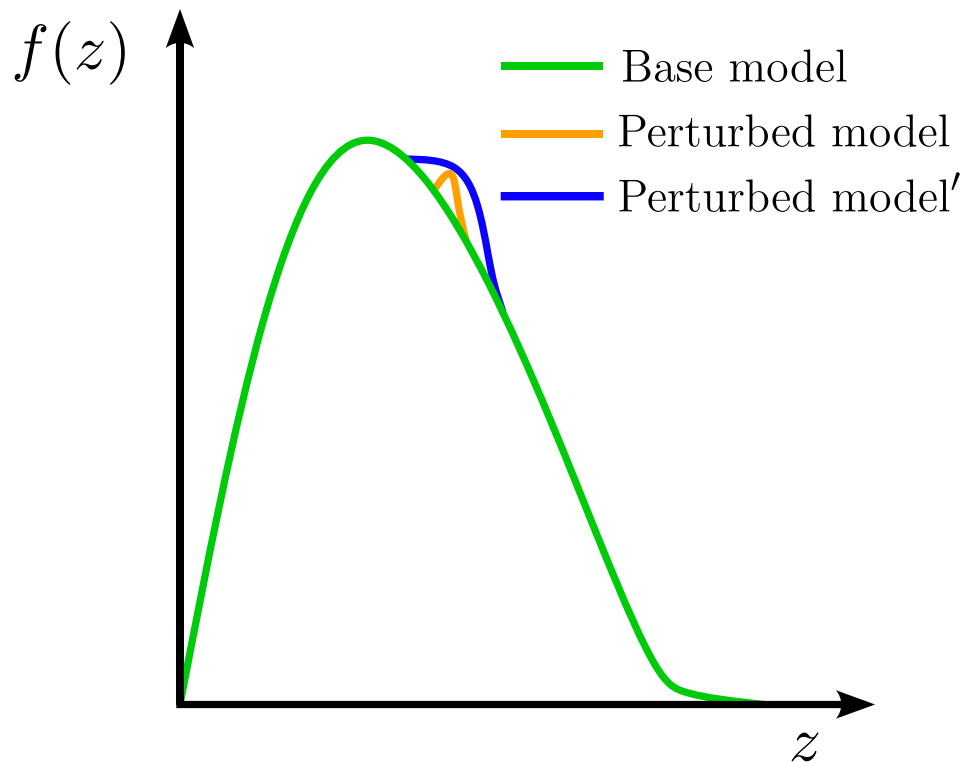
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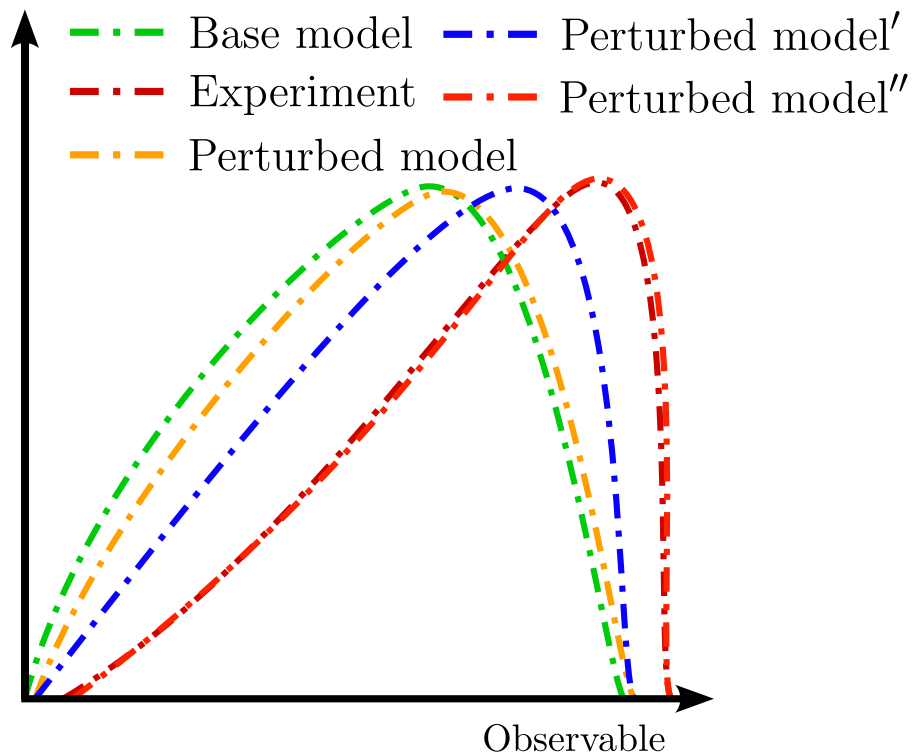
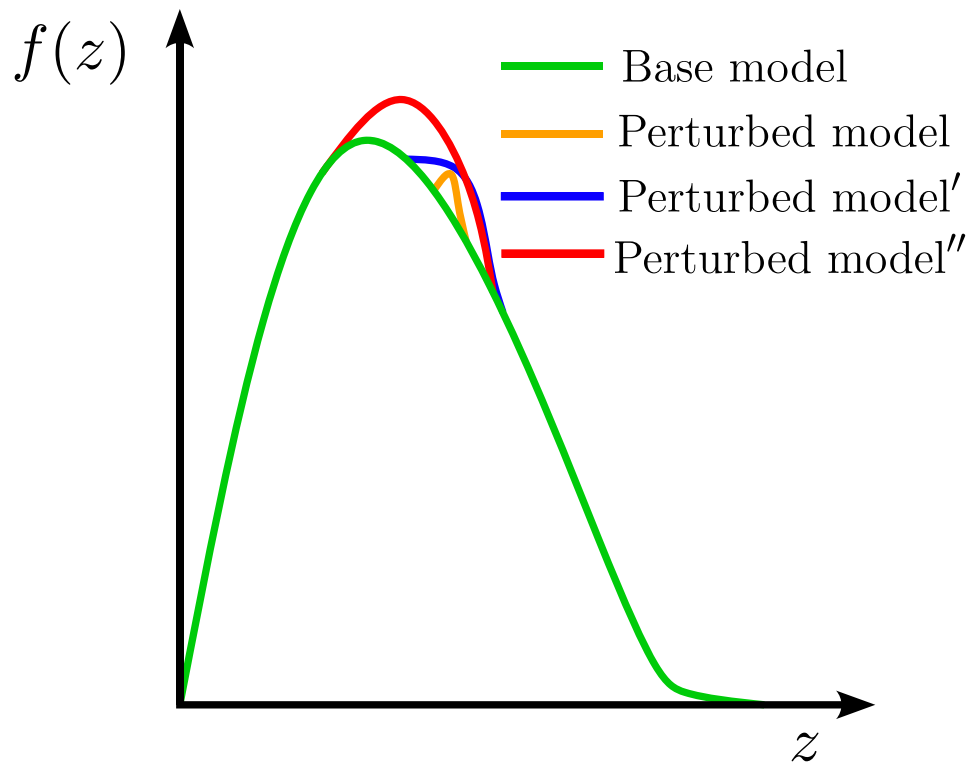
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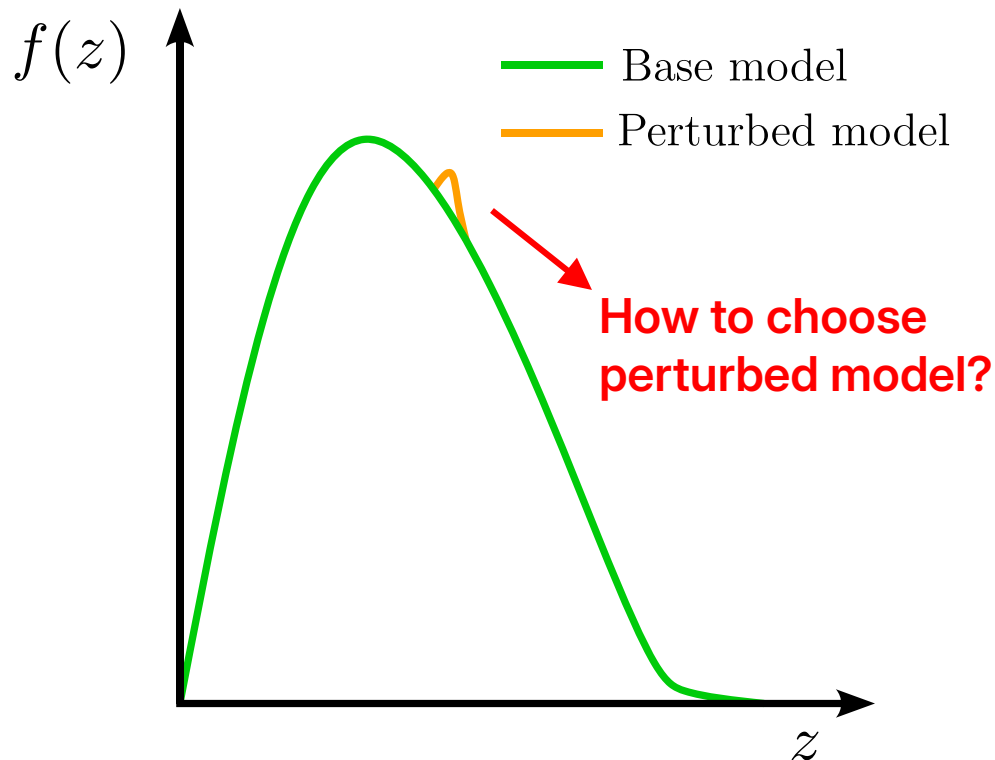
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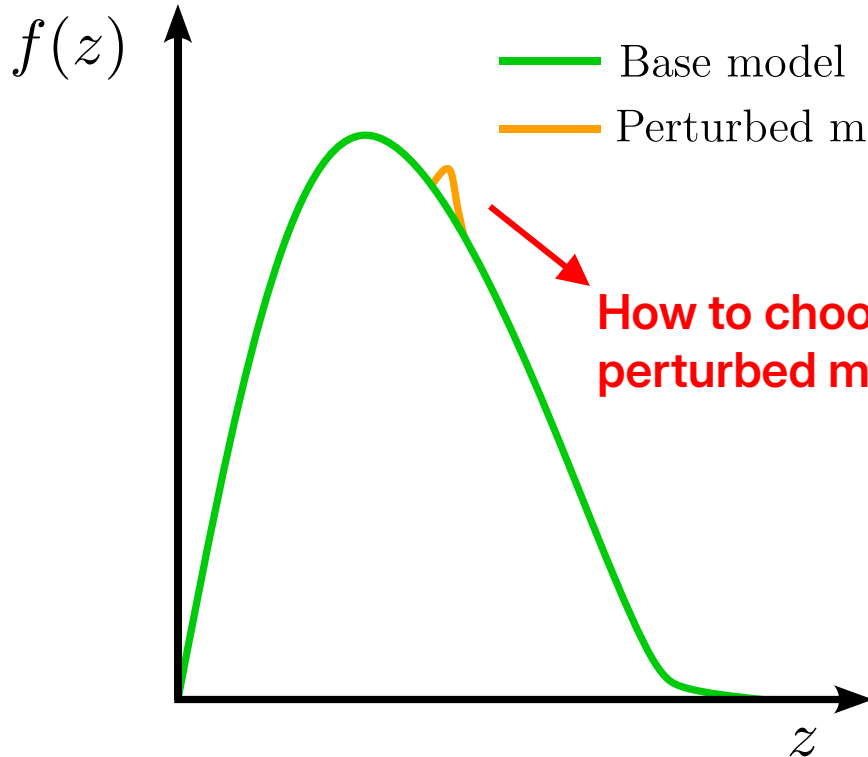
MLHAD efforts: big picture

Solving the “inverse problem of hadronization”



MLHAD efforts: big picture

Solving the “inverse problem of hadronization”



— Base model
— Perturbed model

~~ML~~

vs

ML

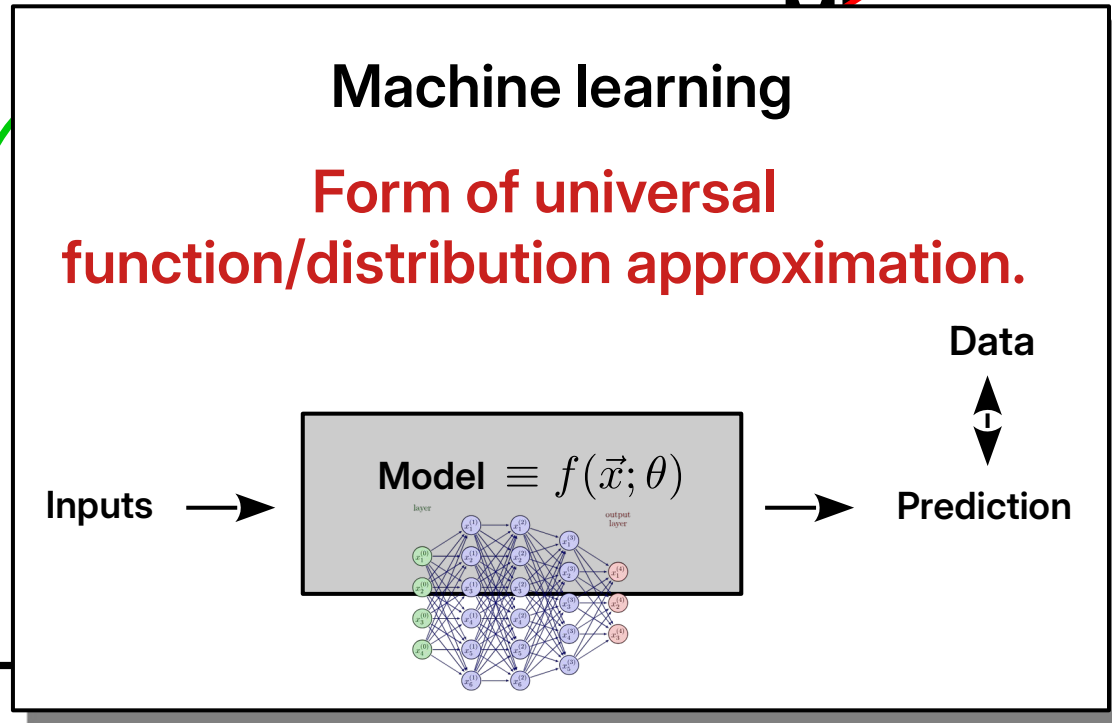
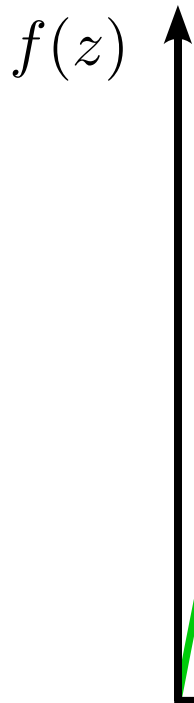
- Vary Lund parameters (traditional “manual”/semi-“manual” tuning)

- ML-based (data-driven) fragmentation function
 - **MAGIC, HOMER**

- Hybrid – keep Lund fragmentation function, promote Lund parameters to differentiable objects
 - **Rejection sampling with autodifferentiation (RSA)**

MLHAD efforts: big picture

Solving the "inverse problem of hadronization"



PS

ML

- ML-based (data-driven) fragmentation function
 - **MAGIC, HOMER**

deep Lund fragmentation function, Lund parameters, tunable objects
on sampling with reweighting (RSA)

The "reweighting revolution"

2308.13459 ,2411.02194, 2505.00142, 26xx.xxxxx

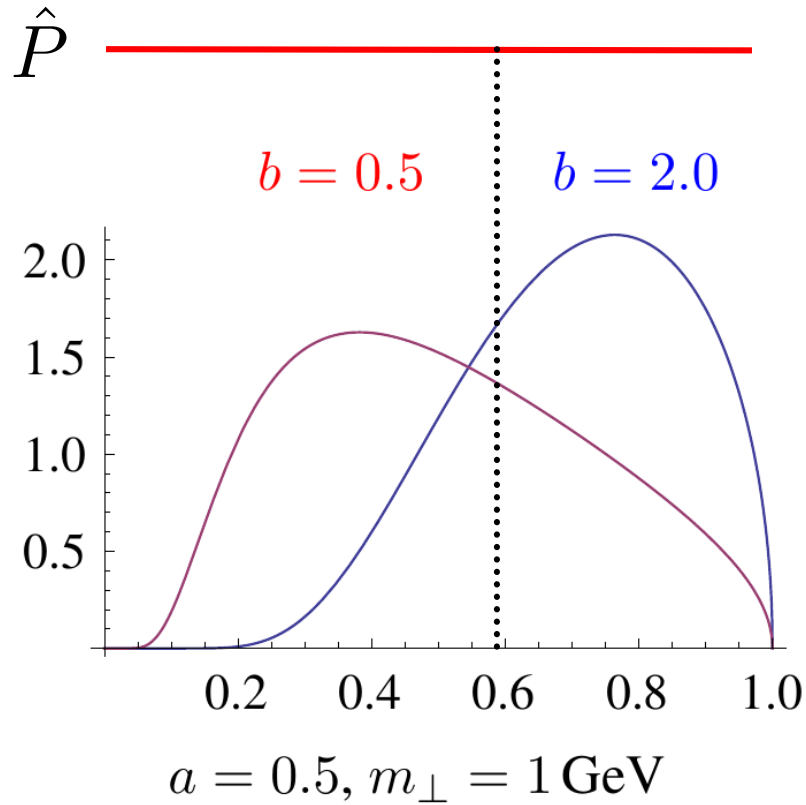
In collaboration w/

MLHAD 

+

Nick Heller, Ben Nachman, Andrzej Siodmok

Kinematic reweighting (2308.13459)



Rejection sampling:

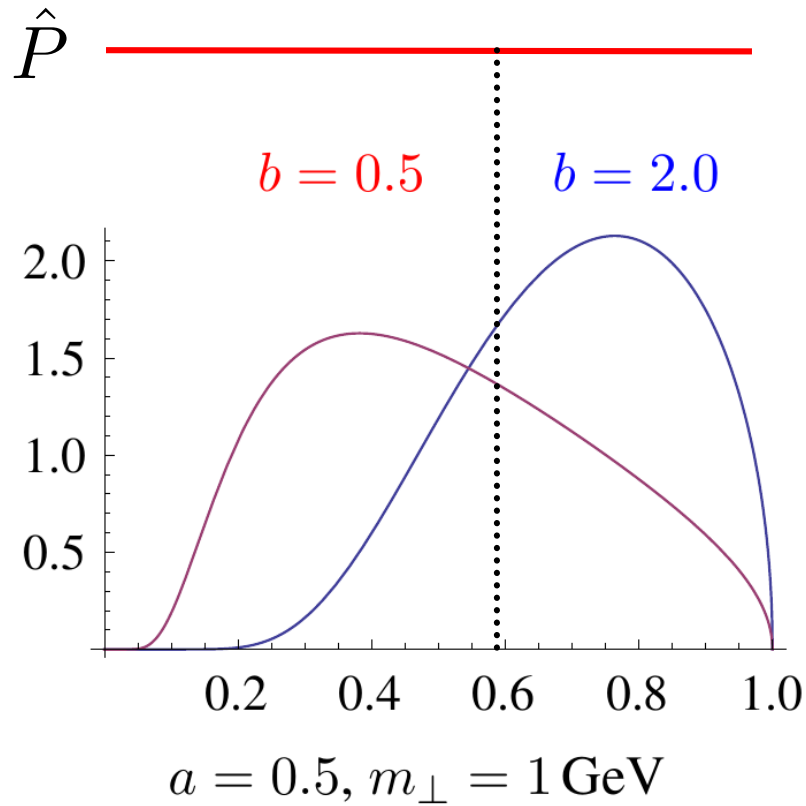
$$\text{Acceptance probability: } P_{\text{accept}} = \frac{p(z, \theta)}{\hat{P}}$$

$$\text{Rejection probability: } P_{\text{reject}} = 1 - P_{\text{accept}}$$

$$w_{\text{accept}} = \frac{P_{\text{accept}}(z; \theta')}{P_{\text{accept}}(z; \theta)}$$

$$w_{\text{reject}} = \frac{1 - P_{\text{accept}}(z; \theta')}{1 - P_{\text{accept}}(z; \theta)}$$

Kinematic reweighting (2308.13459)



Rejection sampling:

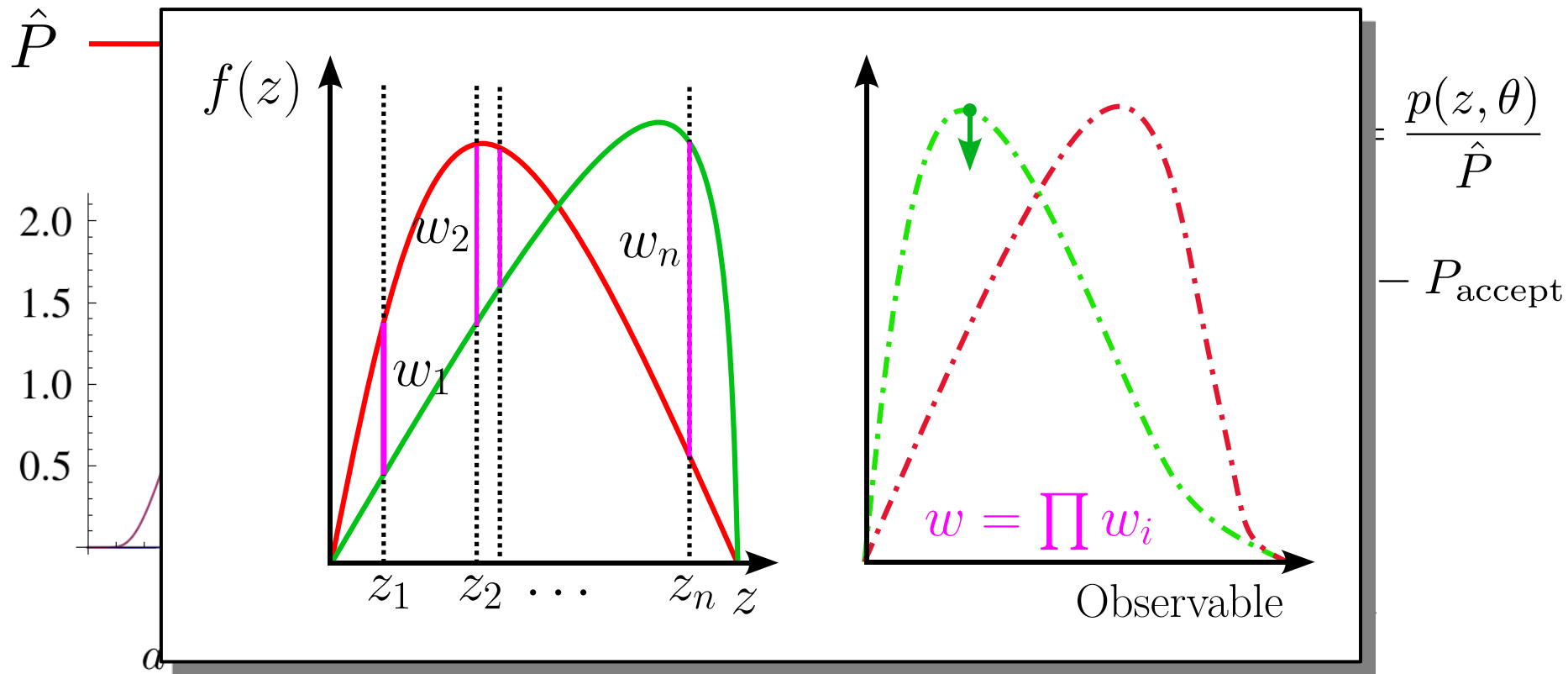
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$$w_{\text{reject}} = \frac{\hat{P} - p(z; \theta')}{\hat{P} - p(z; \theta)}$$

Kinematic reweighting (2308.13459)



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\hat{P}

Rejection sampling:

Data-structure:

$$z = \begin{pmatrix} z_1 \\ z_2 \\ \vdots \\ z_N = \dots \end{pmatrix} = \begin{pmatrix} \left\{ m_T^{h_1}, z_{\text{accept}}^{h_1}, z_{\text{reject}}^{1,h_1}, \dots, z_{\text{reject}}^{n_{h_1},h_1} \right\} \\ \left\{ m_T^{h_2}, z_{\text{accept}}^{h_2}, z_{\text{reject}}^{1,h_2}, \dots, z_{\text{reject}}^{n_{h_2},h_2} \right\} \\ \left\{ m_T^{h_3}, z_{\text{accept}}^{h_3}, z_{\text{reject}}^{1,h_3}, \dots, z_{\text{reject}}^{n_{h_3},h_3} \right\} \\ \left\{ m_T^{h_4}, z_{\text{accept}}^{h_4}, z_{\text{reject}}^{1,h_4}, \dots, z_{\text{reject}}^{n_{h_4},h_4} \right\} \\ \vdots \\ \left\{ m_T^{h_N}, z_{\text{accept}}^{h_N}, z_{\text{reject}}^{1,h_N}, \dots, z_{\text{reject}}^{n_{h_N},h_N} \right\} \end{pmatrix}$$

$$w_n = \prod_{i=1}^{\tilde{N}_{h,n}} \left(\frac{f(z_{\text{accept}}^{h_i}; \{a, b\}_P)}{f(z_{\text{accept}}^{h_i}; \{a, b\}_B)} \right) \times \prod_{j=1}^{n_{h_i}} \left(\frac{\hat{f} - f(z_{\text{reject}}^{j,h_i}; \{a, b\}_P)}{\hat{f} - f(z_{\text{reject}}^{j,h_i}; \{a, b\}_B)} \right)$$

θ

accept

$a = 0.5, m_{\perp} = 1 \text{ GeV}$

$1 - P(z, \theta)$

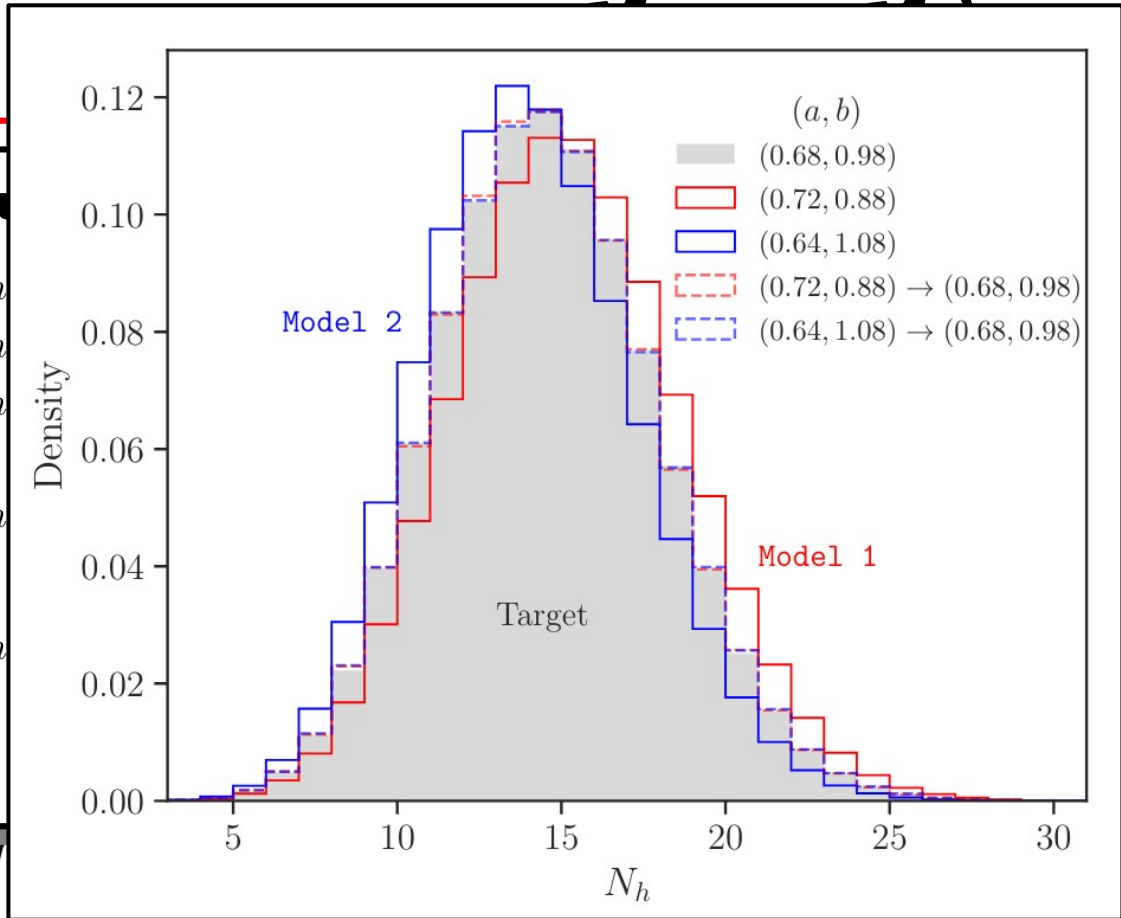
Kinematic reweighting (2308.13459)

\hat{P}

Data-structure

$$z = \begin{pmatrix} z_1 \\ z_2 \end{pmatrix} = \begin{pmatrix} \{m_1\} \\ \{m_2\} \\ \{m_3\} \\ \{m_4\} \\ \{m_5\} \\ \{m_6\} \\ \{m_7\} \\ \{m_8\} \\ \{m_9\} \\ \{m_{10}\} \\ \{m_{11}\} \\ \{m_{12}\} \\ \{m_{13}\} \\ \{m_{14}\} \\ \{m_{15}\} \\ \{m_{16}\} \\ \{m_{17}\} \\ \{m_{18}\} \\ \{m_{19}\} \\ \{m_{20}\} \\ \{m_{21}\} \\ \{m_{22}\} \\ \{m_{23}\} \\ \{m_{24}\} \\ \{m_{25}\} \\ \{m_{26}\} \\ \{m_{27}\} \\ \{m_{28}\} \\ \{m_{29}\} \\ \{m_{30}\} \\ \{m_{31}\} \\ \{m_{32}\} \\ \{m_{33}\} \\ \{m_{34}\} \\ \{m_{35}\} \\ \{m_{36}\} \\ \{m_{37}\} \\ \{m_{38}\} \\ \{m_{39}\} \\ \{m_{40}\} \\ \{m_{41}\} \\ \{m_{42}\} \\ \{m_{43}\} \\ \{m_{44}\} \\ \{m_{45}\} \\ \{m_{46}\} \\ \{m_{47}\} \\ \{m_{48}\} \\ \{m_{49}\} \\ \{m_{50}\} \\ \{m_{51}\} \\ \{m_{52}\} \\ \{m_{53}\} \\ \{m_{54}\} \\ \{m_{55}\} \\ \{m_{56}\} \\ \{m_{57}\} \\ \{m_{58}\} \\ \{m_{59}\} \\ \{m_{60}\} \\ \{m_{61}\} \\ \{m_{62}\} \\ \{m_{63}\} \\ \{m_{64}\} \\ \{m_{65}\} \\ \{m_{66}\} \\ \{m_{67}\} \\ \{m_{68}\} \\ \{m_{69}\} \\ \{m_{70}\} \\ \{m_{71}\} \\ \{m_{72}\} \\ \{m_{73}\} \\ \{m_{74}\} \\ \{m_{75}\} \\ \{m_{76}\} \\ \{m_{77}\} \\ \{m_{78}\} \\ \{m_{79}\} \\ \{m_{80}\} \\ \{m_{81}\} \\ \{m_{82}\} \\ \{m_{83}\} \\ \{m_{84}\} \\ \{m_{85}\} \\ \{m_{86}\} \\ \{m_{87}\} \\ \{m_{88}\} \\ \{m_{89}\} \\ \{m_{90}\} \\ \{m_{91}\} \\ \{m_{92}\} \\ \{m_{93}\} \\ \{m_{94}\} \\ \{m_{95}\} \\ \{m_{96}\} \\ \{m_{97}\} \\ \{m_{98}\} \\ \{m_{99}\} \\ \{m_{100}\} \end{pmatrix}$$

$a = 0.5, \tau$



$$\left(\frac{\text{pt}; \{a, b\}_P}{\text{pt}; \{a, b\}_B} \right)$$

(z, σ)

cept

Kinematic reweighting (2308.13459)

\hat{P}

Rejection sampling:

Data-structure:

$$z = \begin{pmatrix} z_1 \\ z_2 \\ \vdots \\ z_N = \dots \end{pmatrix} = \begin{pmatrix} \left\{ m_T^{h_1}, z_{\text{accept}}^{h_1}, z_{\text{reject}}^{1,h_1}, \dots, z_{\text{reject}}^{n_{h_1},h_1} \right\} \\ \left\{ m_T^{h_2}, z_{\text{accept}}^{h_2}, z_{\text{reject}}^{1,h_2}, \dots, z_{\text{reject}}^{n_{h_2},h_2} \right\} \\ \left\{ m_T^{h_3}, z_{\text{accept}}^{h_3}, z_{\text{reject}}^{1,h_3}, \dots, z_{\text{reject}}^{n_{h_3},h_3} \right\} \\ \left\{ m_T^{h_4}, z_{\text{accept}}^{h_4}, z_{\text{reject}}^{1,h_4}, \dots, z_{\text{reject}}^{n_{h_4},h_4} \right\} \\ \vdots \\ \left\{ m_T^{h_N}, z_{\text{accept}}^{h_N}, z_{\text{reject}}^{1,h_N}, \dots, z_{\text{reject}}^{n_{h_N},h_N} \right\} \end{pmatrix}$$

Differentiable! → RSA

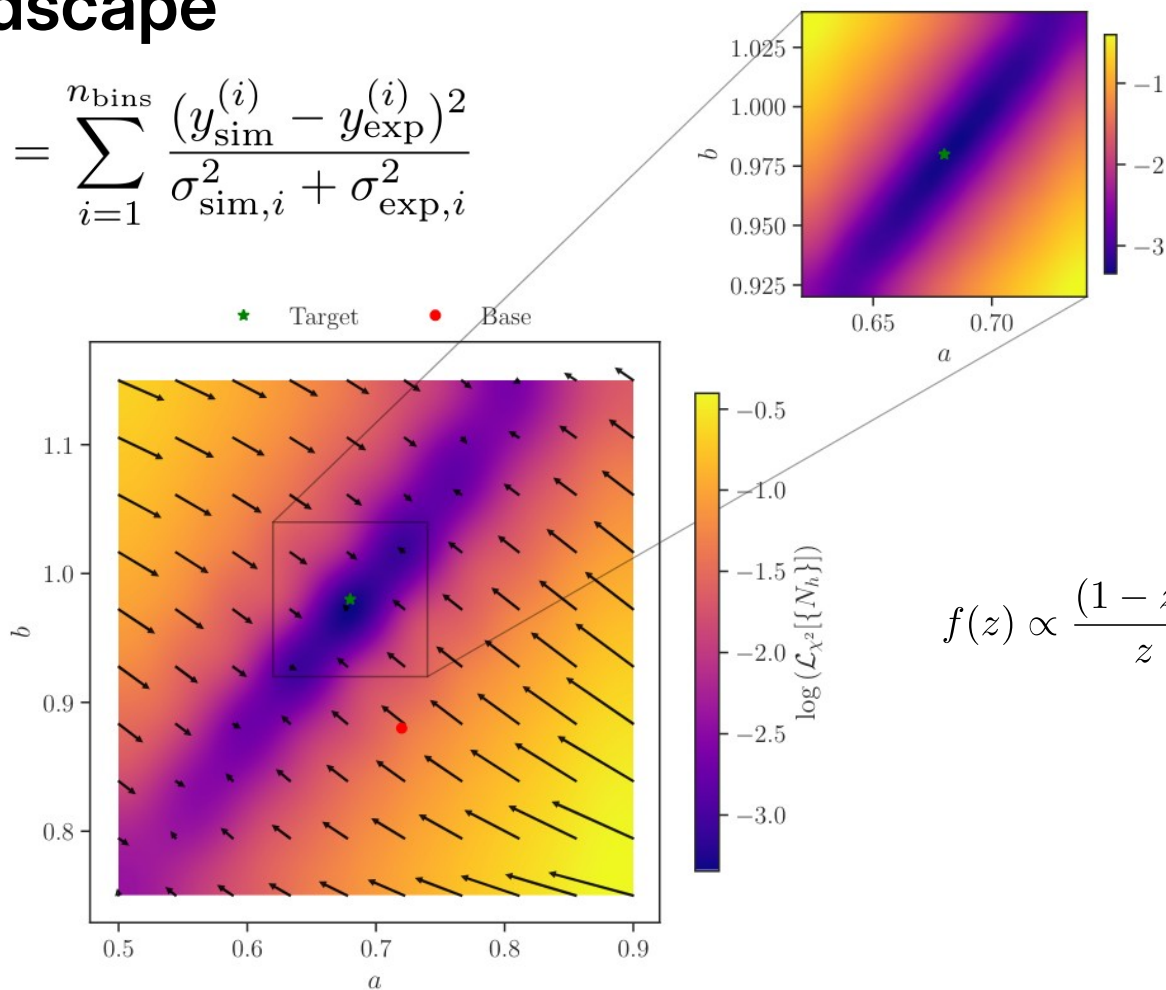
$$w_n = \prod_{i=1}^{\tilde{N}_{h,n}} \left(\frac{f(z_{\text{accept}}^{h_i}; \{a, b\}_P)}{f(z_{\text{accept}}^{h_i}; \{a, b\}_B)} \right) \times \prod_{j=1}^{n_{h_i}} \left(\frac{\hat{f} - f(z_{\text{reject}}^{j,h_i}; \{a, b\}_P)}{\hat{f} - f(z_{\text{reject}}^{j,h_i}; \{a, b\}_B)} \right)$$

$a = 0.5, m_{\perp} = 1 \text{ GeV}$

$1 - P(z, \theta)$

χ^2 -loss landscape

$$\mathcal{L}_{\chi^2}(\mathbf{y}_{\text{sim}}, \mathbf{y}_{\text{exp}}; \mathbf{w}) = \sum_{i=1}^{n_{\text{bins}}} \frac{(y_{\text{sim}}^{(i)} - y_{\text{exp}}^{(i)})^2}{\sigma_{\text{sim},i}^2 + \sigma_{\text{exp},i}^2}$$

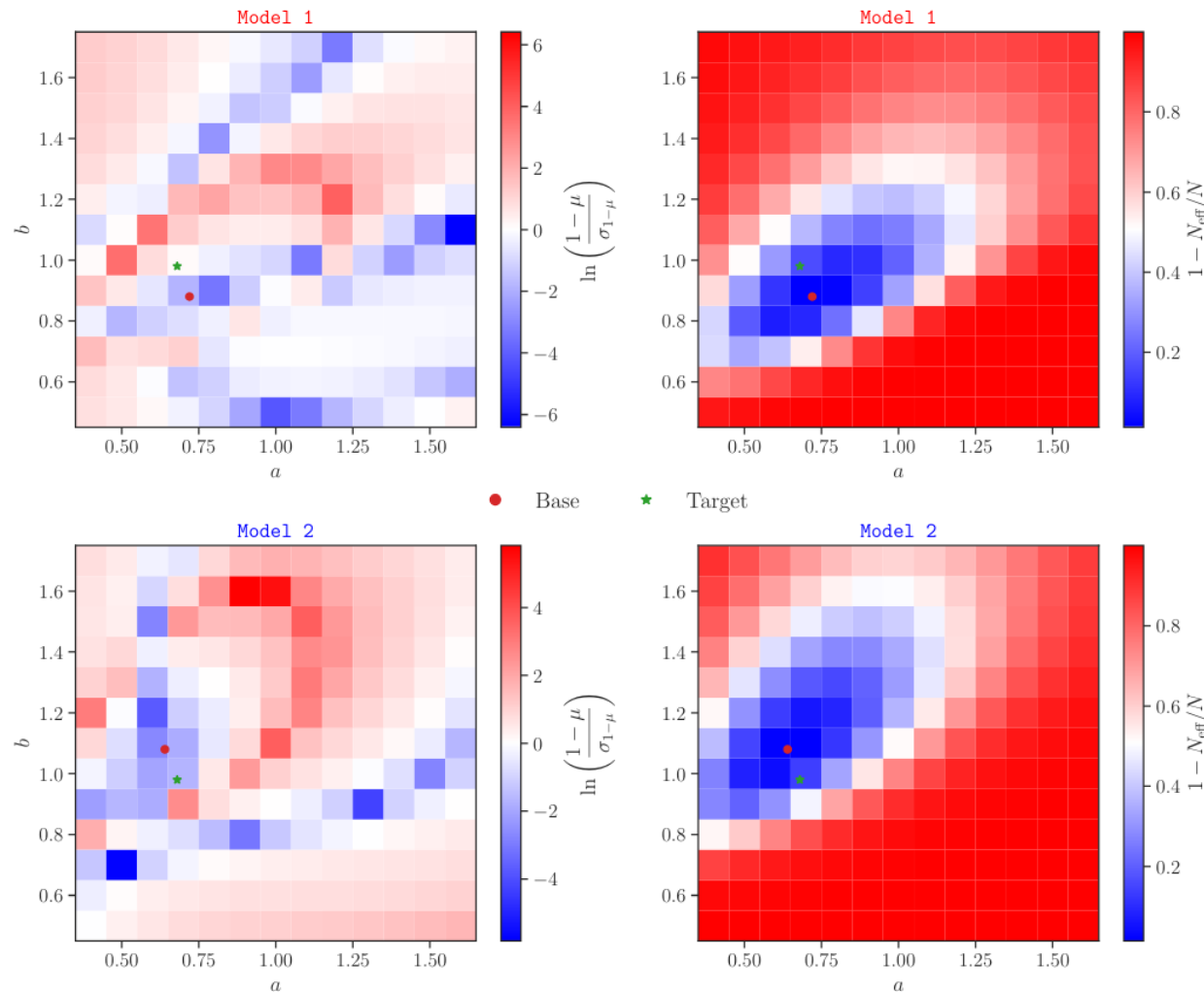


$$f(z) \propto \frac{(1-z)^a}{z} \exp\left(\frac{-bm_T^2}{z}\right)$$

No free lunch

Statistical power drops off quickly as you move away from the base parameterization

$$\mu \equiv \sum_{i=1}^N \frac{w_i}{N}, \quad N_{\text{eff}} = \frac{\left(\sum_{i=1}^N w_i\right)^2}{\sum_{i=1}^N w_i^2}$$



ML-based observables

- Train a deepsets classifier to distinguish simulation from experiment

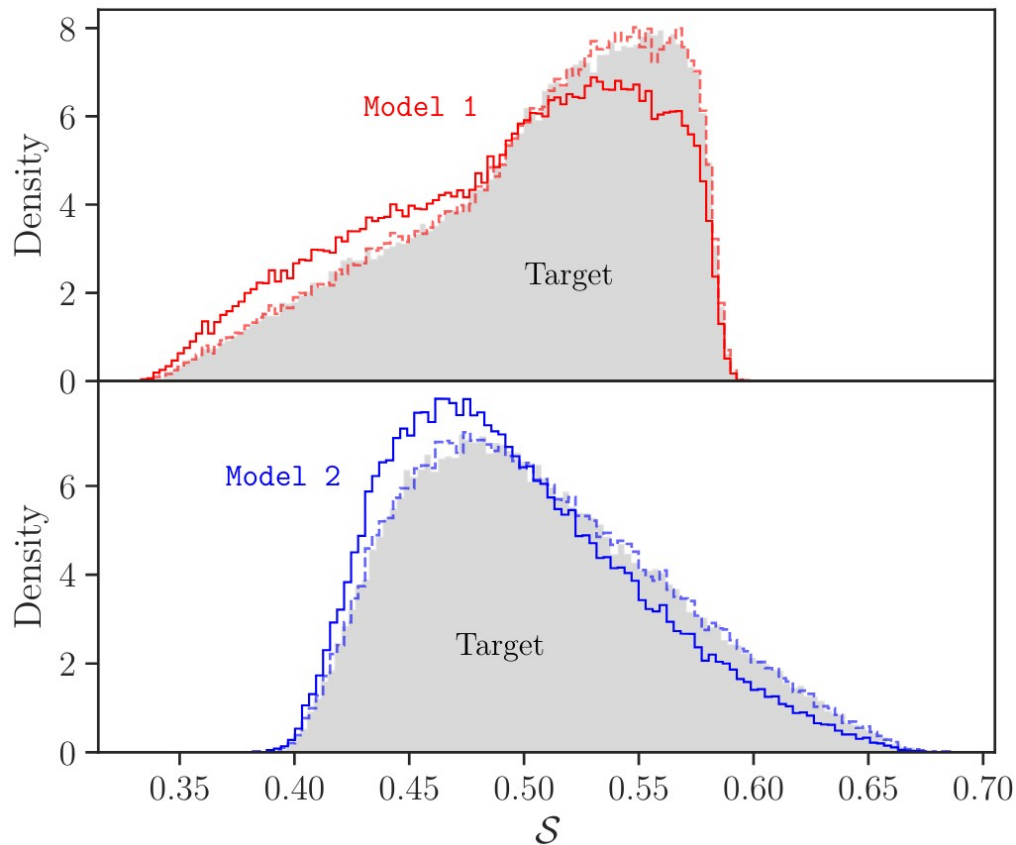
- Takes full event information as input

$(E, p_x, p_y, p_z)_1$

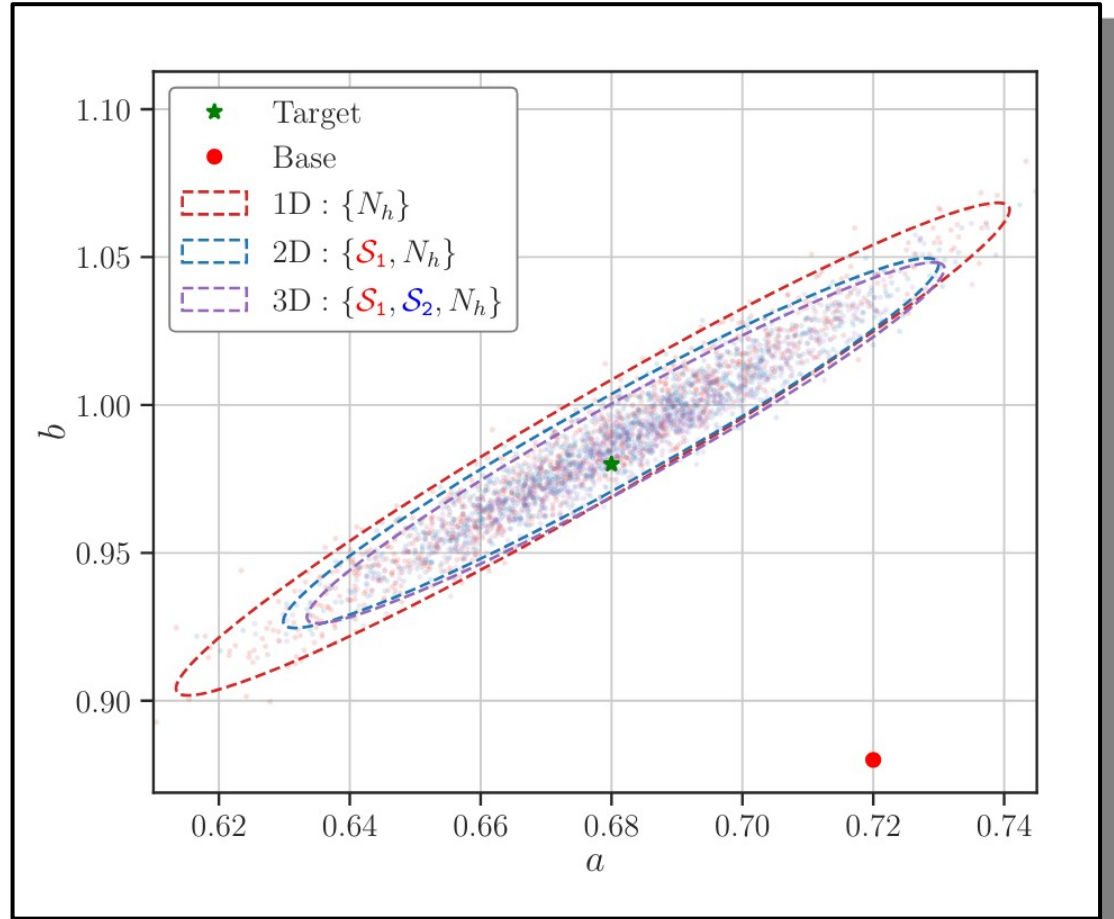
$(E, p_x, p_y, p_z)_2$

...

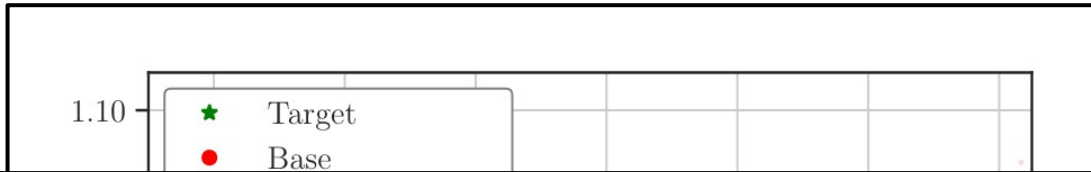
Use trained classifier output
(score) as an observable



Classifier score with full event info improves tuning convergence

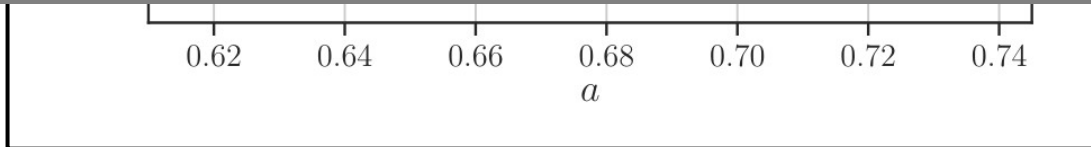


Clas
info
con



Analytic form of $f(z)$ is useful but potentially limiting when trying to describe data.

Can we extract the fragmentation function directly from data?



Microscopic Alterations Generated from IR Collections

+ (2311.09296, 25xx.xxxx)

Histories and Observables for Monte Carlo Event Reweighting

(2410.06342, 2503.05667, 2509.03592)

MLHAD 

MAGIC

+ (2311.09296, 25xx.xxxx)

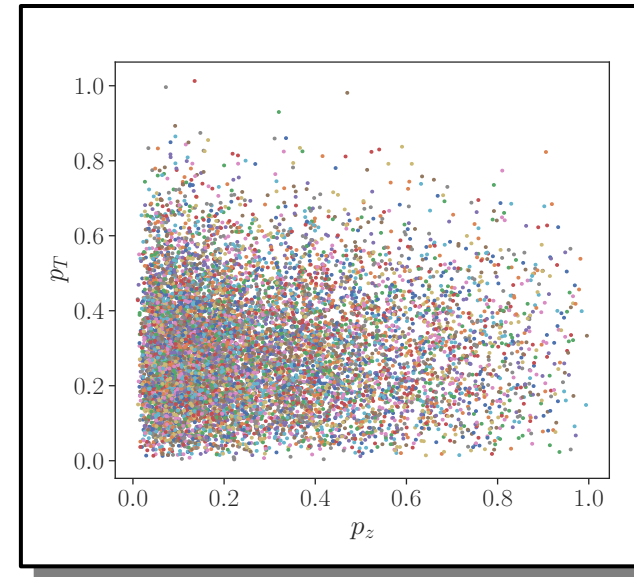
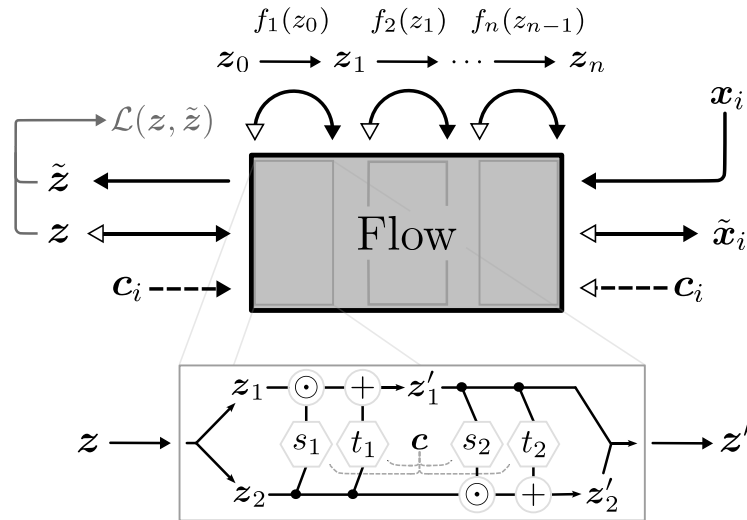
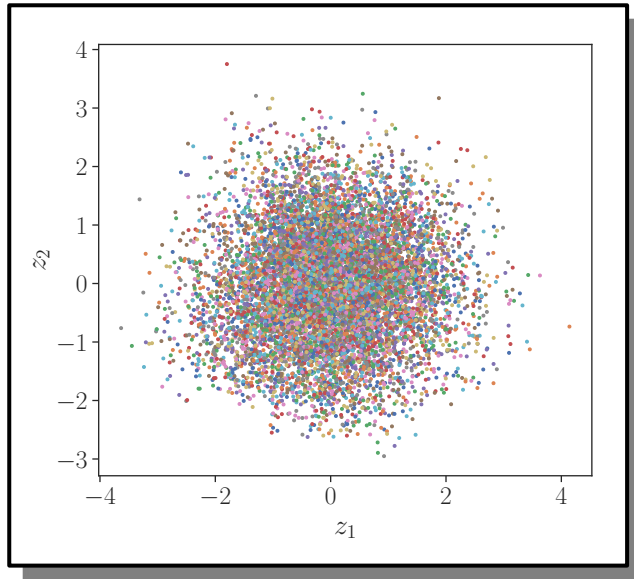
HOMER

(2410.06342, 2503.05667, 2509.03592)

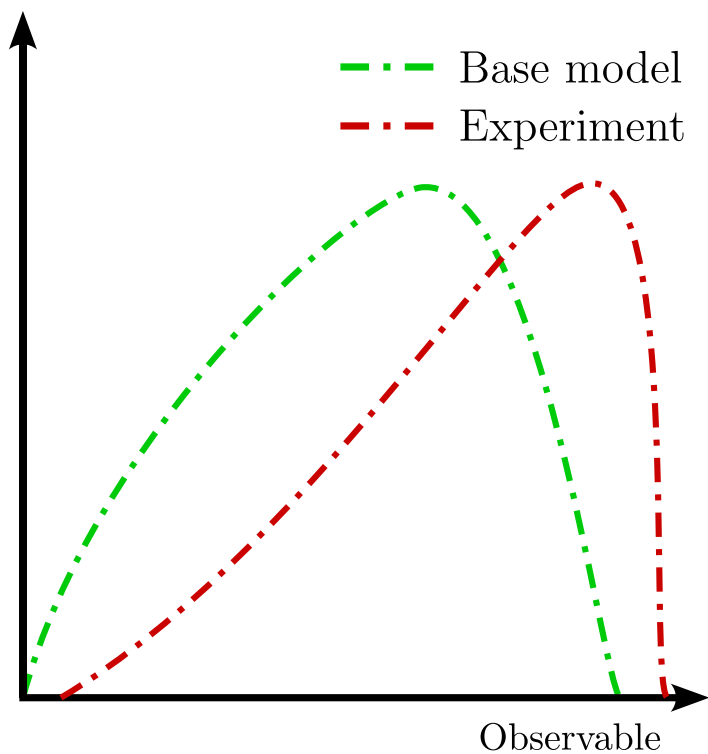
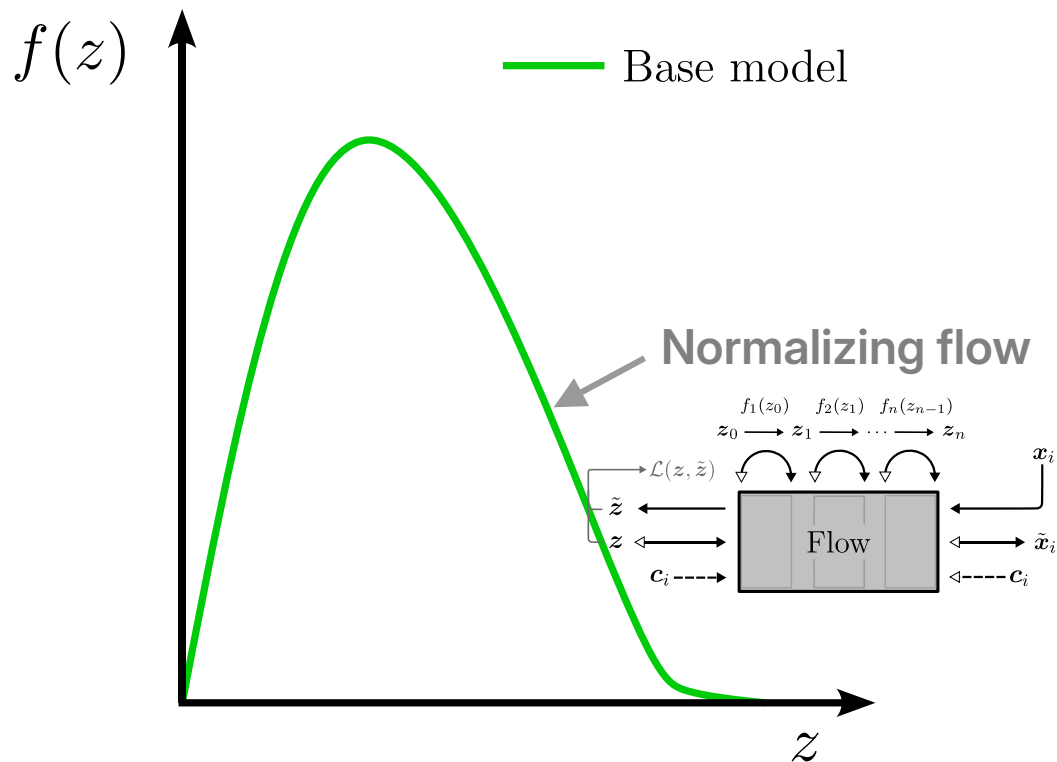
MLHAD 

MAGIC: Invertible neural networks (INN)

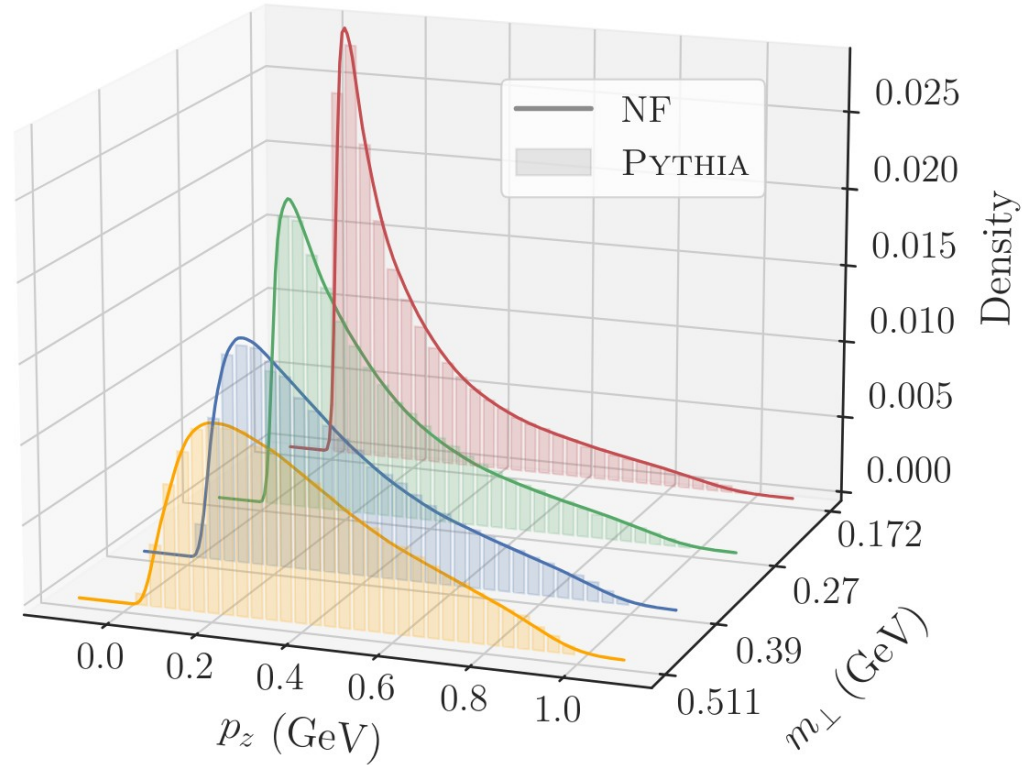
a.k.a normalizing flow



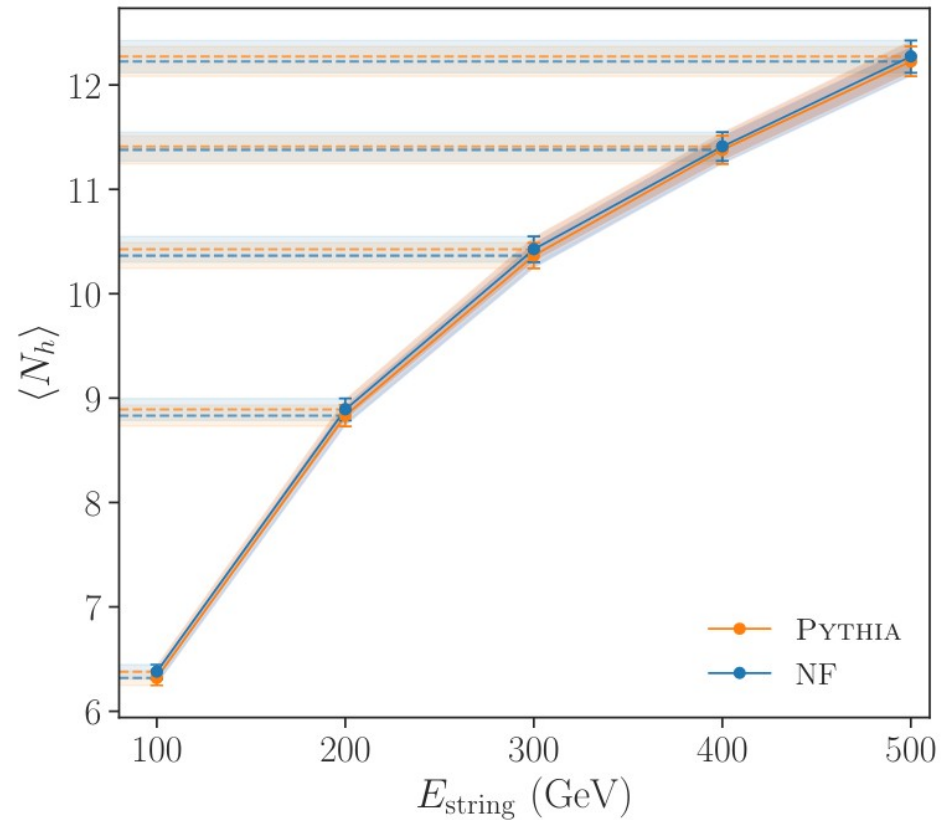
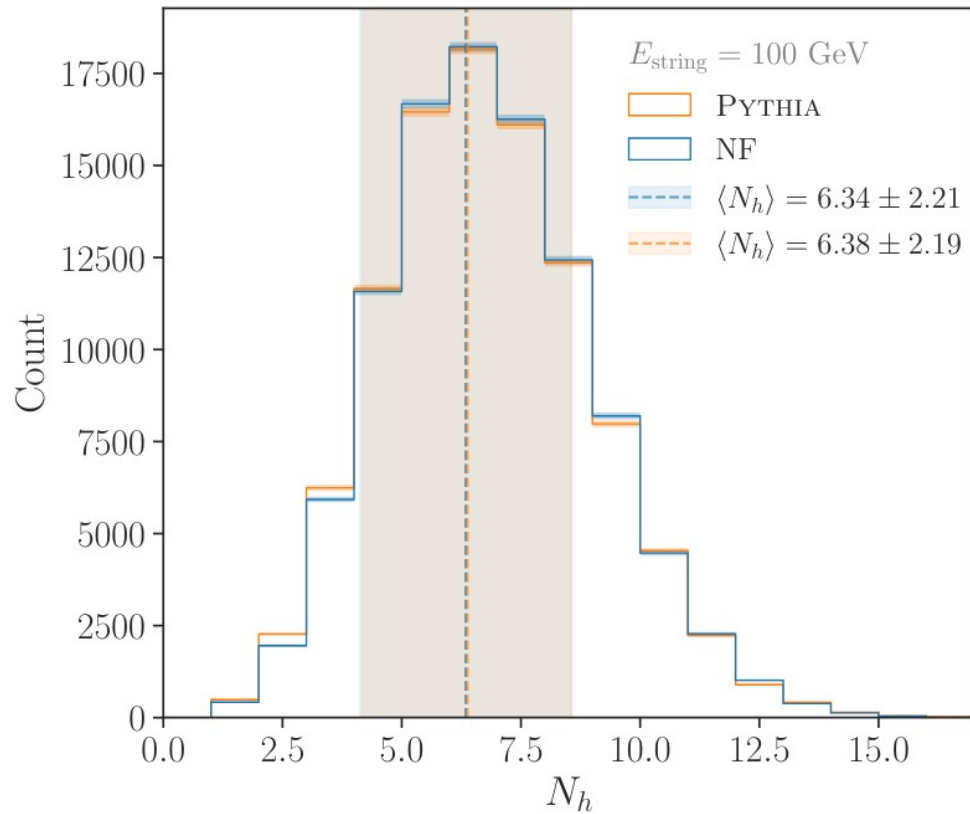
MLHAD efforts: MAGIC



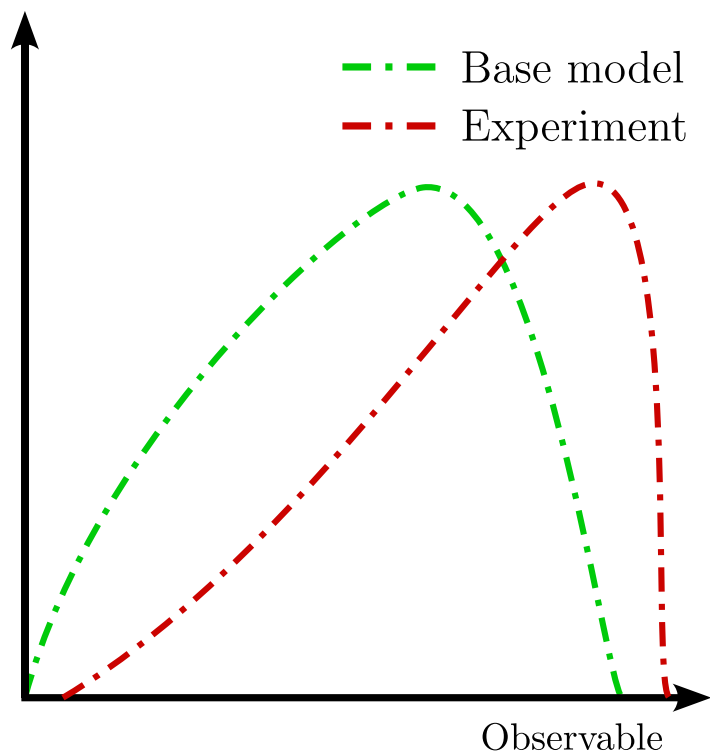
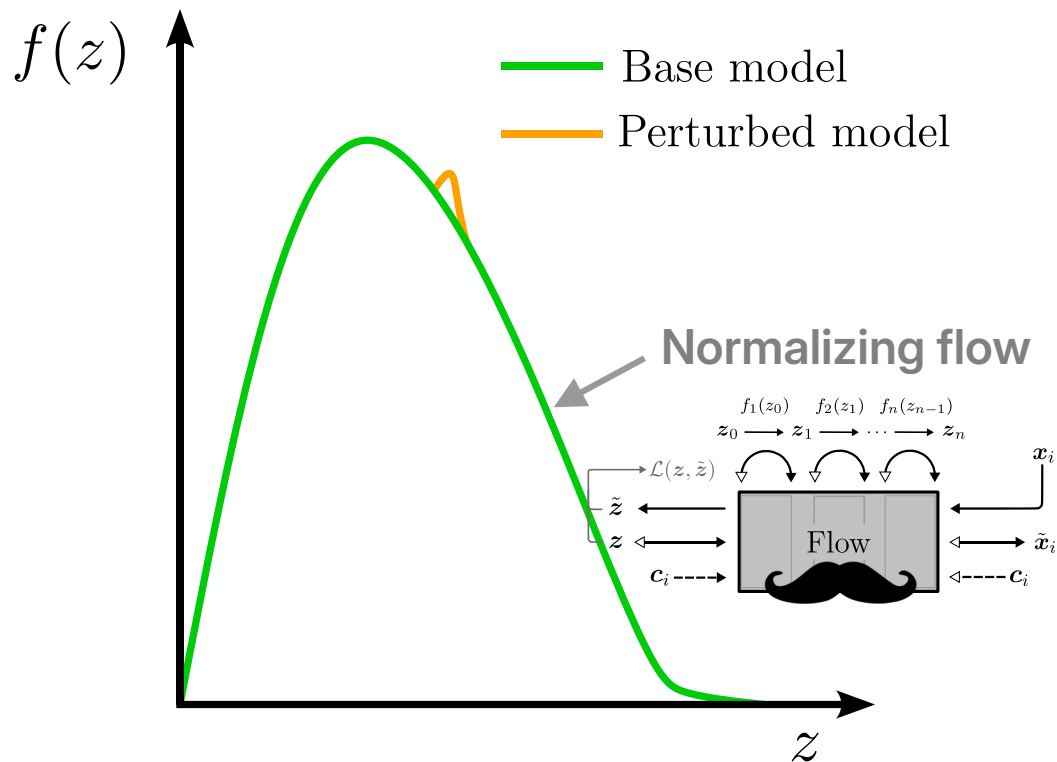
MAGIC: Base model



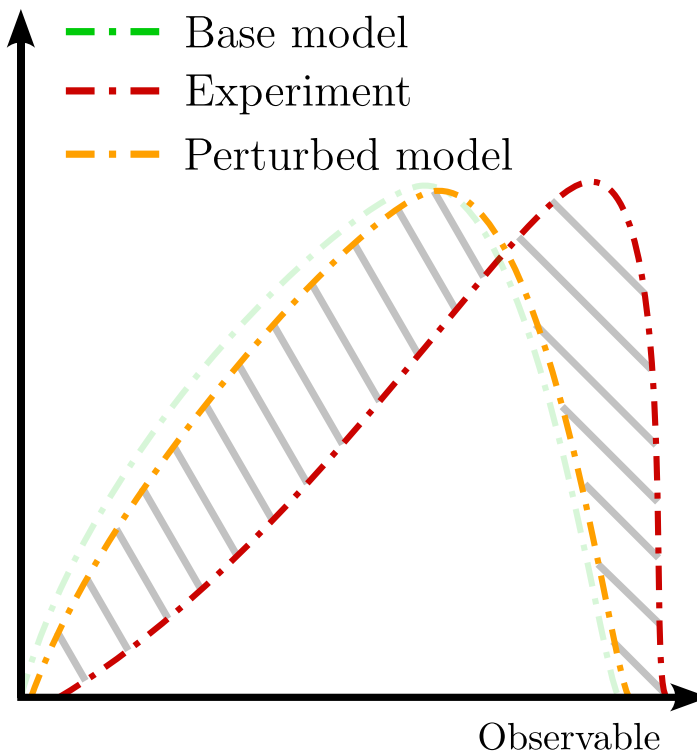
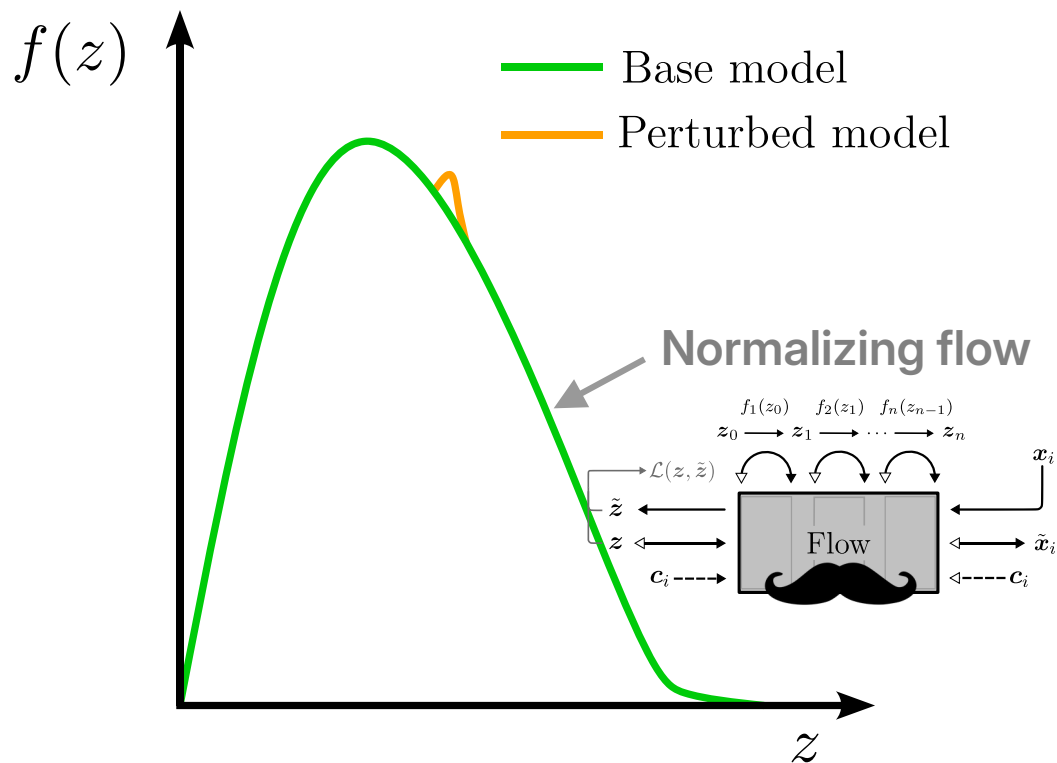
MAGIC: Base model



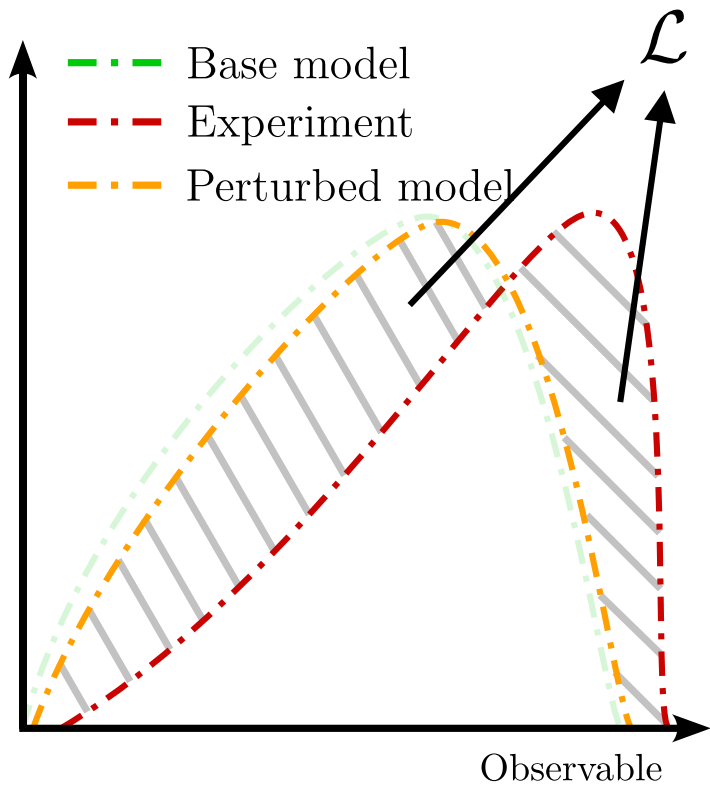
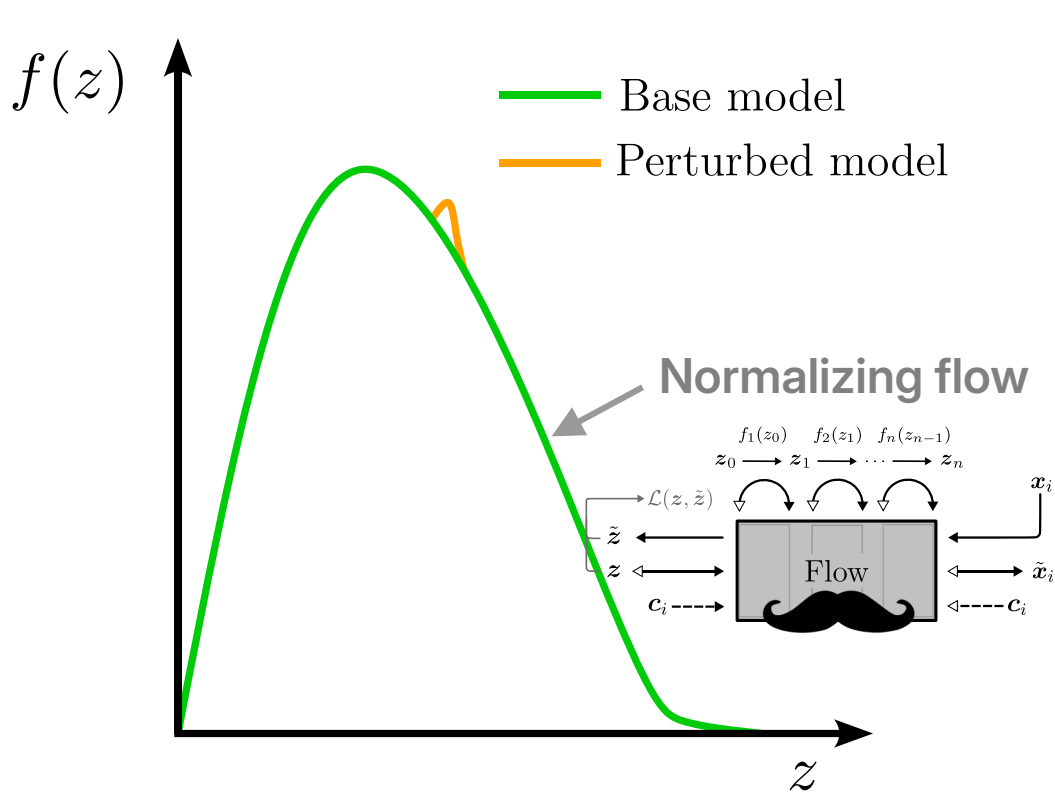
MLHAD efforts: **MAGIC**



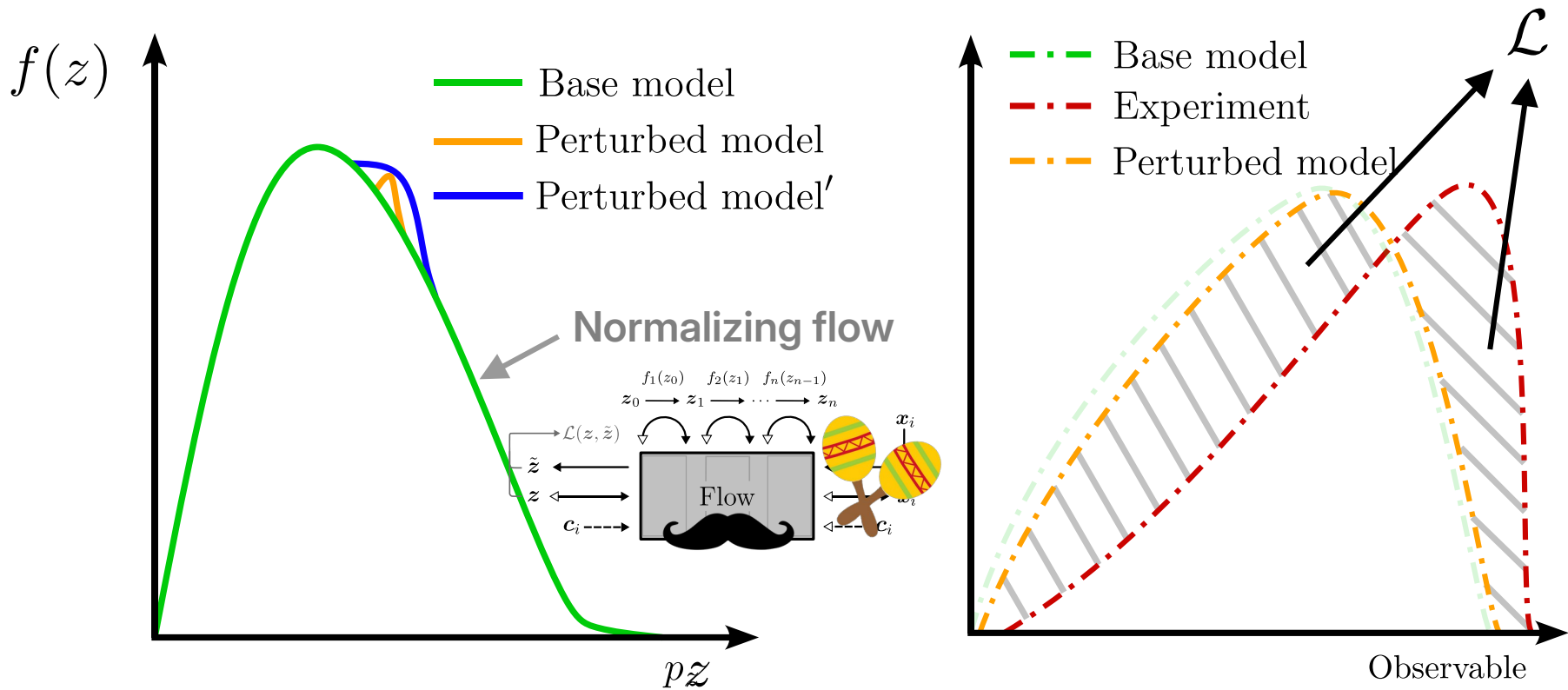
MLHAD efforts: MAGIC



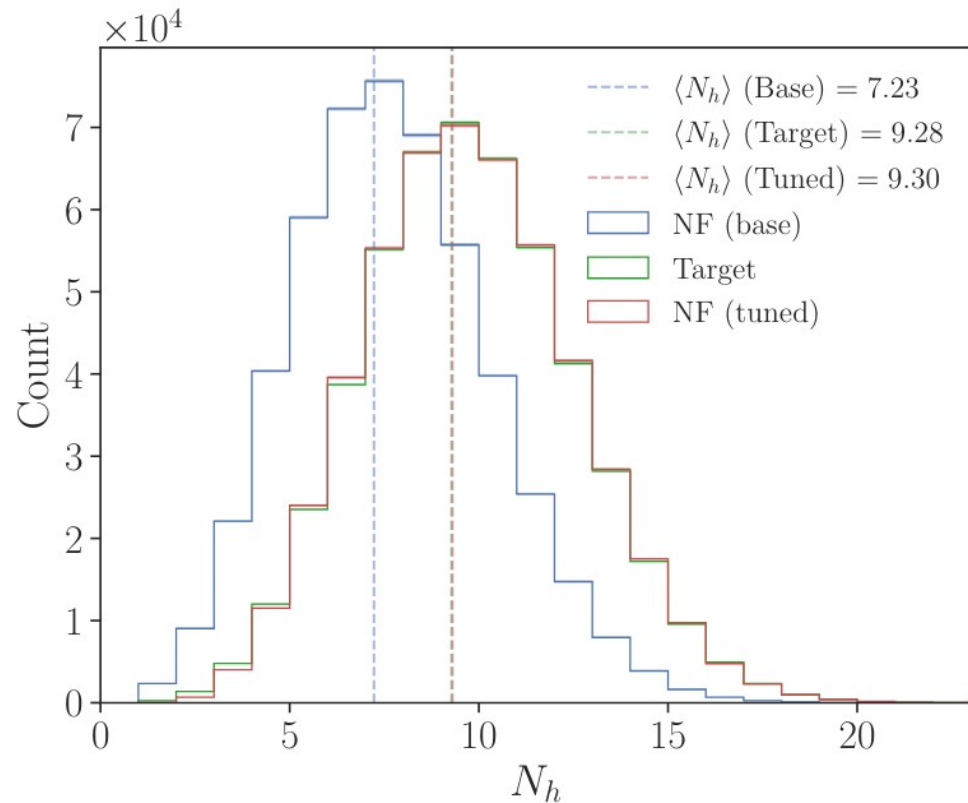
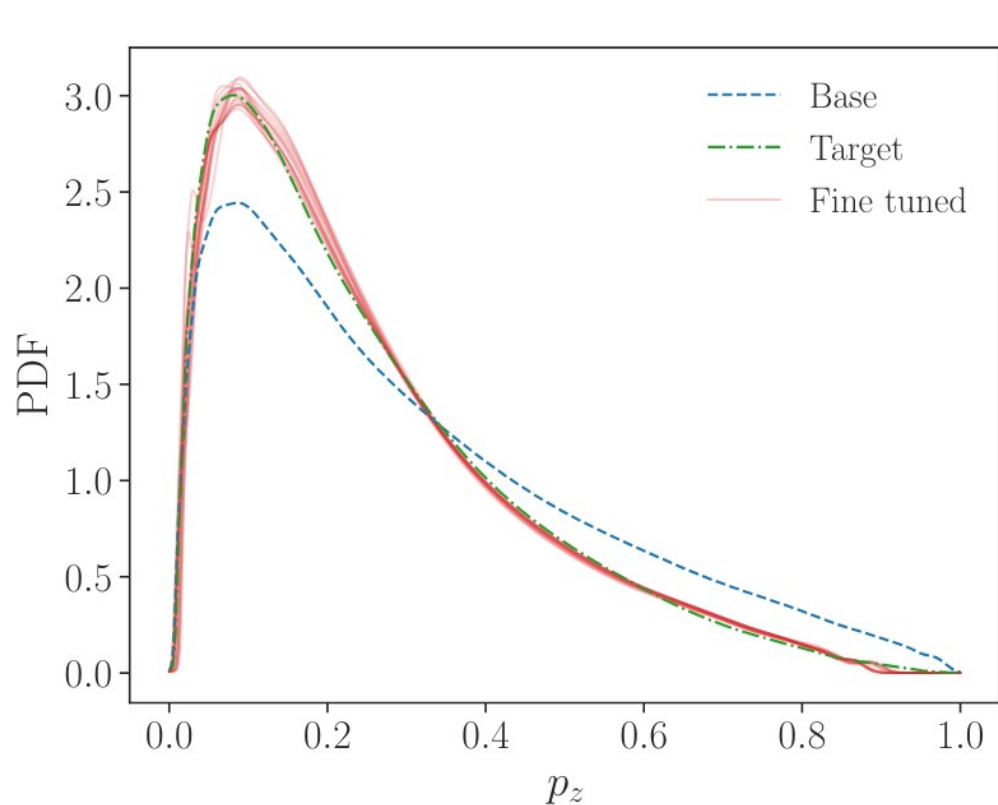
MLHAD efforts: MAGIC



MLHAD efforts: MAGIC



MAGIC: $q\bar{q}$

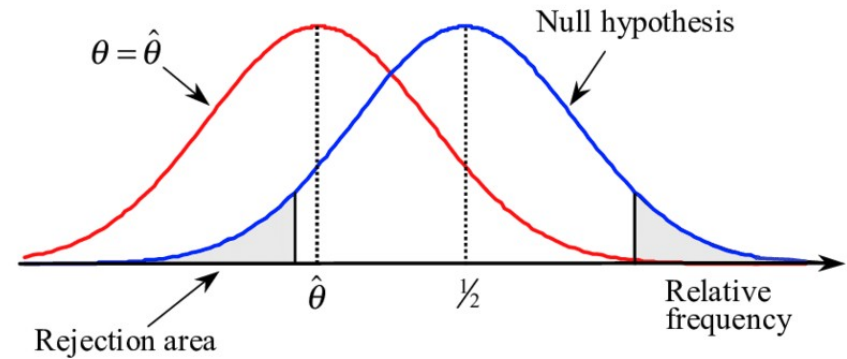


HOMER

$$\operatorname{argmin}_f L[f] = \frac{p(x | \theta_0)}{p(x | \theta_1)} = \mathcal{L}(x)$$

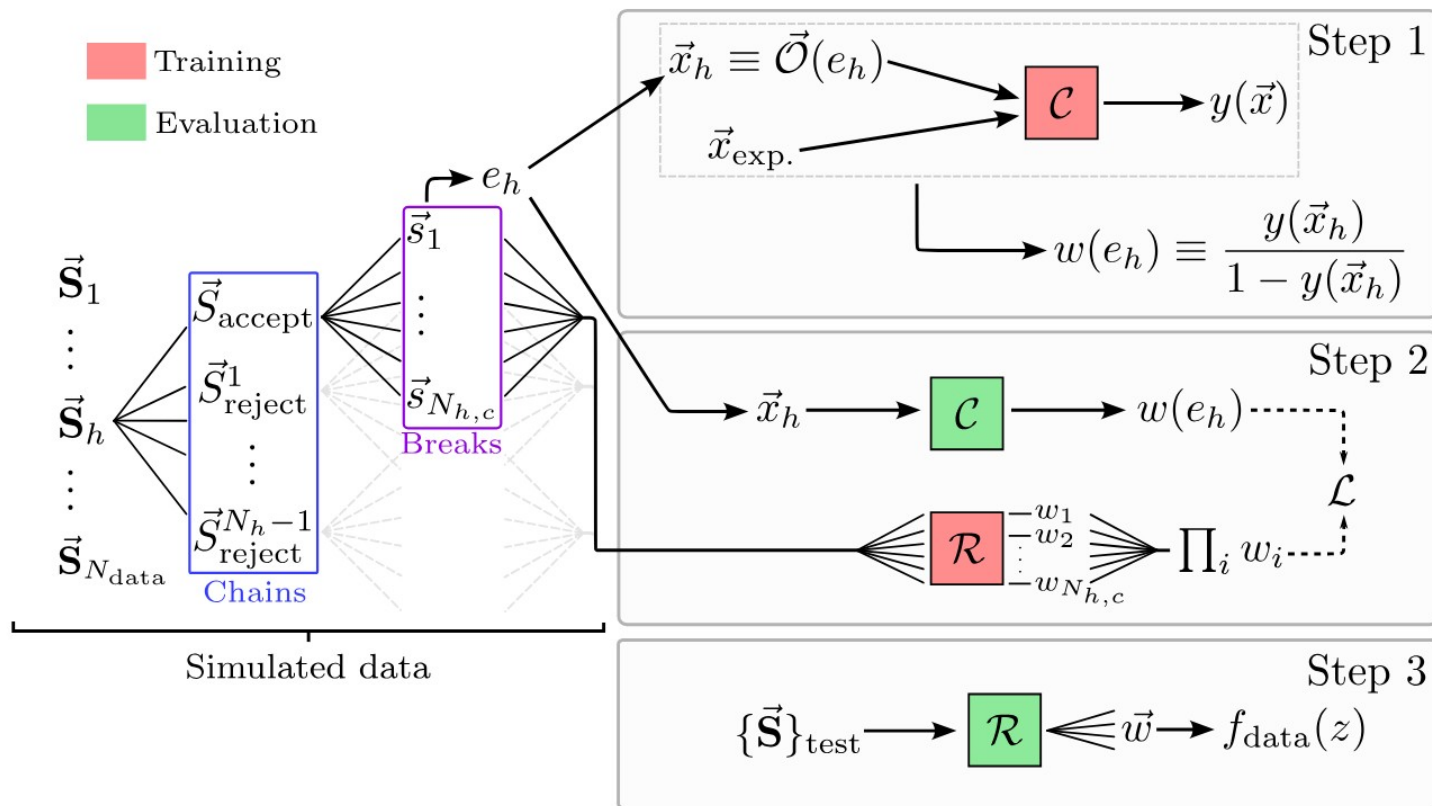
Minimize binary cross-entropy

$$f(x) = \frac{p(x | \theta_0)}{p(x | \theta_0) + p(x | \theta_1)}$$

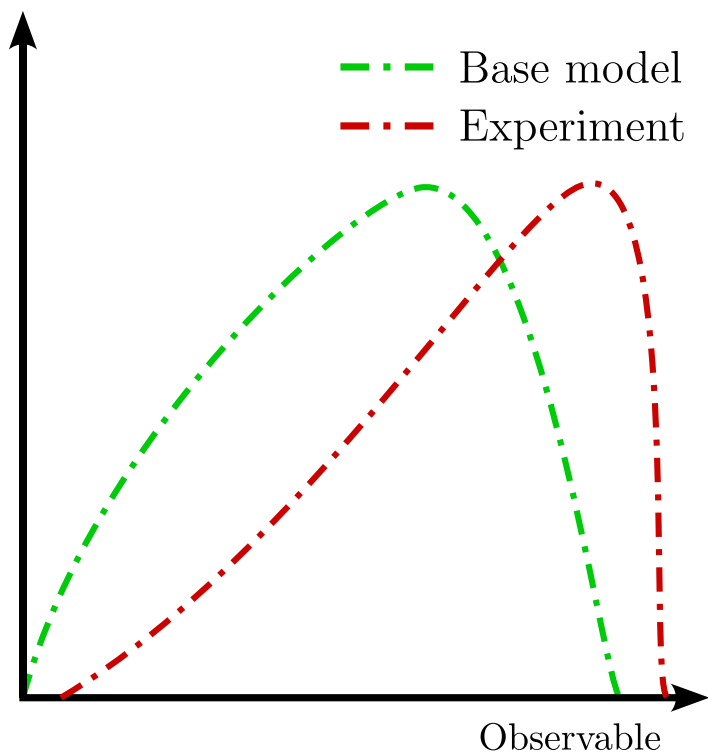
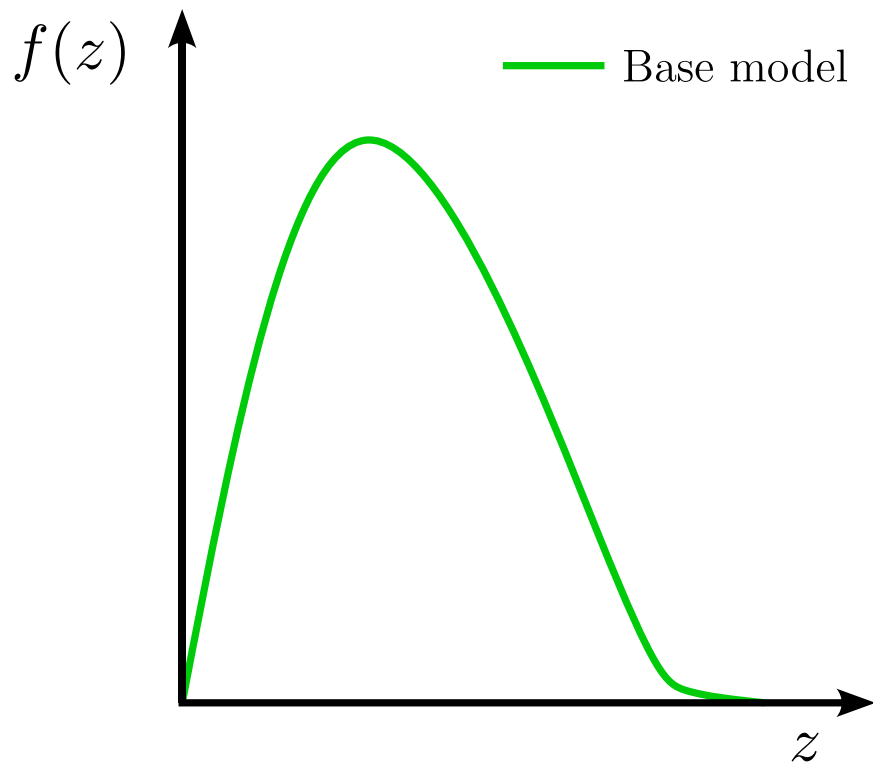


$$\frac{f(x)}{1 - f(x)} = \frac{\frac{p(x|\theta_0)}{p(x|\theta_0)+p(x|\theta_1)}}{1 - \frac{p(x|\theta_0)}{p(x|\theta_0)+p(x|\theta_1)}} = \frac{p(x | \theta_0)}{p(x | \theta_0) + p(x | \theta_1) - p(x | \theta_0)} = \frac{p(x | \theta_0)}{p(x | \theta_1)} = \mathcal{L}(x)$$

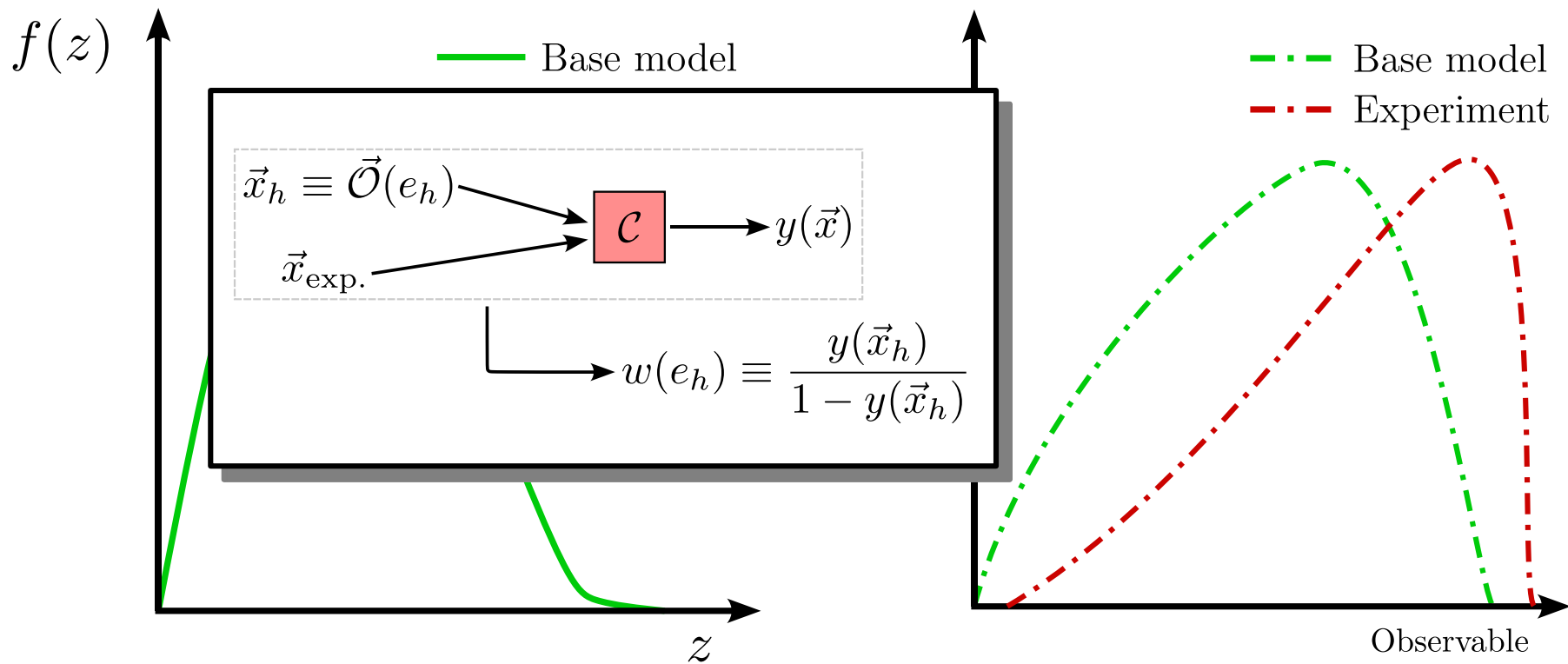
HOMER: $q\bar{q}$



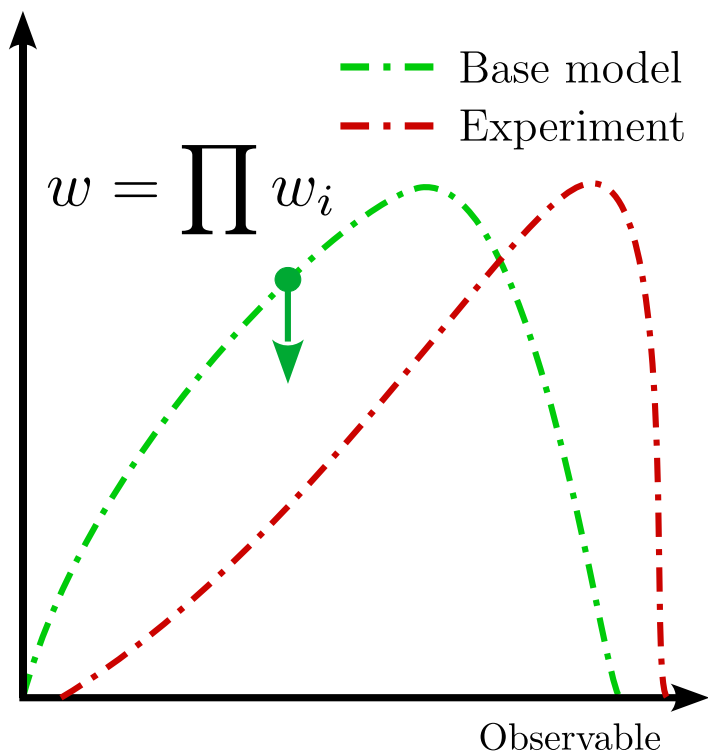
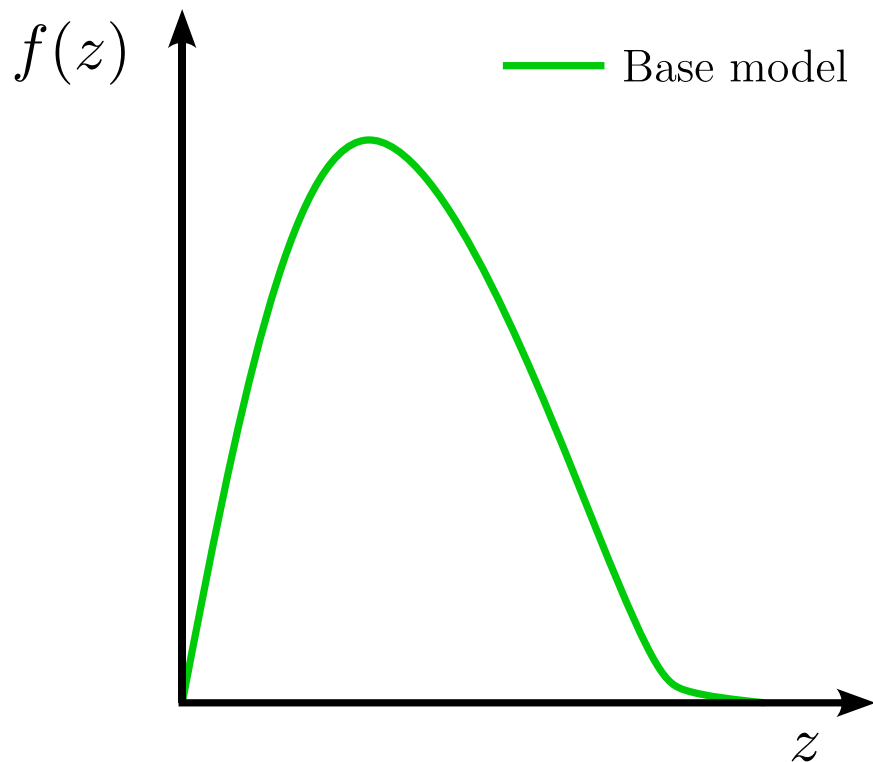
MLHAD efforts: HOMER



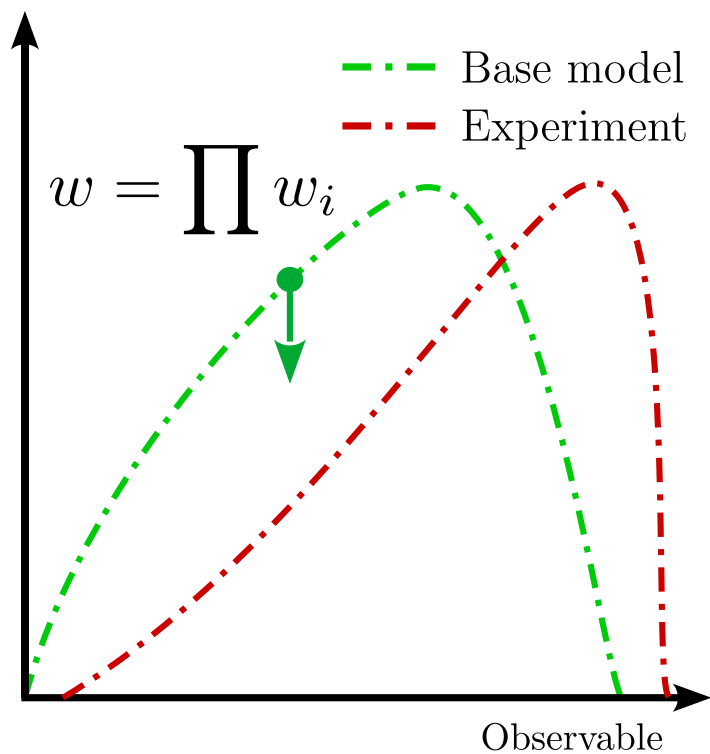
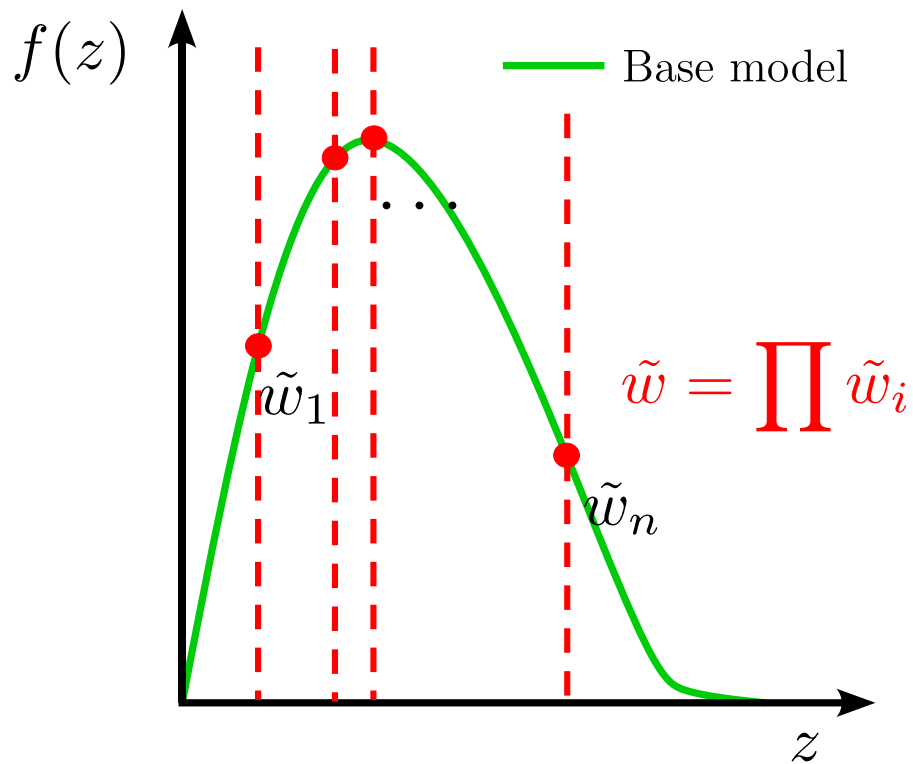
MLHAD efforts: HOMER



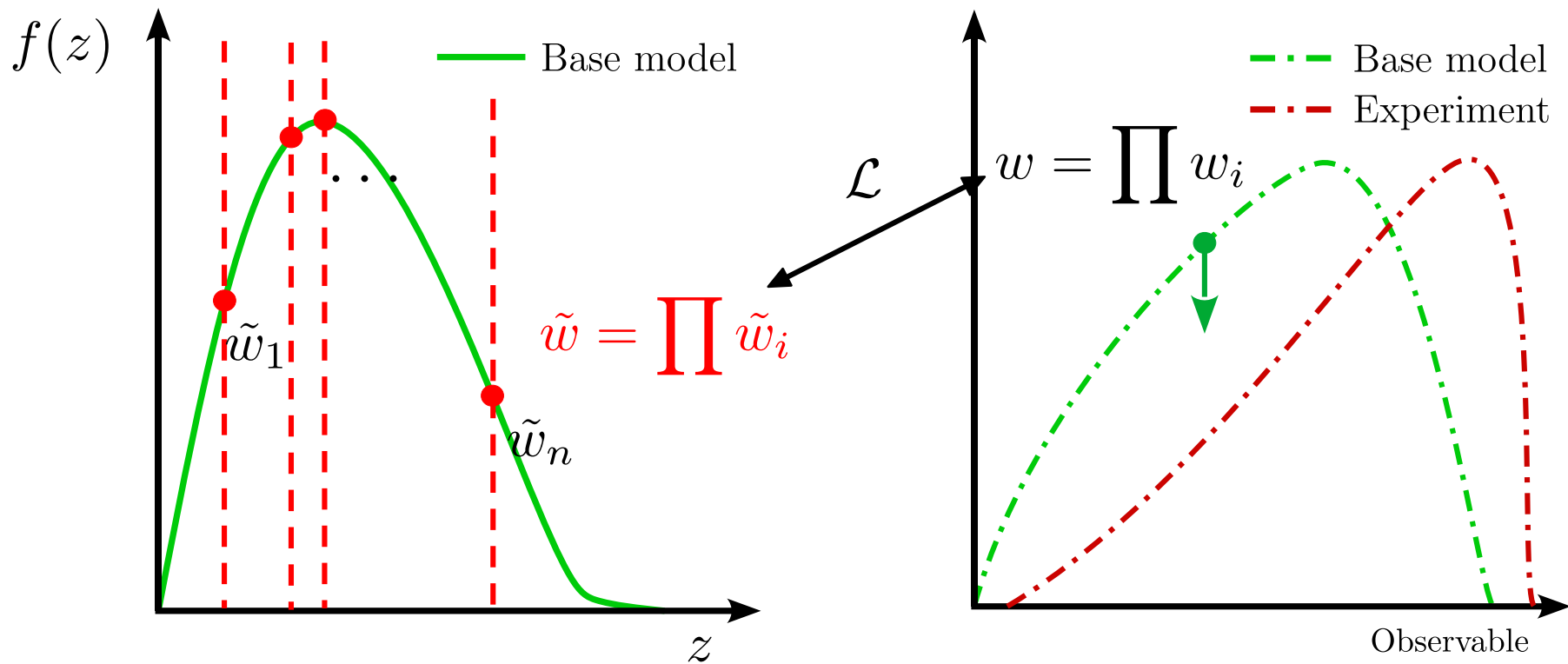
MLHAD efforts: HOMER



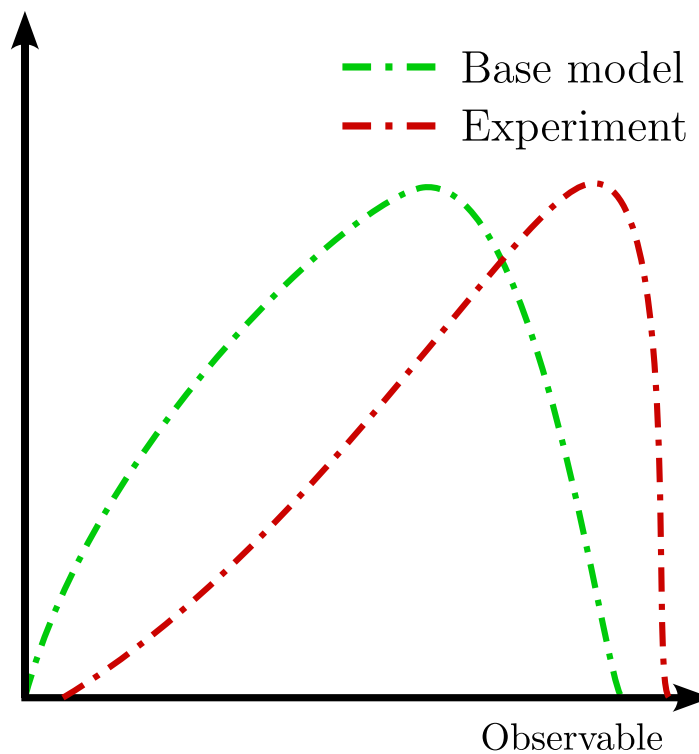
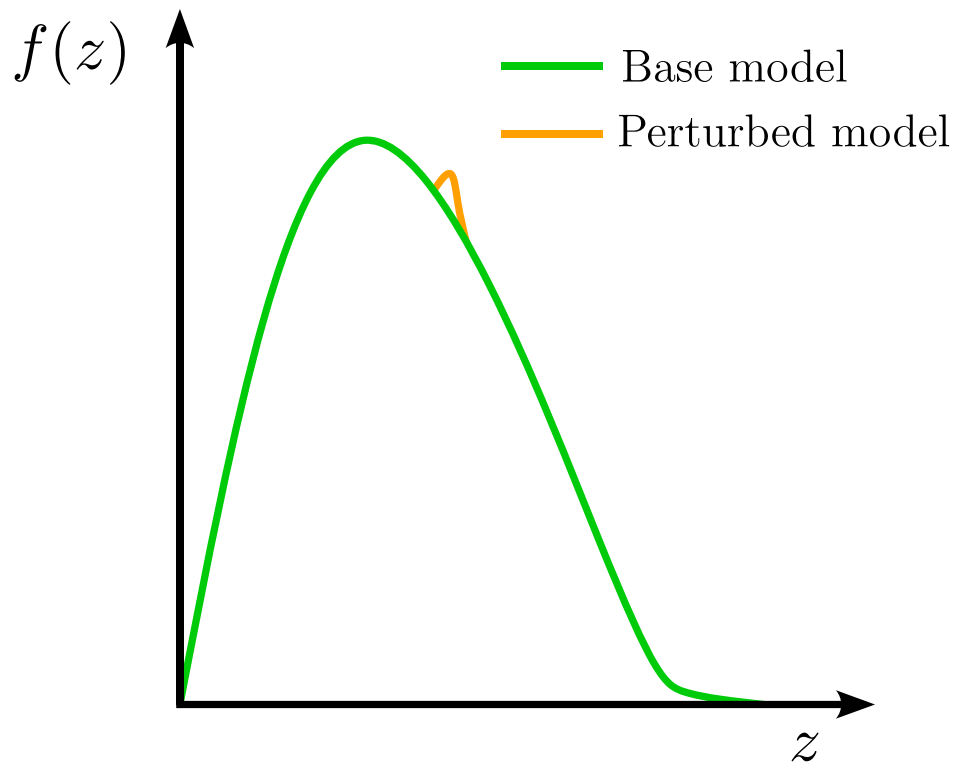
MLHAD efforts: HOMER



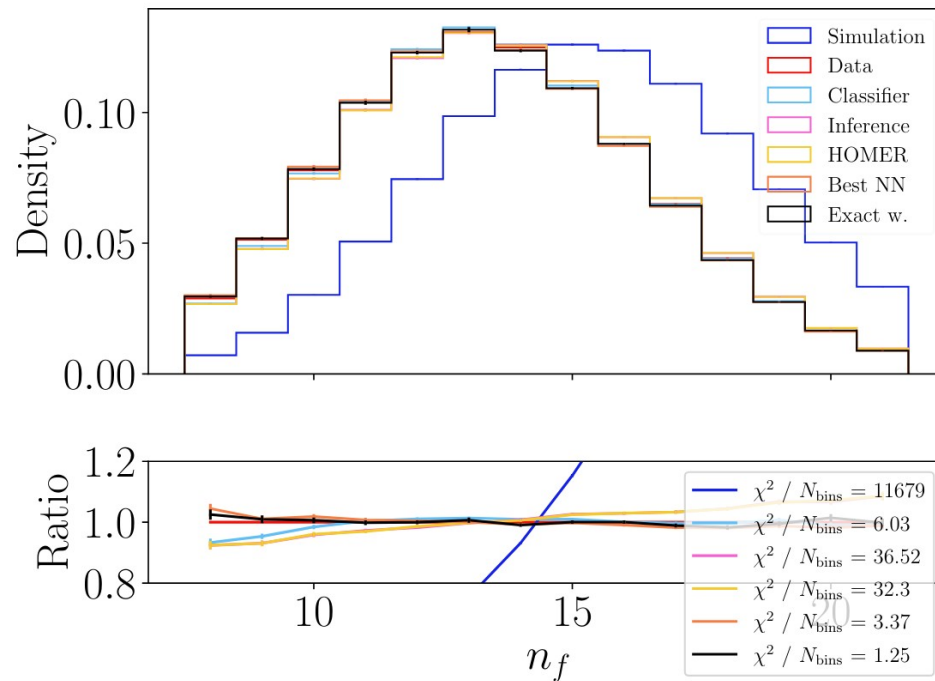
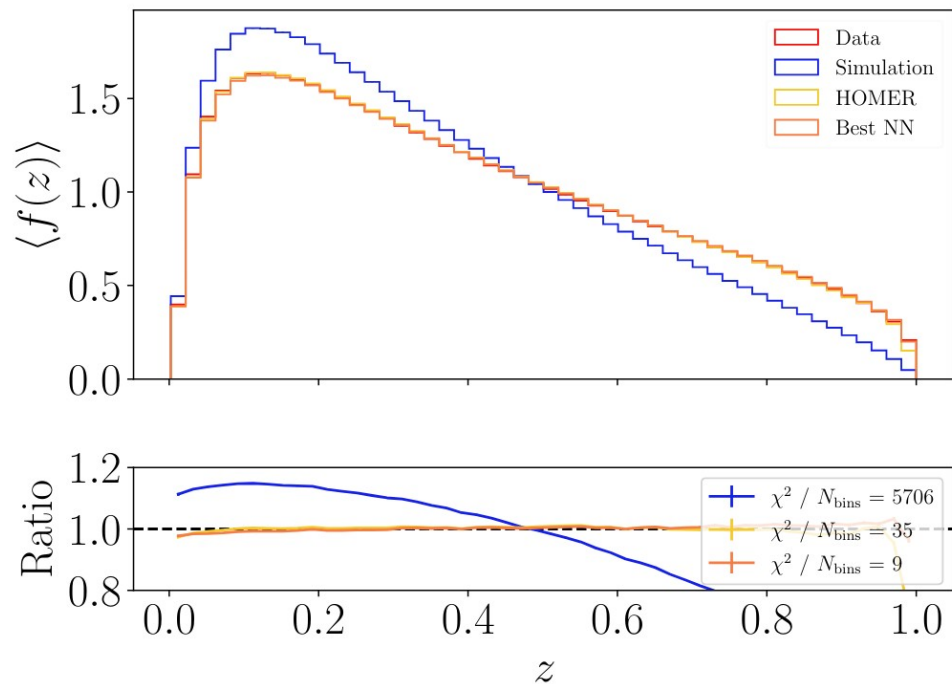
MLHAD efforts: HOMER



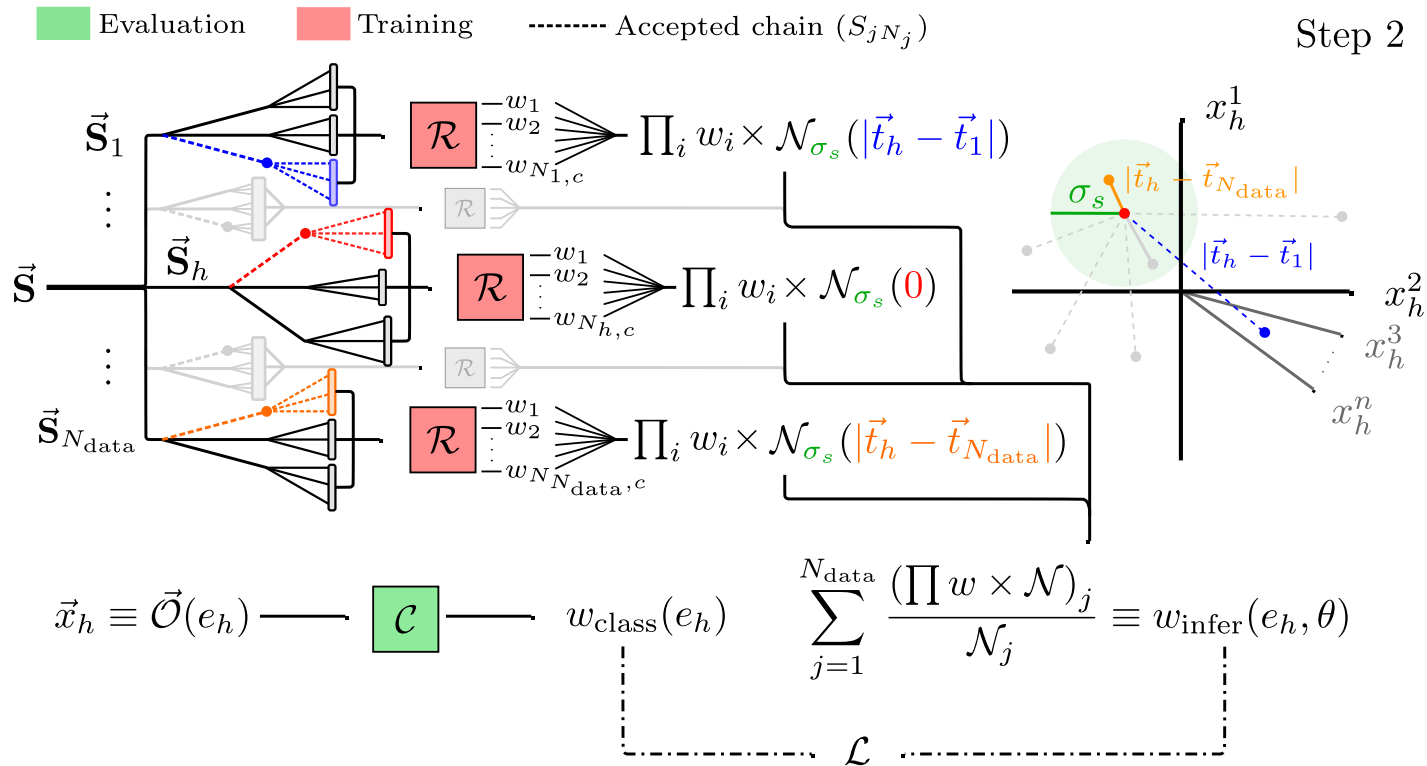
MLHAD efforts: HOMER



HOMER: $q\bar{q}$



HOMER: w/ gluons



Two “solutions” to the inverse problem of hadronization

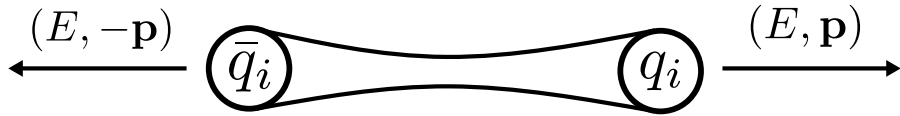
MAGIC

- “Top-down”
- Relies on the construction of an explicit likelihood (normalizing flow)
- Must be embedded into the simulation pipeline

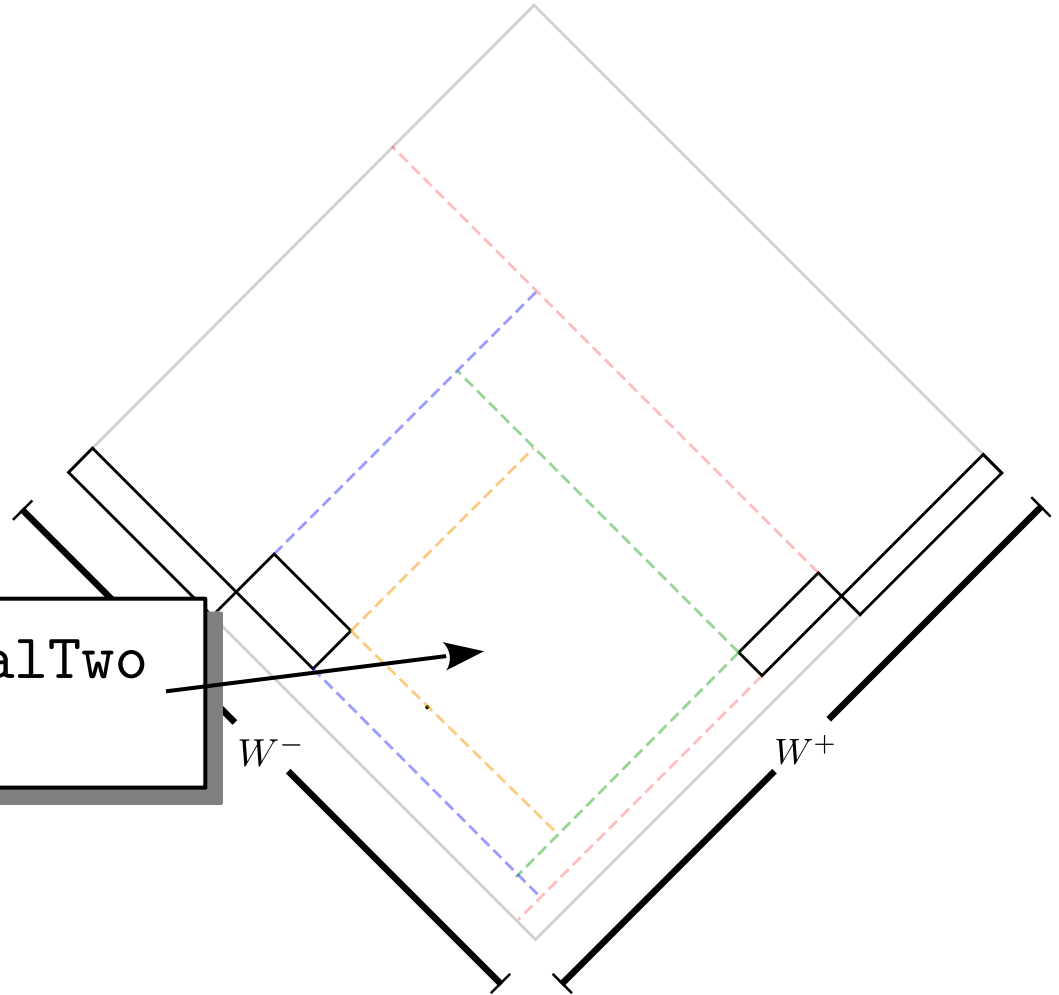
HOMER

- “Bottom-up”
- Relies on the reconstruction of a classifier weight (derived from the score) from a product of string-break weights
- No modifications to the simulation pipeline required

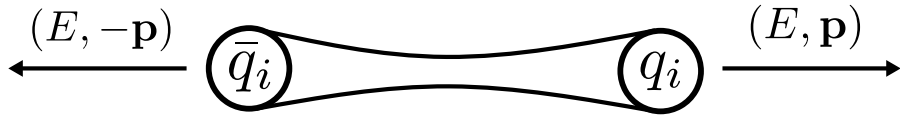
The road ahead



When E_{CM} goes below E_{cut} , finalTwo is called.



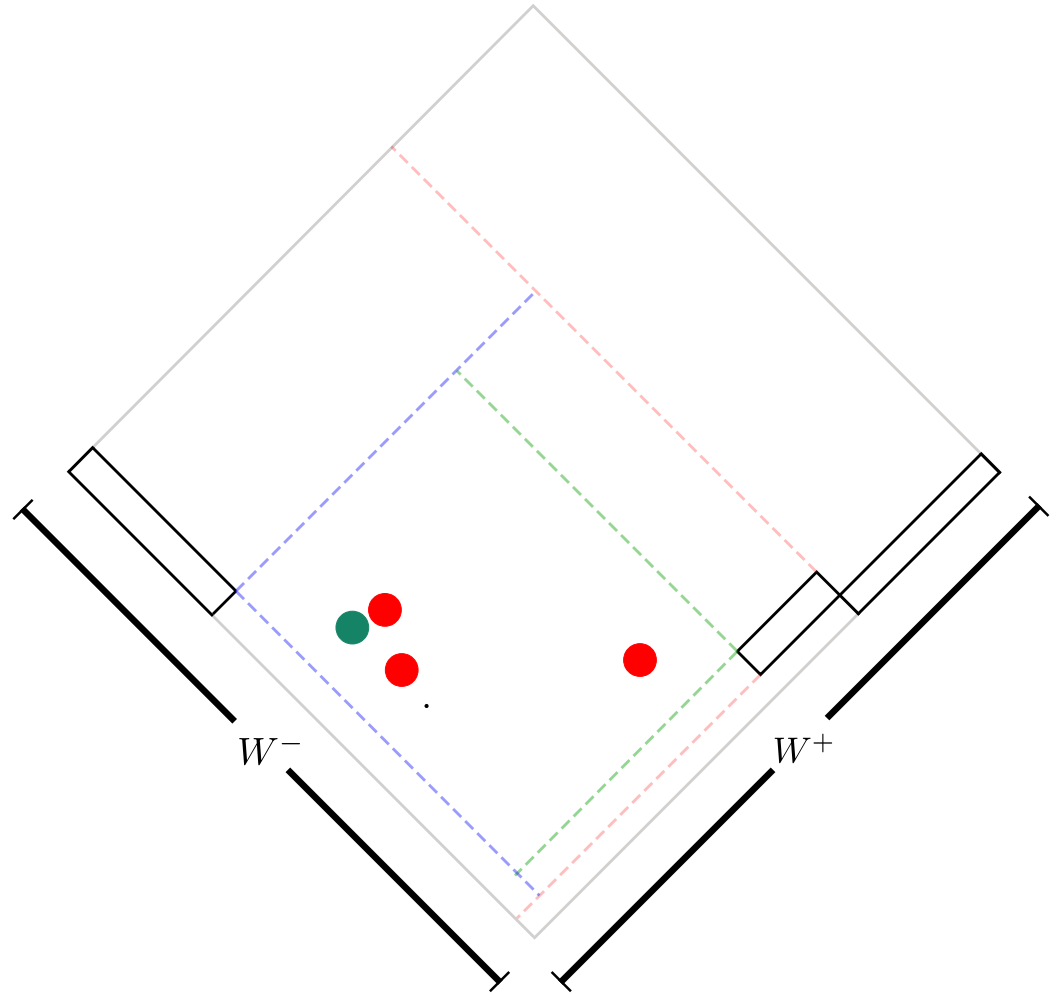
The road ahead



This system has two limits:

1. $E_{\text{CM}} \gg m_h$
- Random walk in $f(z)$

2. $E_{\text{CM}} \sim m_h$



The road ahead



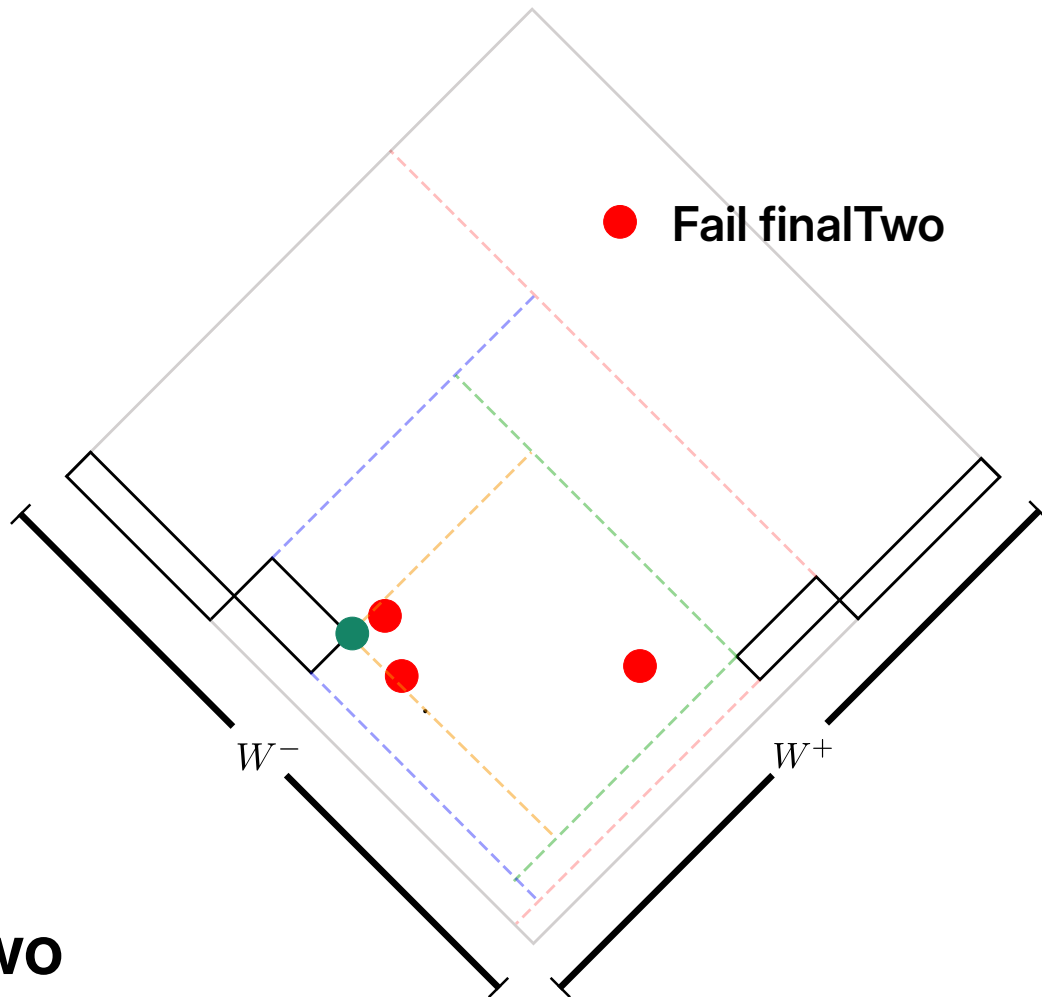
This system has two limits:

1. $E_{\text{CM}} \gg m_h$

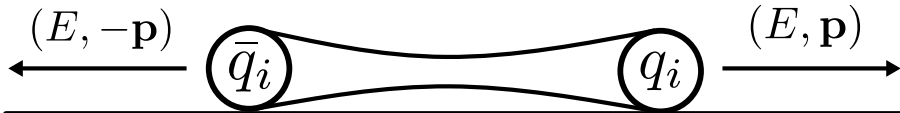
- Random walk in $f(z)$

2. $E_{\text{CM}} \sim m_h$

- $f'(z)$ influenced by finalTwo



The road ahead



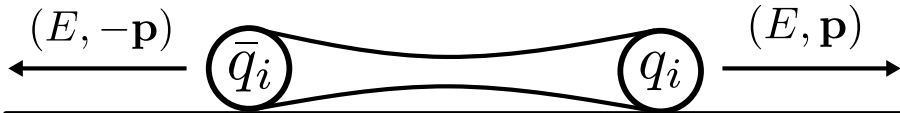
● Fail finalTwo

We want to extract $f(\mathbf{z})$, not $f'(\mathbf{z})$ – cleaner extraction would factorize these two pieces.

How?

- $f'(\mathbf{z})$ influenced by finalTwo

The road ahead



● Fail finalTwo

We want to extract $f(z)$, not $f'(z)$ – cleaner extraction would factorize these two pieces.

How? Maybe another time... :)

- $f'(z)$ influenced by finalTwo

Conclusions

- **RSA: flexible, scalable, replicable** approach to tuning
 - **ML-based observables, optimizers, infrastructure**
 - **Uncertainty quantification, scaling with parameter dimension**
- **Data-driven fragmentation function:**
 - **HOMER and MAGIC offer as complementary methods**
 - **HOMER + flavor, tackle real data soon?**
 - **Better observables needed**
 - **HOMER x MAGIC**
 - **Better likelihood models? (MAGIC+)**

MLHAD 

<https://uchep.gitlab.io/mlhad-docs/>

Conclusions

- **RSA: flexible, scalable, replicable** approach to tuning
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MLHAD 

<https://uchep.gitlab.io/mlhad-docs/>

Thank you :)

Back-up

Stringy hadronization: overview

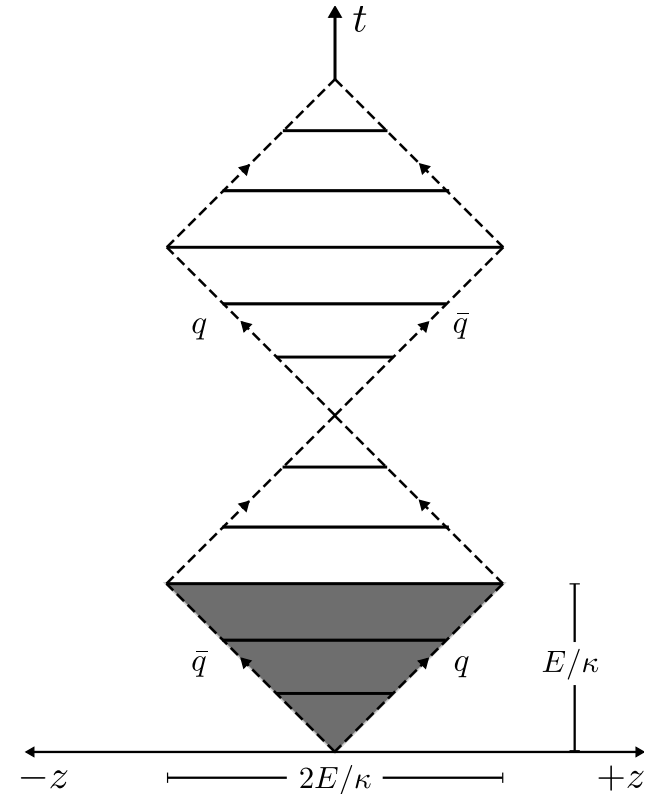
Consider the simplest hadronizing system:

A $q\bar{q}$ pair oriented along the z -axis, with equal and opposite momentum.

Treat this as a semi-classical system with potential

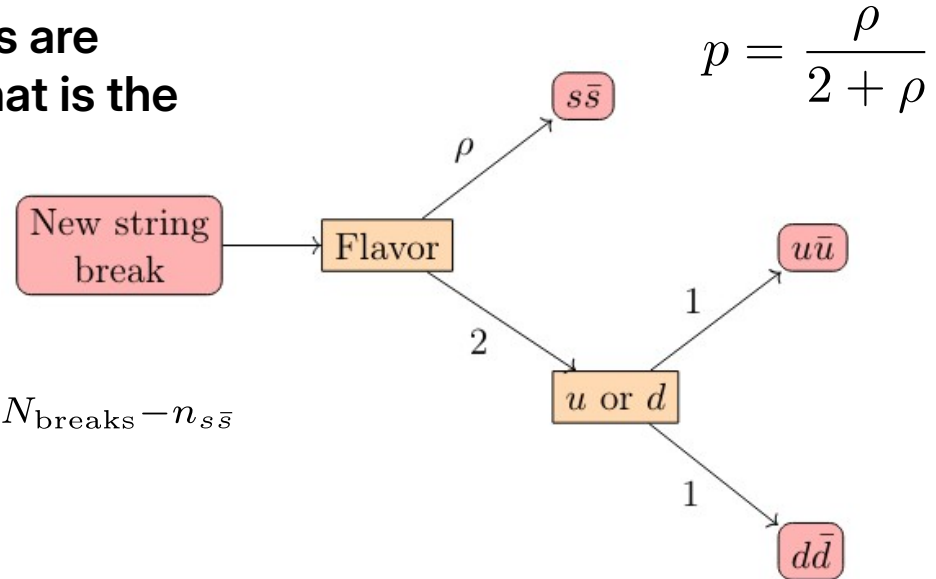
$$V(r) = \kappa r$$

A color flux tube forms between the $q\bar{q}$ pair and in the absence of string breaks the system follows a 'yo-yo' motion



Flavor reweighting (2505.00142)

Consider N_{breaks} string breaks. If strange breaks are suppressed by ρ compared to light quarks, what is the probability to producing $n_{s\bar{s}}$ breaks?



$$P(n_{s\bar{s}} | N_{\text{breaks}}) = \binom{N_{\text{breaks}}}{n_{s\bar{s}}} p^{n_{s\bar{s}}} (1 - p)^{N_{\text{breaks}} - n_{s\bar{s}}}$$

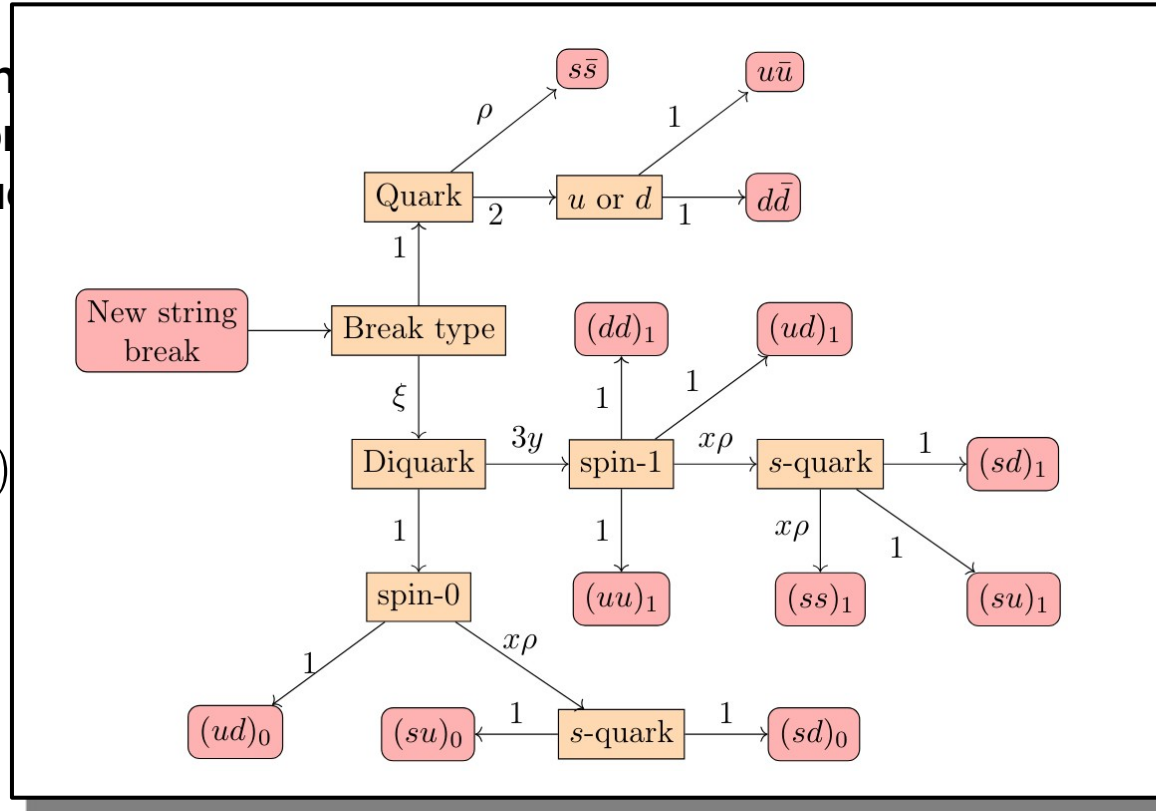
$$w = \frac{P(\rho')}{P(\rho)} = \left(\frac{\rho'}{\rho}\right)^{n_{s\bar{s}}} \left(\frac{1 - \rho'}{1 - \rho}\right)^{N_{\text{breaks}} - n_{s\bar{s}}}$$

Flavor reweighting (2505.00142)

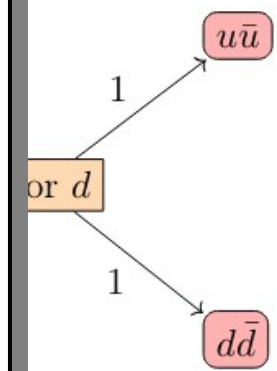
Consider N_{breaks} string breaks suppressed by ρ compared to probability to produce

$$P(n_{s\bar{s}} | N_{\text{breaks}})$$

$$w = \frac{P(\rho')}{P(\rho)}$$



$$p = \frac{\rho}{2 + \rho}$$



MAGIC: Invertible neural networks (INN)

a.k.a normalizing flow

Invertible Real NVP transformations:

$$z'_1 = z_1 \odot \exp(s_1(z_2)) + t_1(z_2),$$

$$z'_2 = z_2 \odot \exp(s_2(z'_1)) + t_2(z'_1),$$

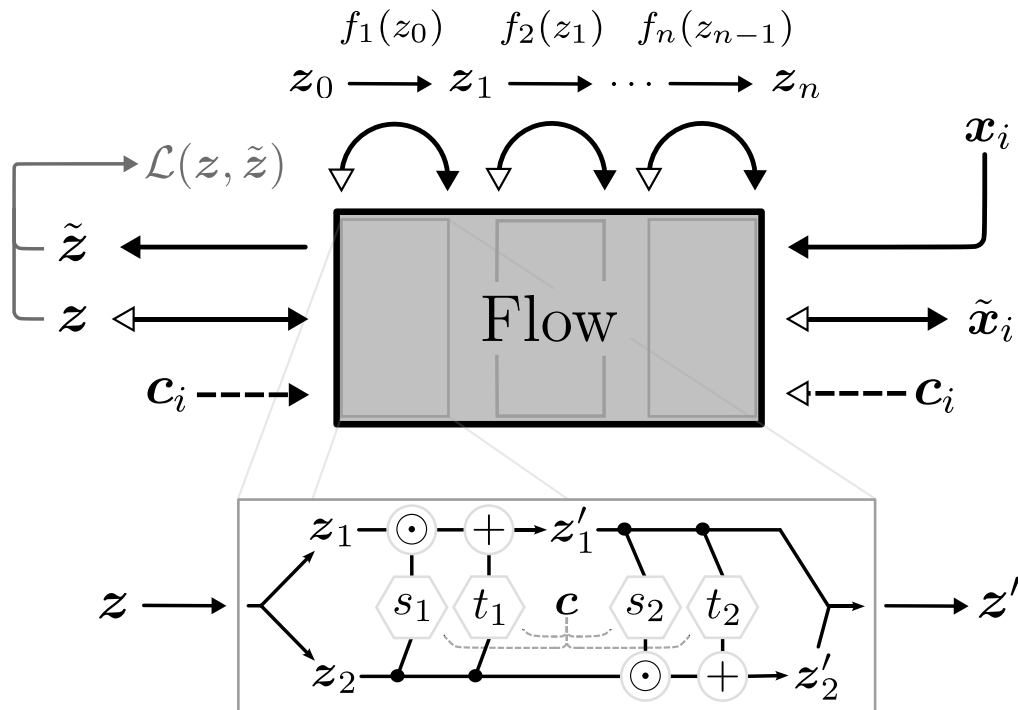
Scale transform

Translation transform

Inverse:

$$z_2 = (z'_2 - t(z'_1)) \odot \exp(-s(z'_1)),$$

$$z_1 = (z'_1 - t(z_2)) \odot \exp(-s(z_2)),$$



Invertible neural networks (INN)

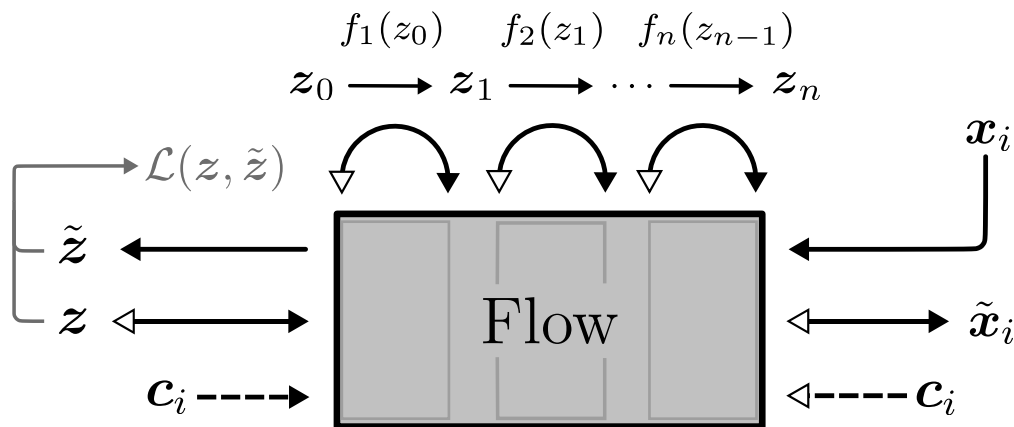
a.k.a normalizing flow

Learn a composition of n independent bijective transformations that relate a probability distribution $p_Z(\mathbf{z})$ on latent space Z to the target distribution $p_X(\mathbf{x})$ on target space X .

The probability distribution for the random variable $\mathbf{x} = f(\mathbf{z})$ is given by

$$p_X^f(\mathbf{x}) = p_Z(\mathbf{z}) |\det J_f(\mathbf{z})|^{-1}$$

$$J_f = \partial f / \partial \mathbf{x}$$



For n iterative transformations:

$$p_X^F(\mathbf{x}) = p_Z(\mathbf{z}_0) \prod_{i=1}^n |\det J_{f_i}(\mathbf{z}_{i-1})|^{-1}$$

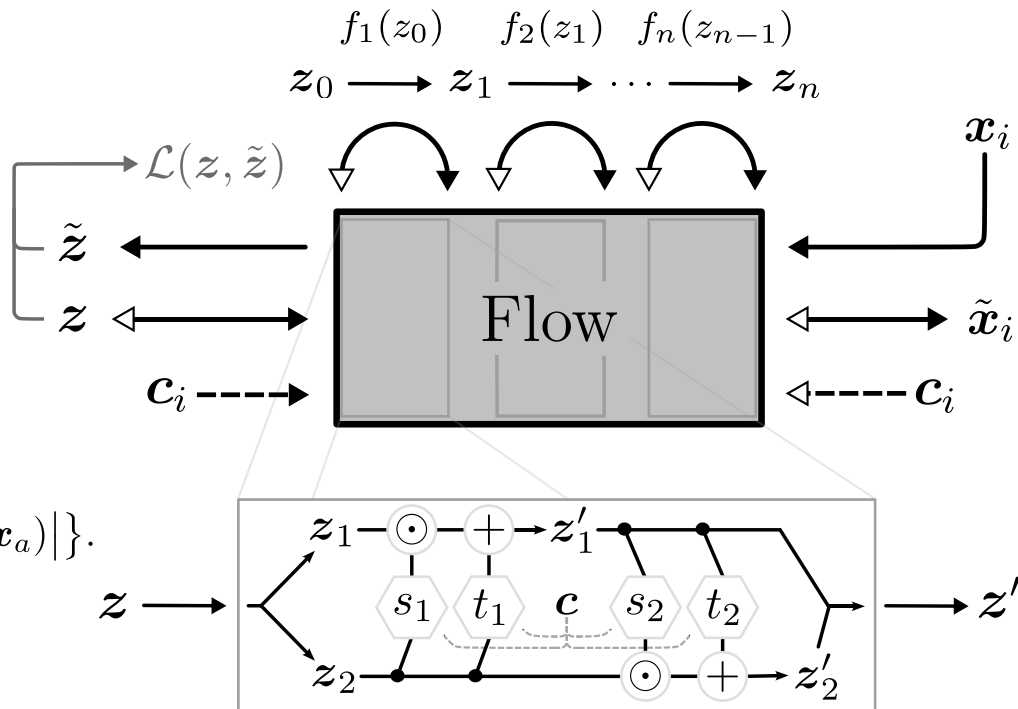
Invertible neural networks (INN)

a.k.a normalizing flow

$$p_X^F(\mathbf{x}) = p_Z(\mathbf{z}_0) \prod_{i=1}^n |\det J_{f_i}(\mathbf{z}_{i-1})|^{-1}$$

Train with the negative log likelihood:

$$\begin{aligned} \mathcal{L}_{\text{NF}} &= - \sum_{a=1}^N \log p_X^F(\mathbf{x}_a; \boldsymbol{\theta}, \mathbf{c}_a) \\ &= \sum_{a=1}^N \{ - \log p_Z(F^{-1}(\mathbf{x}_a; \boldsymbol{\theta}, \mathbf{c}_a)) + \log |\det J_{F^{-1}}(\mathbf{x}_a)| \}. \end{aligned}$$

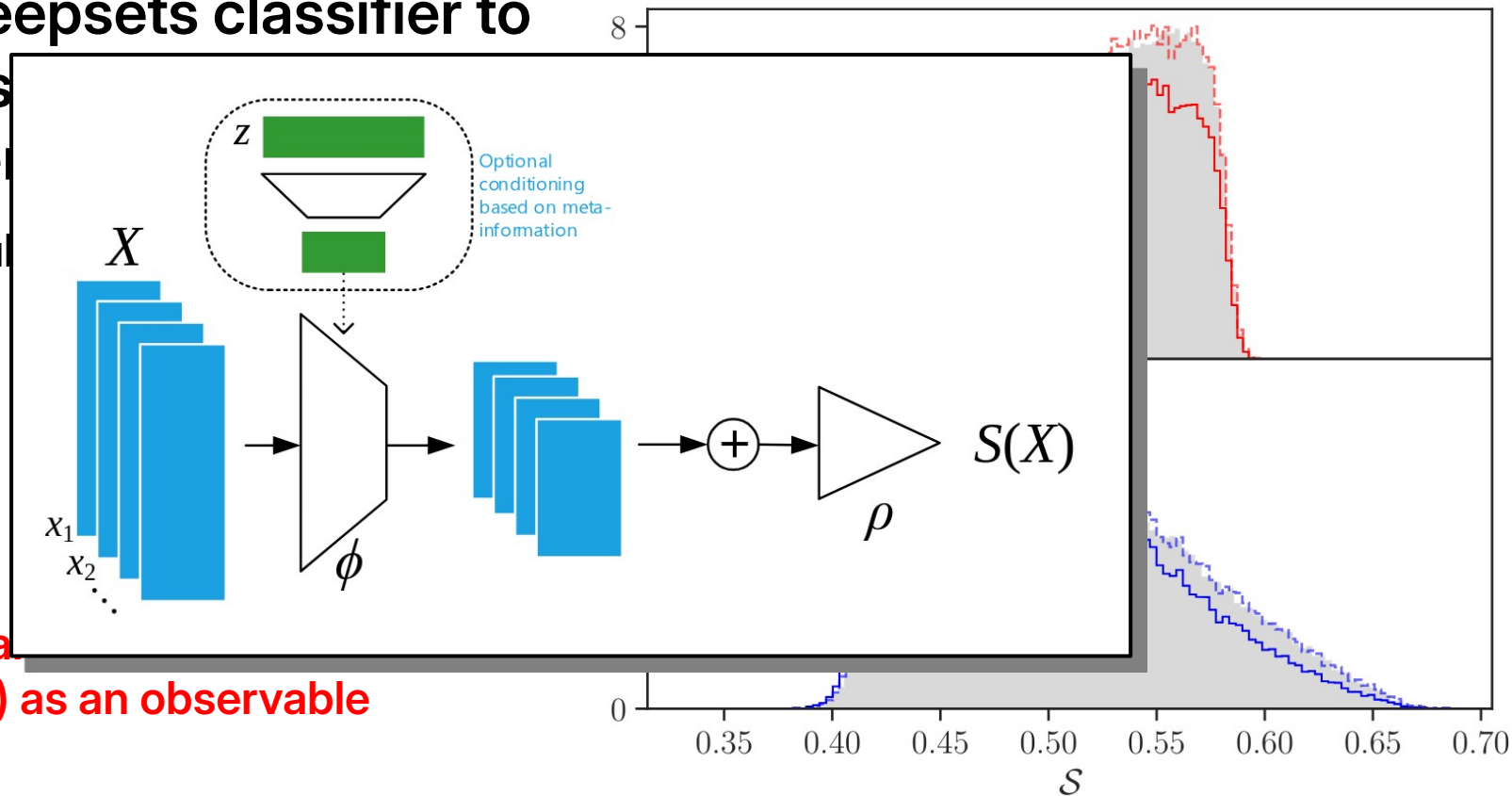


ML-based observables

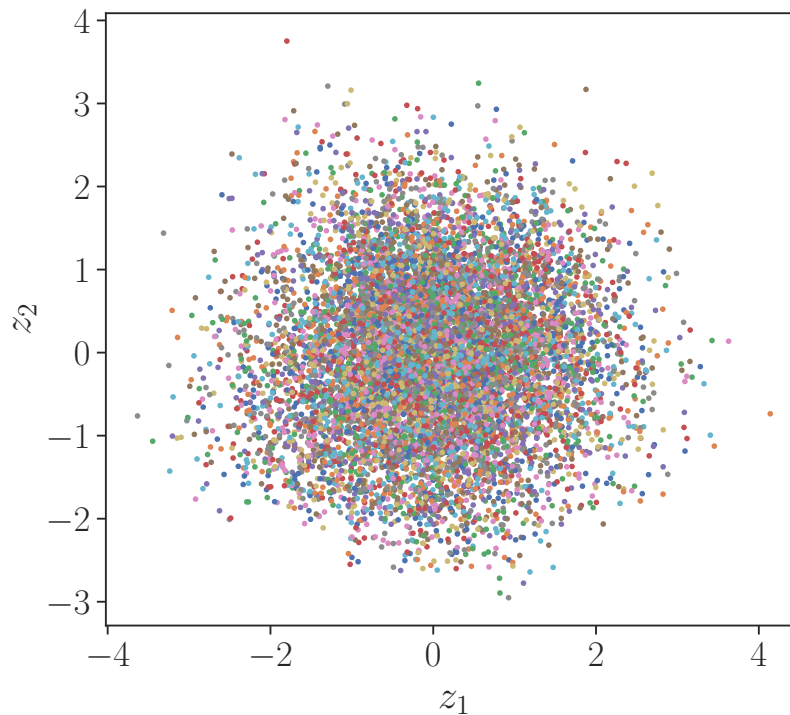
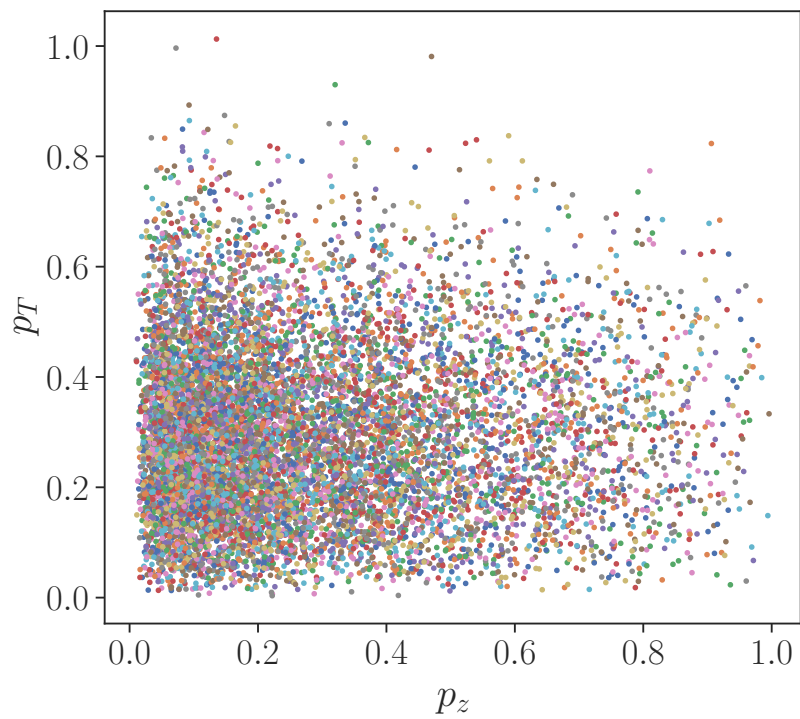
- Train a deepsets classifier to distinguish experimental

– Takes full input

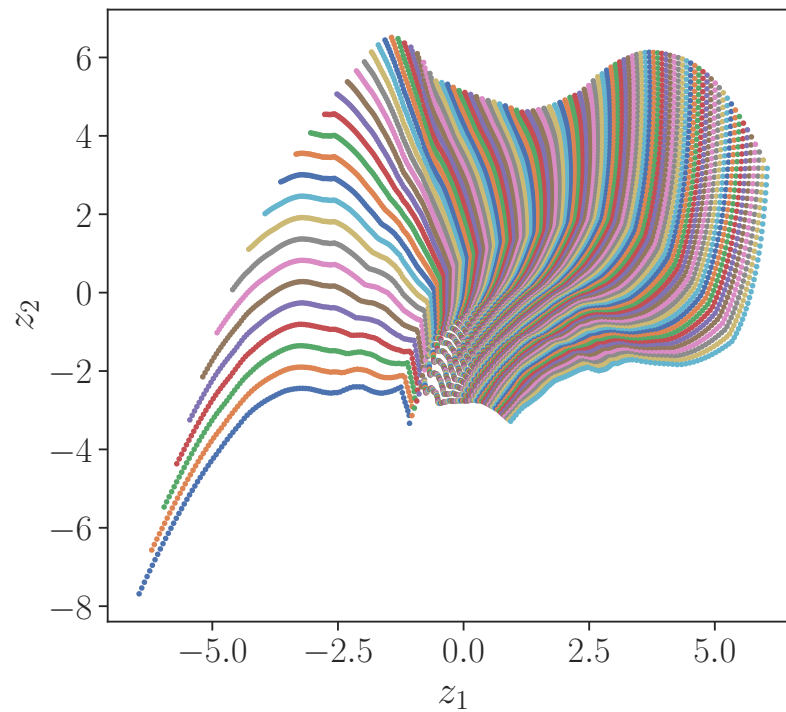
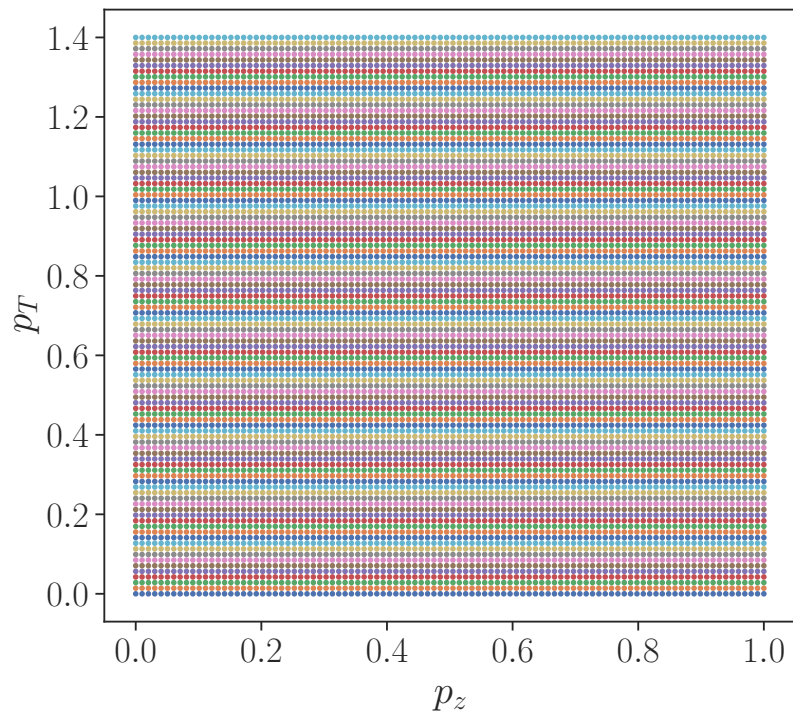
Use training (score) as an observable



INN learned mapping

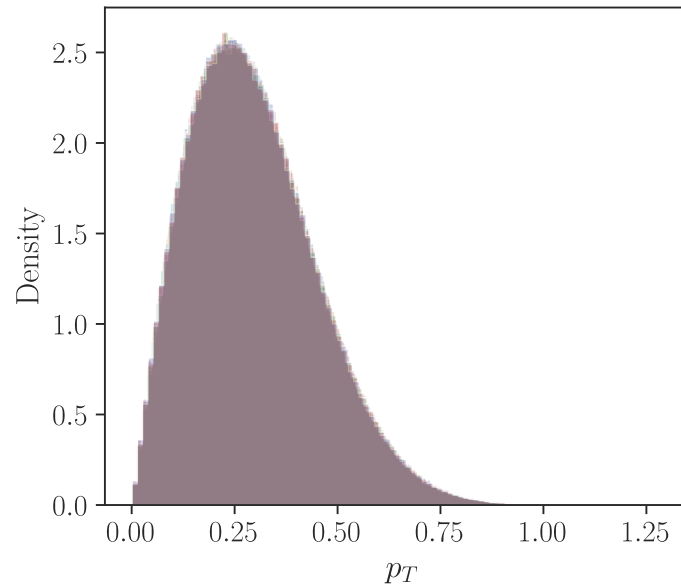
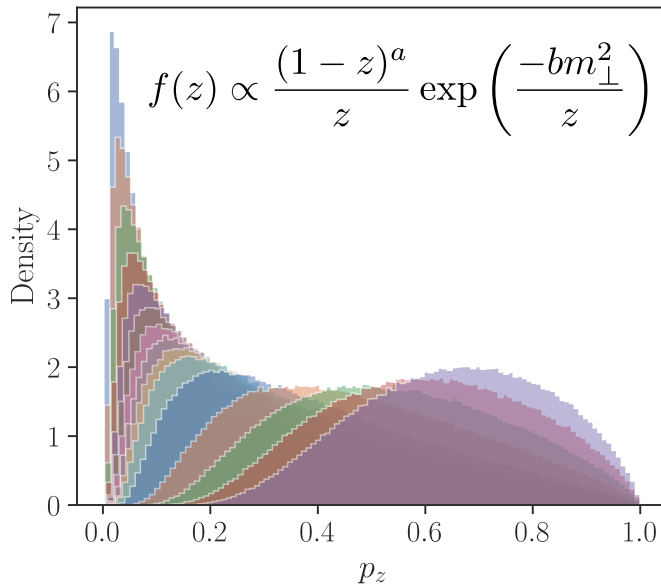


INN learned mapping



Training data

The implementation of full hadronization event using single emission kinematics requires an independent p_T sampling followed by a p_T -dependent sampling of p_z sampling (due to the dependence of $f(z)$ of transverse mass).



Generate Pythia $q\bar{q} \rightarrow h$ events, first with no p_T kicks at different values of the hadron mass, record p_z . Generate events again, with kicks turned on, record p_T .

MAGIC: $q\bar{q}$

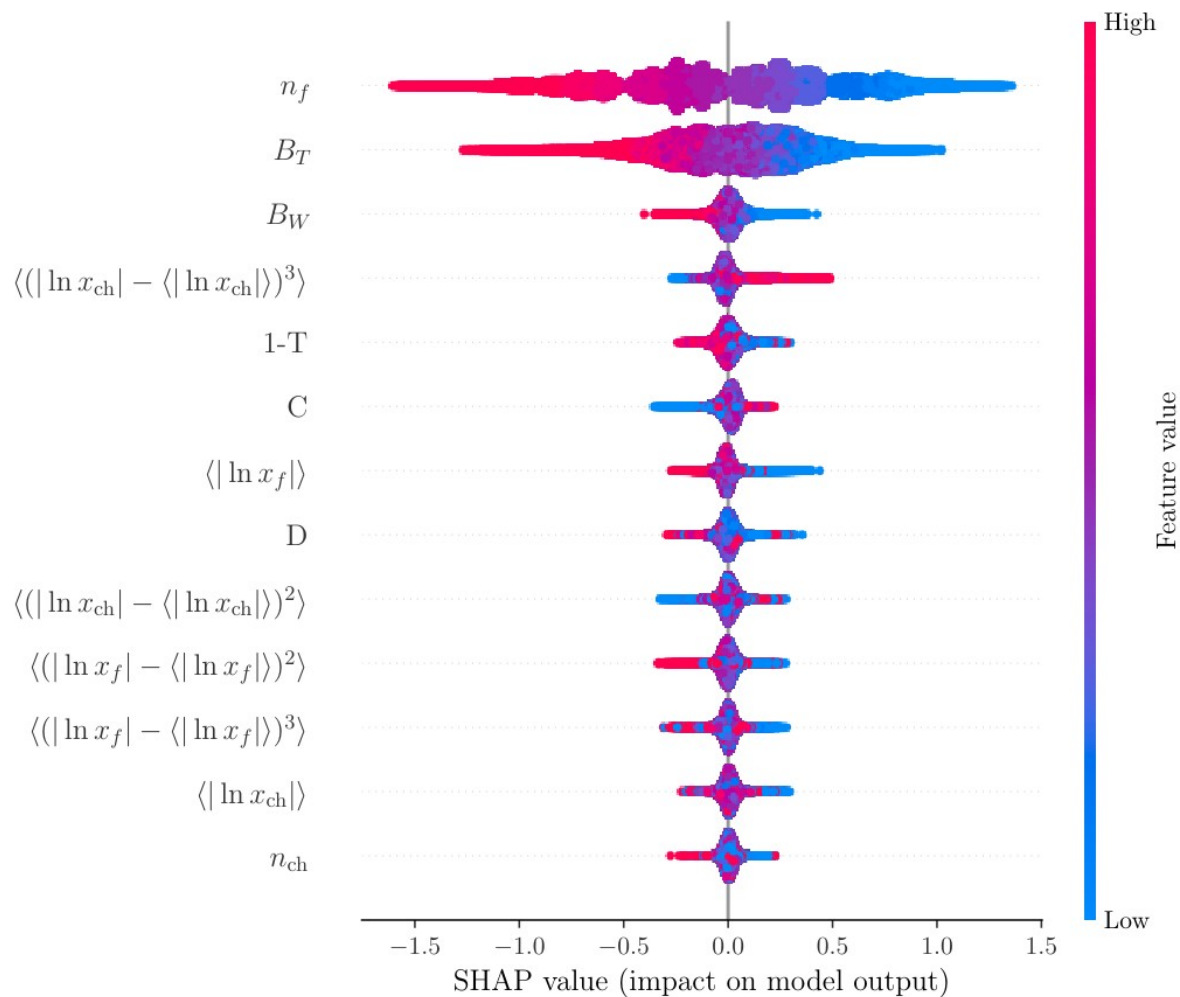
Use hadron multiplicity N as global event-level observable

$$\mathbf{X} = \begin{pmatrix} [\{p_z^{h_1}, p_T^{h_1}\}, \{p_z^{h_2}, p_T^{h_2}\}, \{p_z^{h_3}, p_T^{h_3}\}]_1 \\ [\{p_z^{h_1}, p_T^{h_1}\}, \{p_z^{h_2}, p_T^{h_2}\}, \{p_z^{h_3}, p_T^{h_3}\}, \{p_z^{h_4}, p_T^{h_4}\}, \{p_z^{h_5}, p_T^{h_5}\}]_2 \\ \vdots \\ [\{p_z^{h_1}, p_T^{h_1}\}, \{p_z^{h_2}, p_T^{h_2}\}]_n \end{pmatrix}, \quad \mathbf{Y}^{\text{sim}} = \begin{pmatrix} N_1 = 3 \\ N_2 = 5 \\ \vdots \\ N_n = 2 \end{pmatrix}, \quad \mathbf{Y}^{\text{exp}} = \begin{pmatrix} N_1 = 7 \\ N_2 = 3 \\ \vdots \\ N_n = 2 \end{pmatrix}$$

Event weights: $w = \begin{pmatrix} \prod_{i=1}^{N_1} w_i \\ \prod_{j=1}^{N_2} w_j \\ \vdots \\ \prod_{k=1}^{N_n} w_k \end{pmatrix}$ where $w_i = \frac{p_X^{F'}(p_z^{h_i}, p_T^{h_i})}{p_X^F(p_z^{h_i}, p_T^{h_i})}$,

Loss (Earth mover's distance): $\mathcal{L}_{\text{EMD}}(\mathbf{Y}^{\text{sim}}, \mathbf{w}, \mathbf{Y}^{\text{exp}})$

SHAP values



Brownian bridge

- Gaussian random walk ($E_{CM} \gg m_h$)
 - Markovian
- Gaussian random walk with termination condition, i.e. Gaussian bridge
 - Pseudo-Markovian

Brownian bridge

- Gaussian random walk ($E_{CM} \gg m_h$)

$$dX_t = dW_t$$

- Gaussian bridge
 - Boundary conditions

$$X_0 = x_0 \quad X_T = x_T$$

$$dX_t = \frac{x_T - X_t}{T - t} dt + dW_t$$

Brownian bridge

- Gaussian random walk ($E_{CM} \gg m_h$)

$$dX_t = dW_t$$

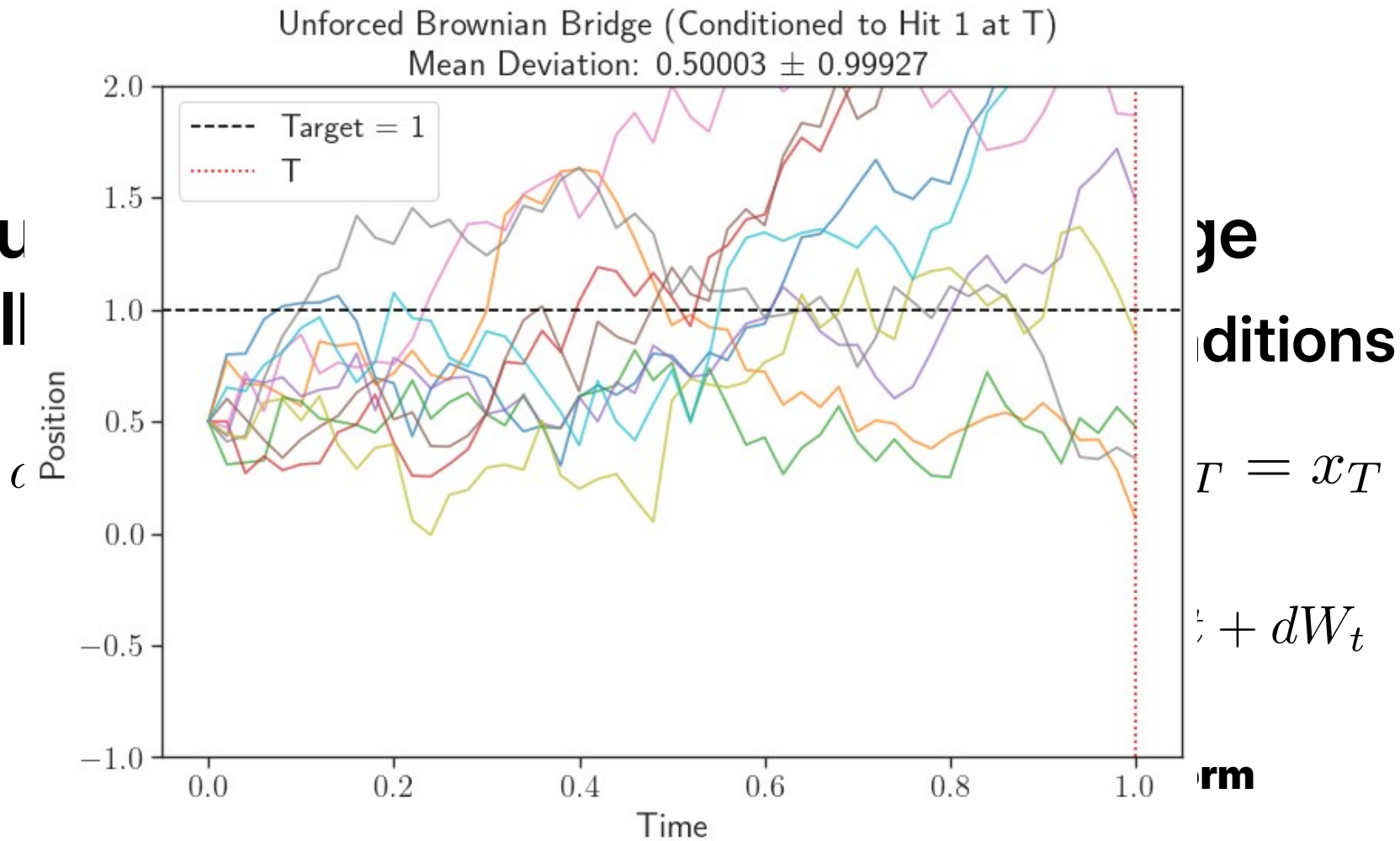
- Gaussian bridge
 - Boundary conditions

$$X_0 = x_0 \quad X_T = x_T$$

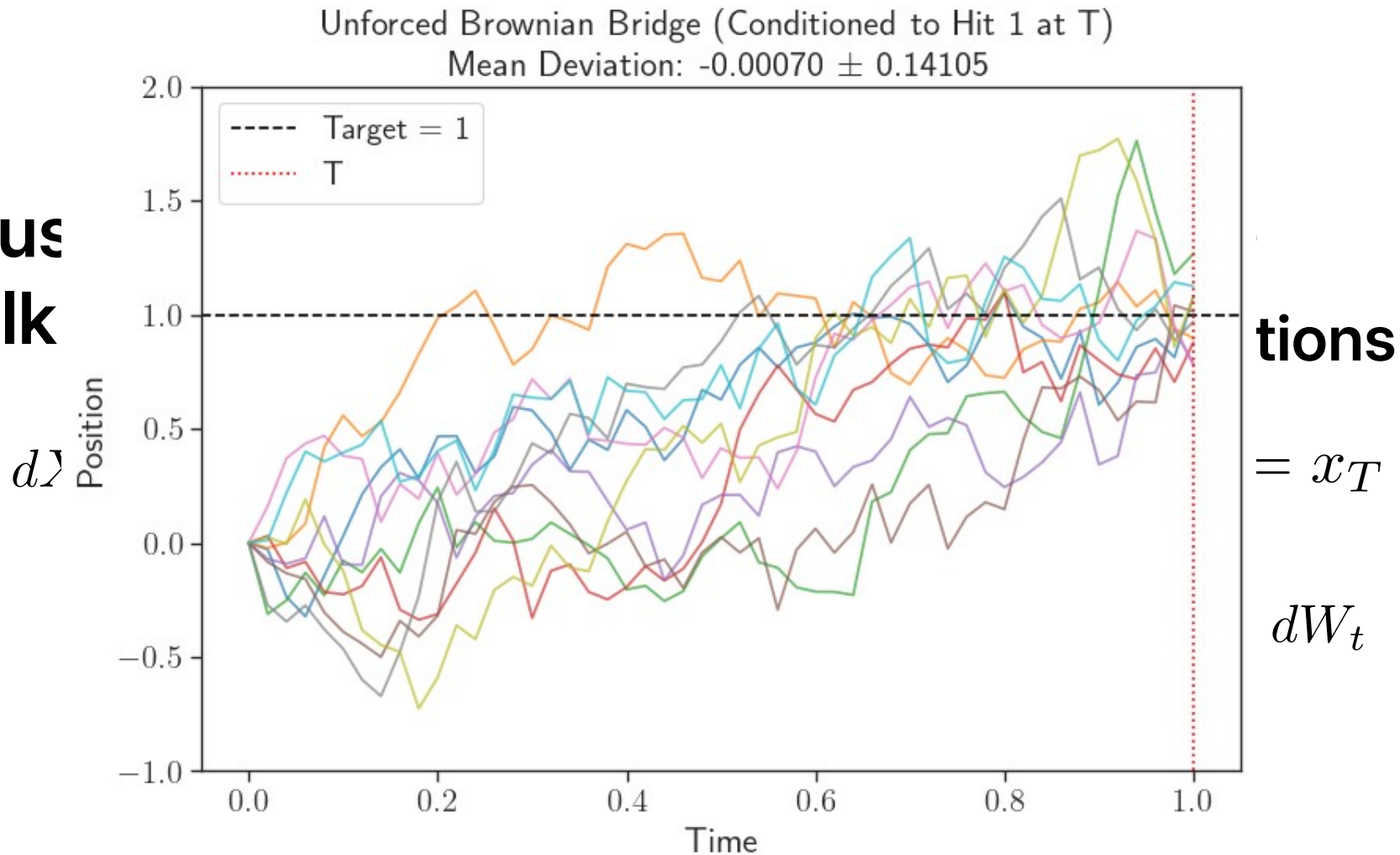
$$dX_t = \frac{x_T - X_t}{T - t} dt + dW_t$$

Doob h-transform

- **Gau**
wall

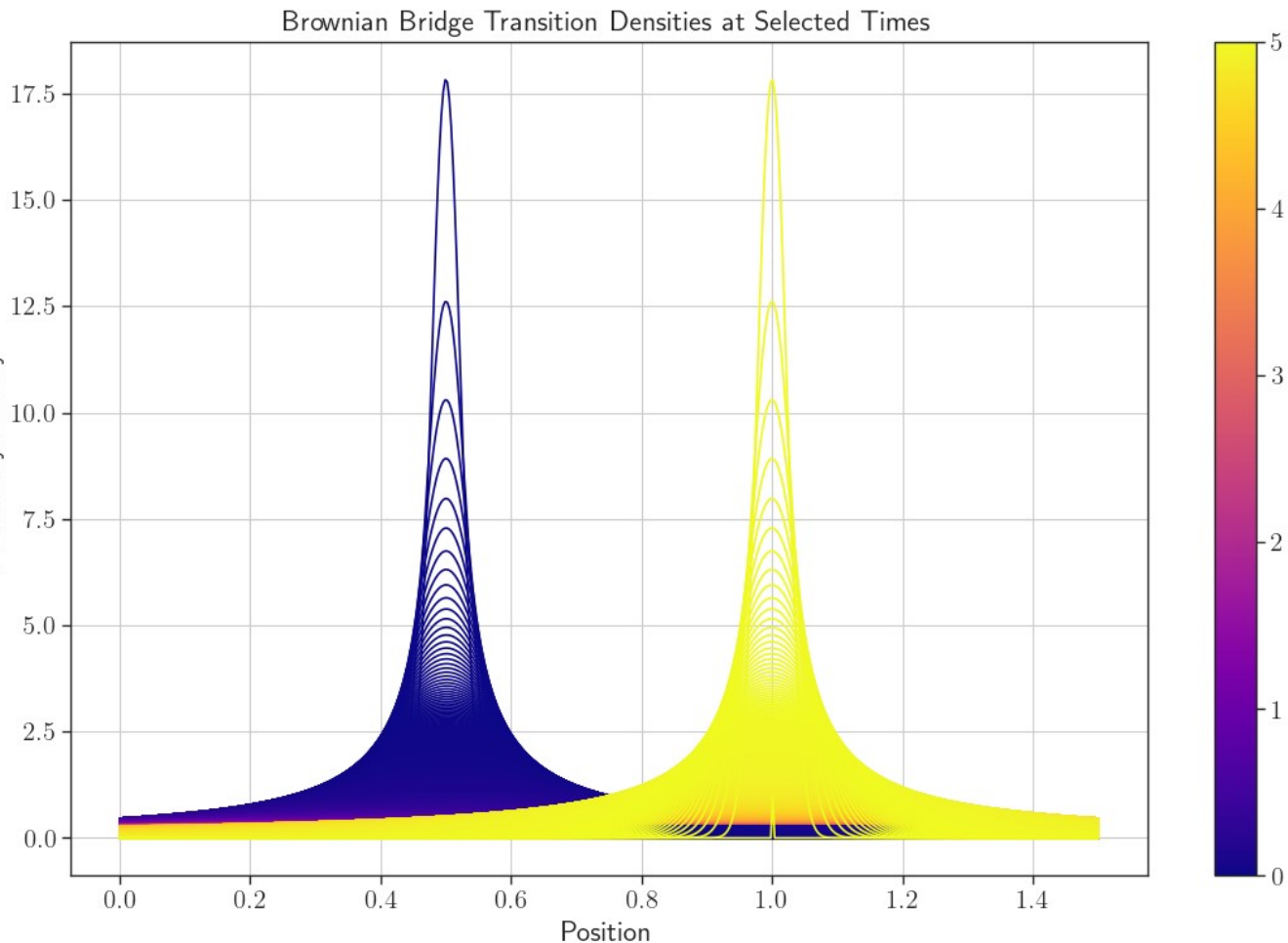


- **Gaus walk**



- Gau
walk

d . Probability Density



e
ditions

$$r = x_T$$

$$+ dW_t$$

m