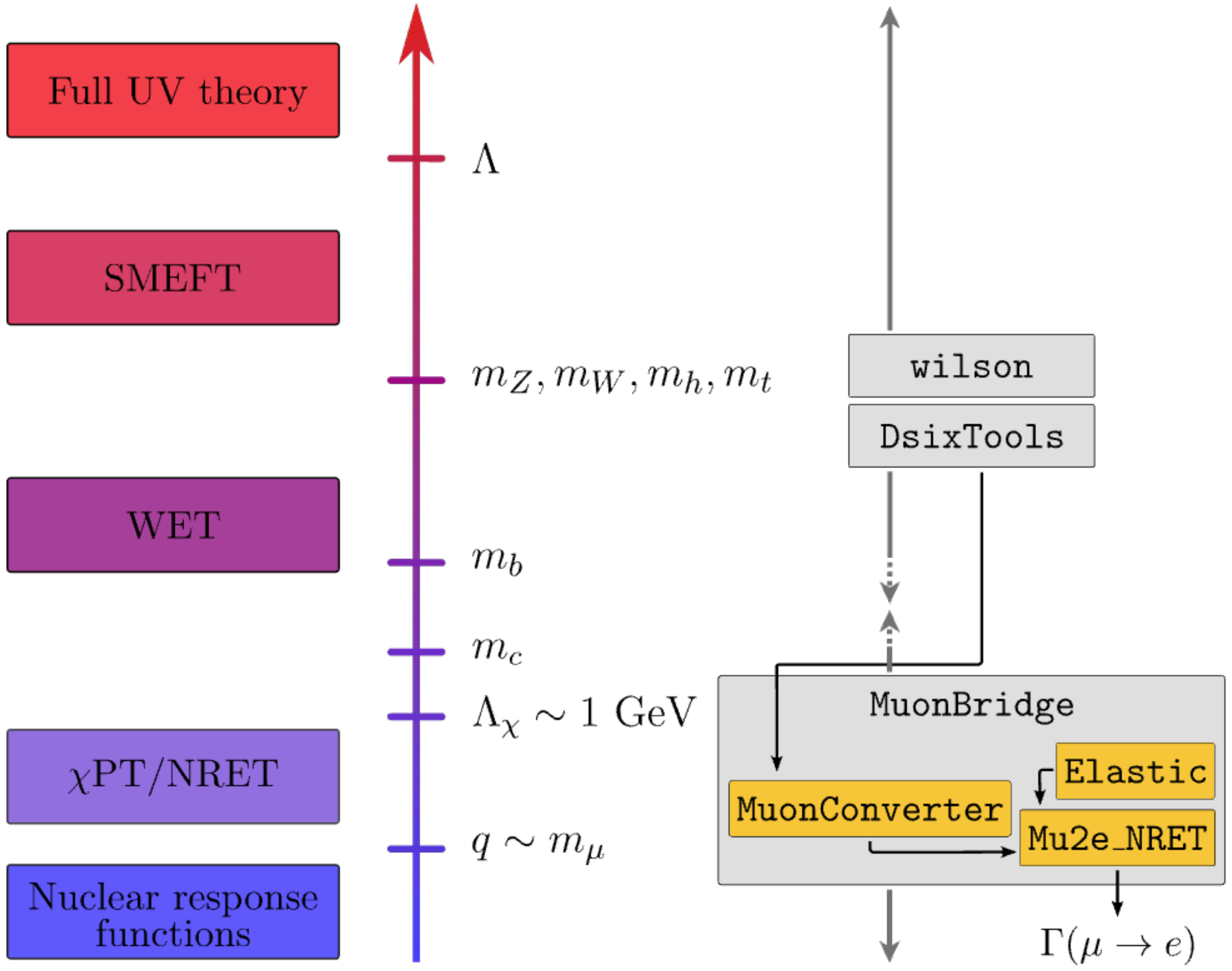
“hands-on”

MuonBridge ^ tutorial

“An effective theory tower
for $\mu \to e$ conversion” (2406.13818)



MIAPbP – Precision lepton physics
April 14th, 2026

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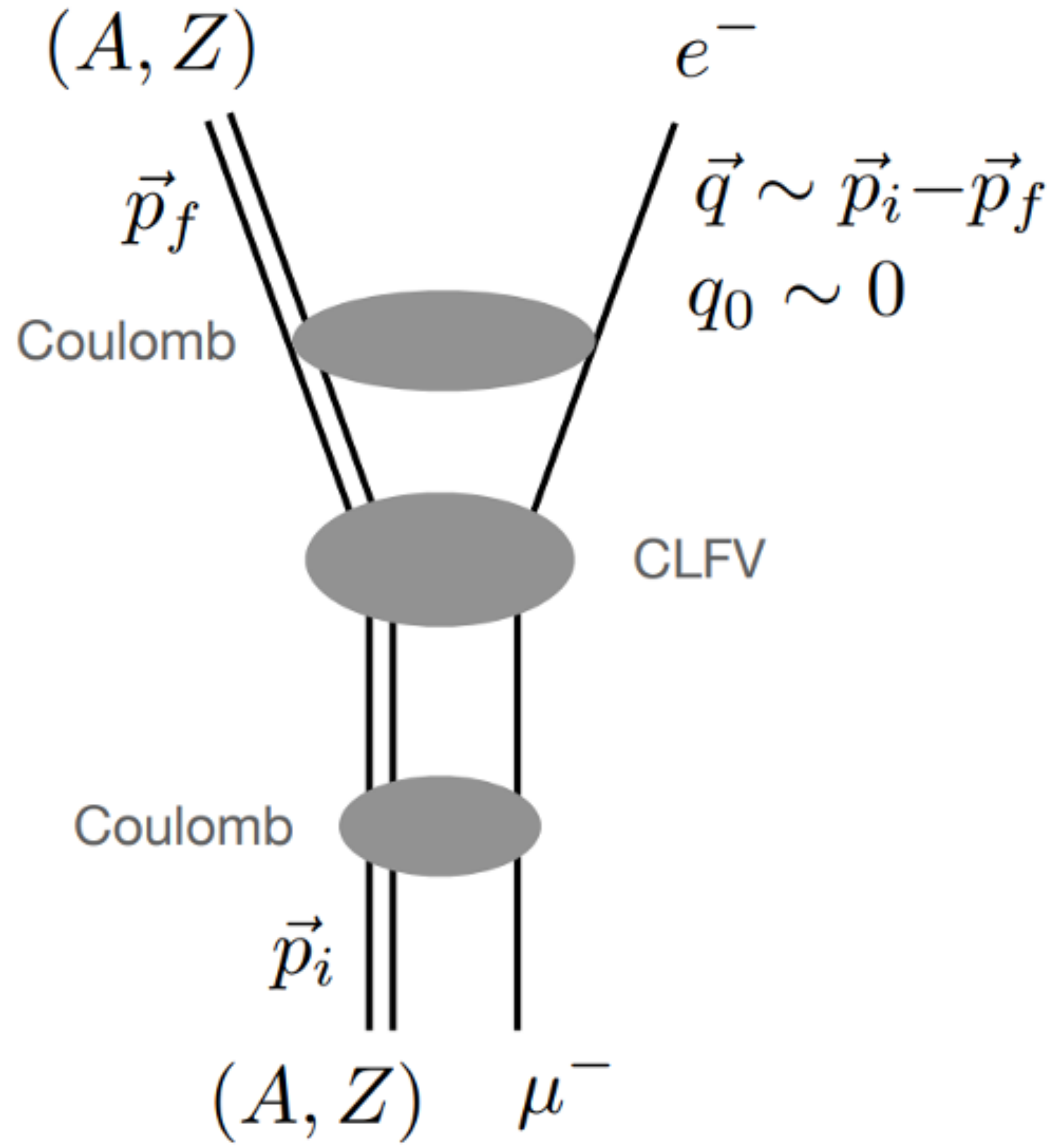
Based on work with Wick Haxton, Evan Rule, Ken McElvain, and Jure Zupan

Outline

1. $\mu \to e$ conversion ingredients
    - **Nuclear effective theory**
    - EFT tower
2. Code overview
3. Demo/tutorial
    - Installation
    - Λ reach
    - UV completion

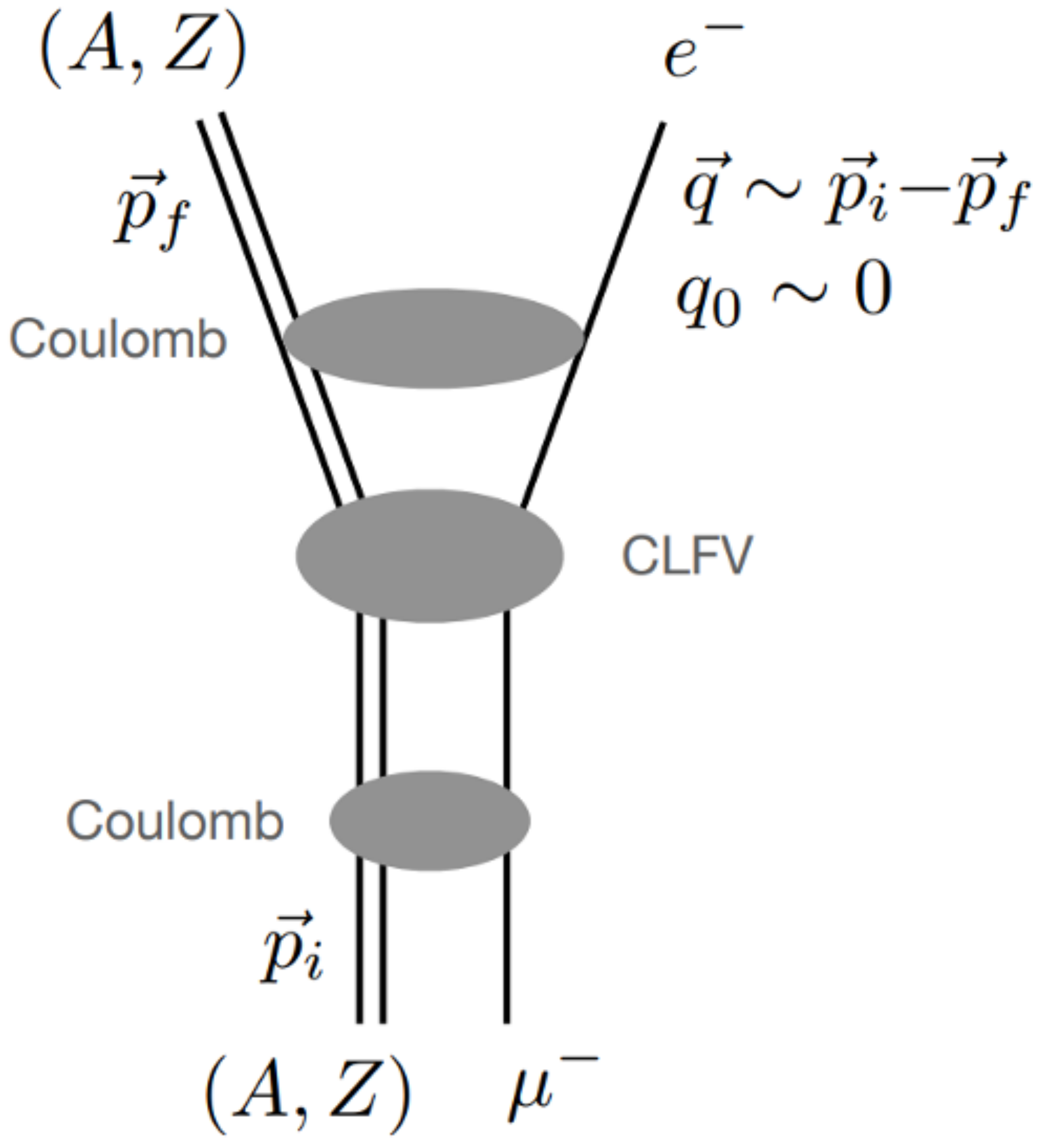
Conversion in the field of a nucleus $(A,Z)\,\mu^- \to (A,Z)\,e^-$



We're interested in computing the rate of $\mu^- \to e^-$ conversion $(\Gamma(\mu \to e))$ encoded experimentally as the ***capture ratio***

$$\mathrm{CR}(\mu \to e) = \frac{\Gamma(\mu^- + (A,Z) \to e^- + (A,Z))}{\Gamma(\mu^- + (A,Z) \to \nu_\mu + (A,Z-1))}$$

Conversion in the field of a nucleus $(A,Z)\,\mu^- \to (A,Z)\,e^-$



Bound (**1s**) μ^- converts to a free e^- with energy

$$E_e = m_\mu - E_\mu^{\text{bind}} - \frac{q^2}{2M_T} \approx m_\mu$$

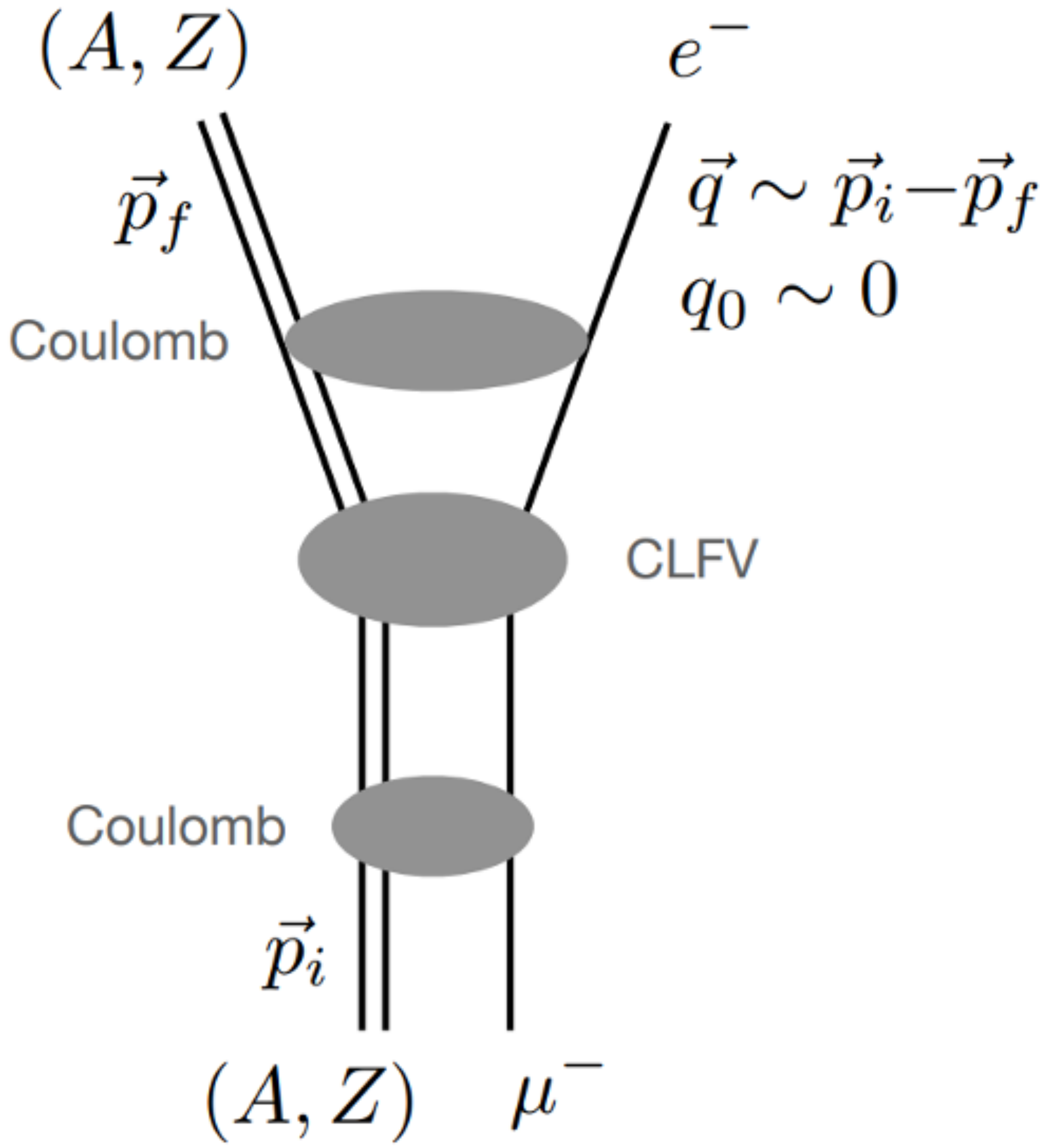
both of which are described by solutions of the Dirac equation

$$\left[-i\,\vec{\alpha}\cdot\vec{\nabla} + \beta\bar{m} + V(r)\right]\psi = E\,\psi$$

with radial and angular parts factorizing

$$\psi(\vec{r}) = \begin{pmatrix} G_\kappa(r)/r \cdot \Omega^{\ell}_{jm}(\hat{r}) \\ iF_\kappa(r)/r \cdot \Omega^{\ell'}_{jm}(\hat{r}) \end{pmatrix}.$$

Conversion in the field of a nucleus $(A,Z)\,\mu^- \to (A,Z)\,e^-$

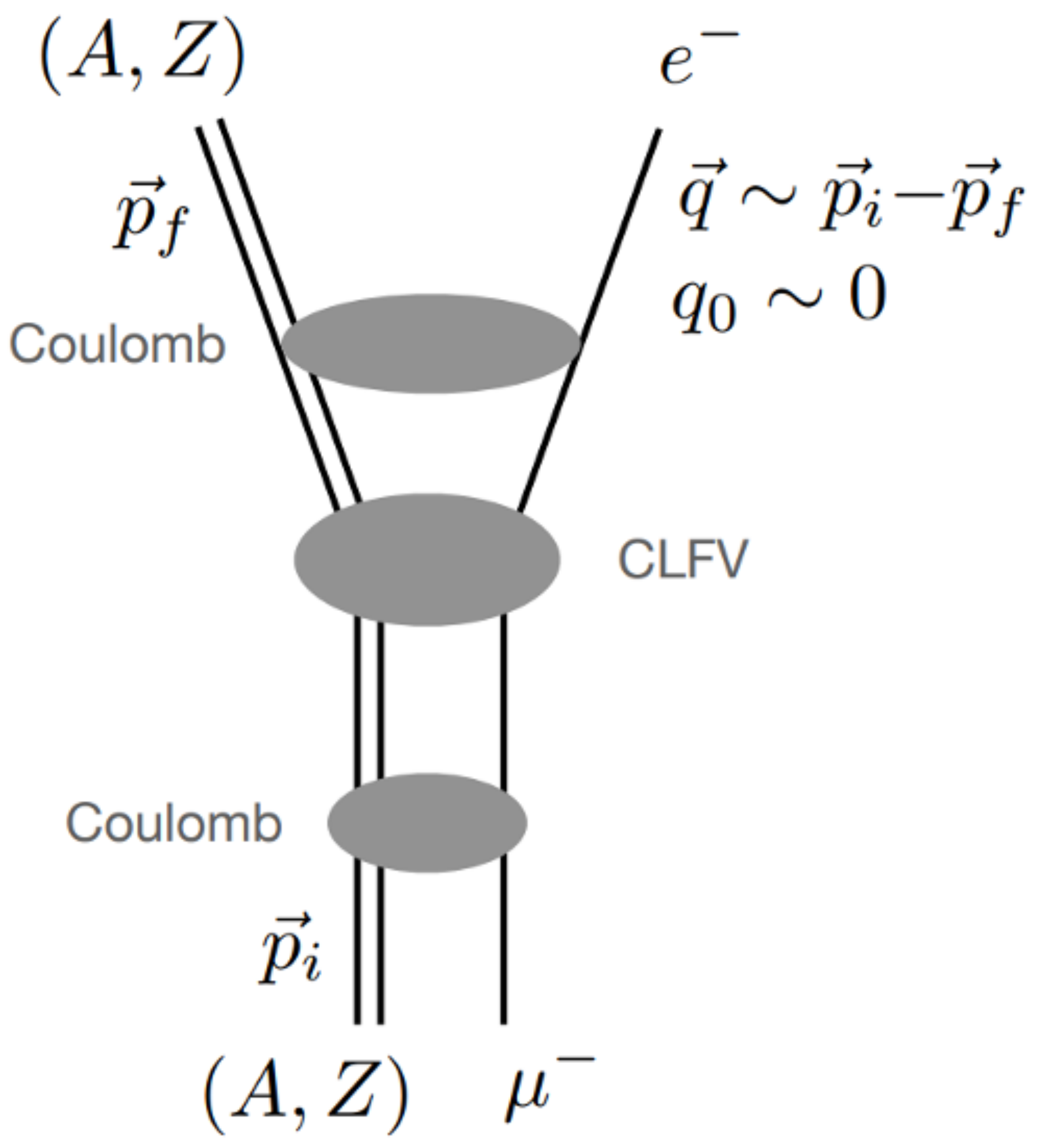


… $\psi(\vec{r}) = \begin{pmatrix} G_\kappa(r)/r \cdot \Omega^{\ell}_{jm}(\hat{r}) \\ iF_\kappa(r)/r \cdot \Omega^{\ell'}_{jm}(\hat{r}) \end{pmatrix}.$

The transition amplitude is just the overlap weighted by the CLFV interaction at each point in space. For an ***elastic*** transition stemming from a local interaction (heavy NP) the amplitude is generically

$$\mathcal{M} = \sum_{i=1}^{A} \int d^3r\, \bar{\psi}_e(\vec{r})\, \mathcal{O}_{\mathrm{lep}}\, \psi_\mu(\vec{r})\, \langle \mathrm{g.s.} \,|\, \mathcal{O}_N(i)\, \delta(\vec{r} - \vec{r}_i) \,|\, \mathrm{g.s.} \rangle \,.$$

Conversion in the field of a nucleus $(A,Z)\,\mu^- \to (A,Z)\,e^-$



For a scalar-scalar interaction $(\mathcal{O}_{\text{lep}} = \mathbf{1}_{\text{lep}}, \mathcal{O}_N = \mathbf{1}_N)$

the δ-function collapses the spatial integral

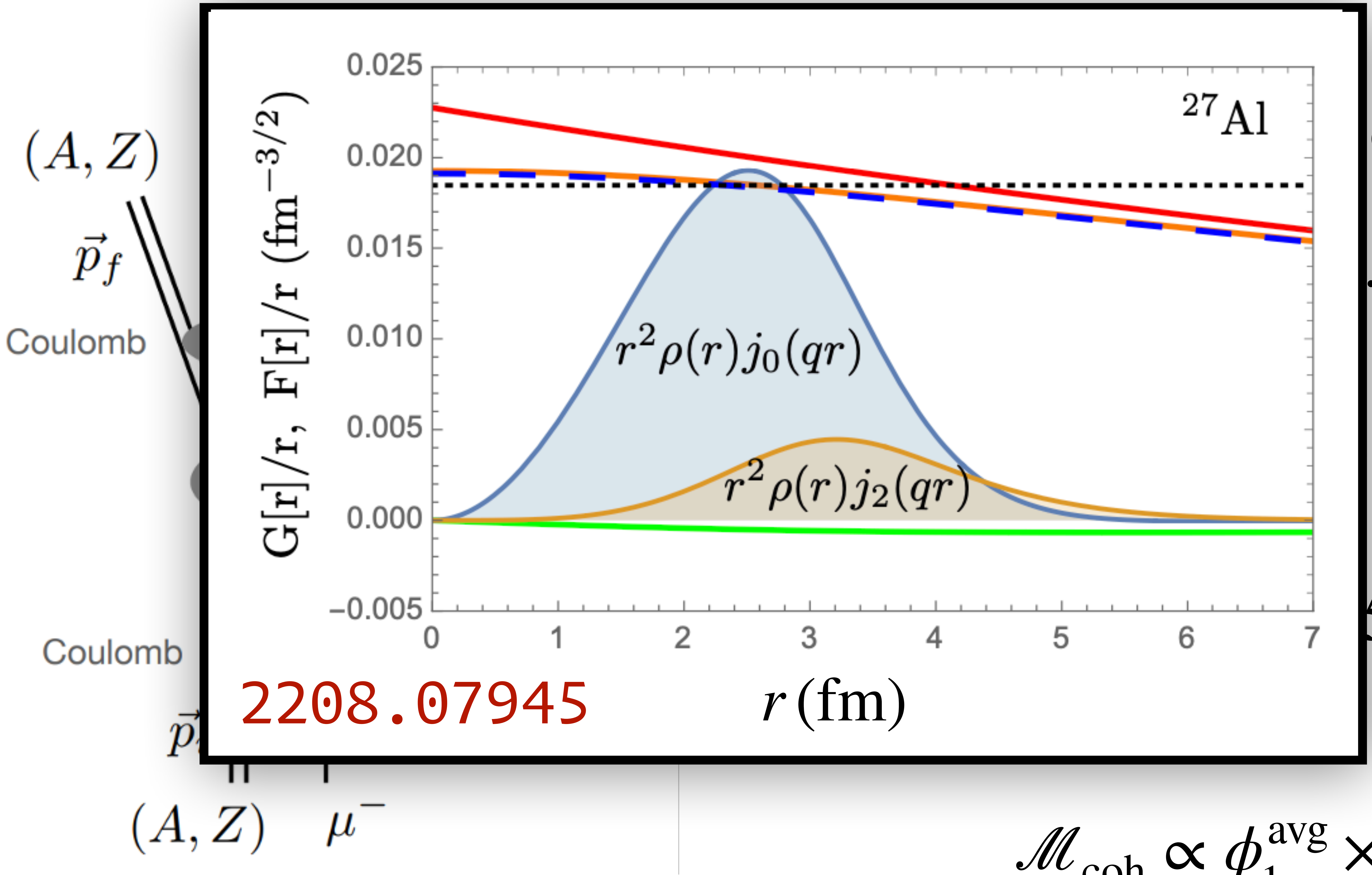
$$\mathcal{M}_{\text{coh}} \propto \langle \text{g.s.} | \sum_{i=1}^{A} \bar{\psi}_e(\vec{r}_i)\, \psi_\mu(\vec{r}_i) \,|\, \text{g.s.} \rangle .$$

The muons effective size is $\lambda_\mu \sim a_\mu \approx 20\text{fm} \gg R_{\text{nuc}} \approx 3\text{fm}$

such that (neglecting the lower component $|F/G| \sim v_\mu \sim Z\alpha/2 = 0.05$) $\psi_\mu(r) \simeq G_{-1}^{(\mu)}(r)/r$ remains $\approx$ constant

$$\mathcal{M}_{\text{coh}} \propto \phi_{1s}^{\text{avg}} \times \langle \text{g.s.} | \sum_{i=1}^{A} \bar{\psi}_e(\vec{r}_i) \,|\, \text{g.s.} \rangle .$$

Conversion in the field of a nucleus $(A,Z)\,\mu^- \to (A,Z)\,e^-$



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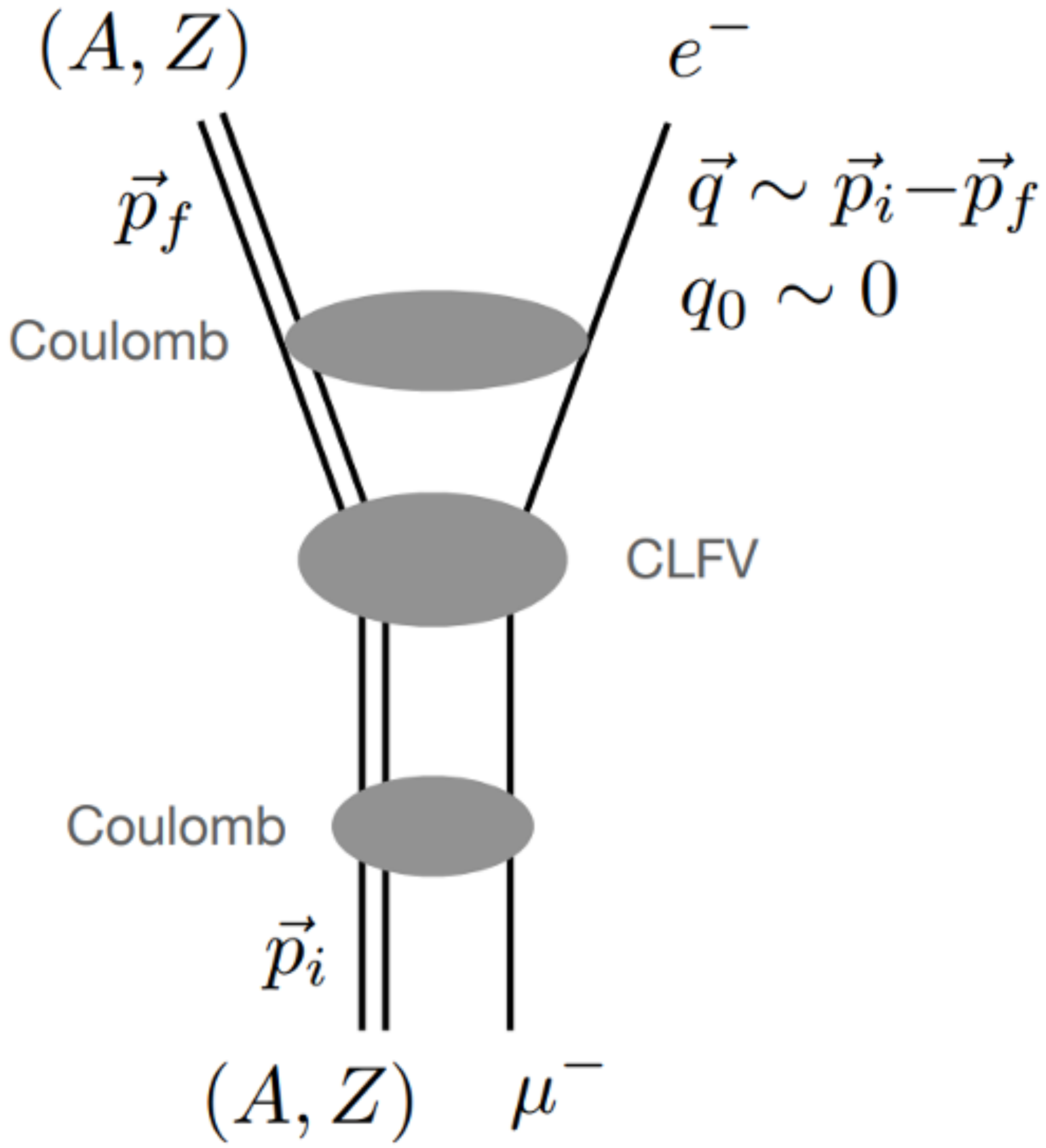


$$\dots\ \mathscr{M}_{\text{coh}} \propto \phi_{1s}^{\text{avg}} \times \langle \text{g.s.} \,|\, \sum_{i=1}^{A} \bar{\psi}_e(\vec{r}_i) \,|\, \text{g.s.} \rangle .$$

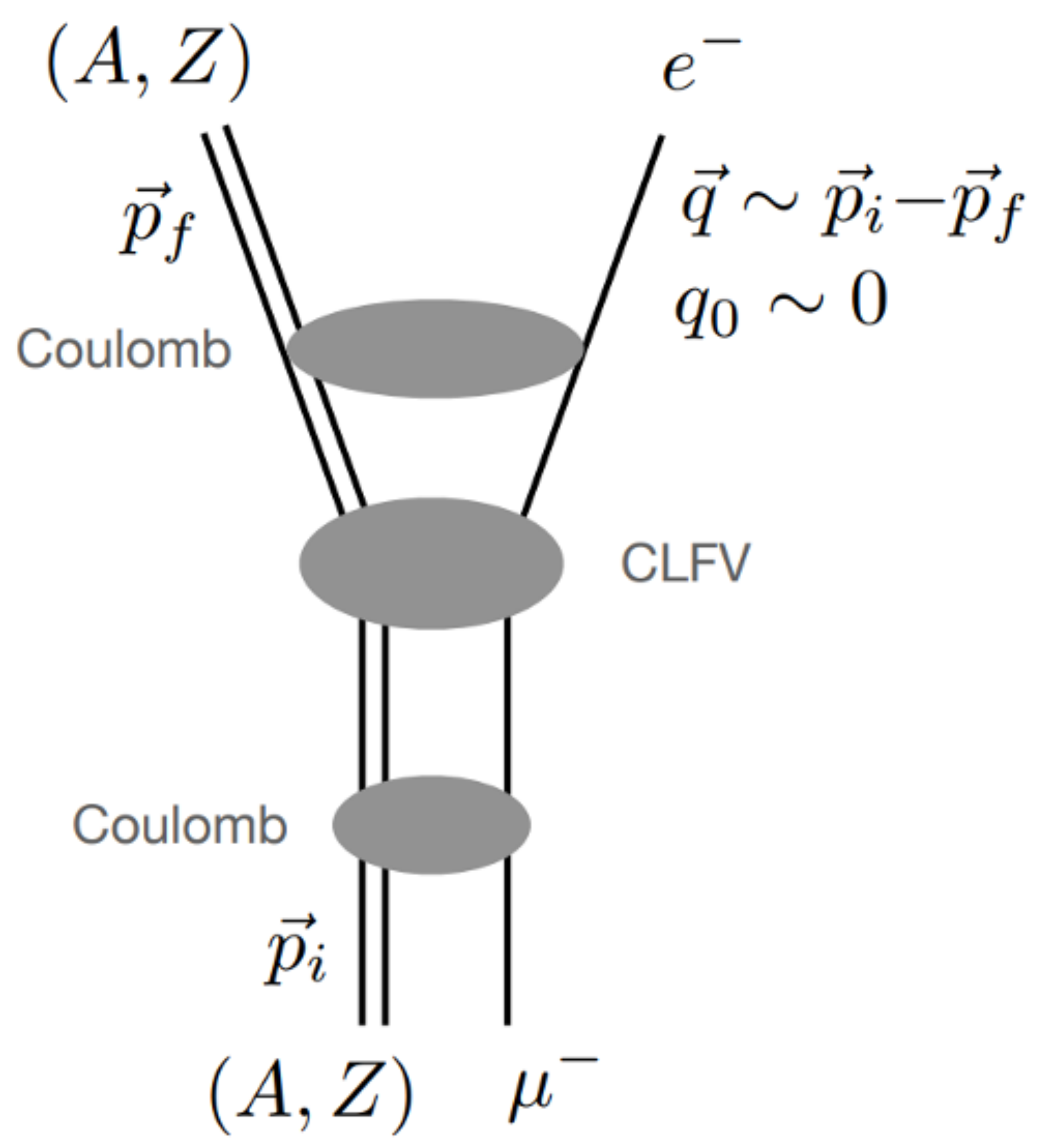
Rewriting in terms of the nuclear density ρ_N

$$\mathscr{M}_{\text{coh}} \propto \phi_{1s}^{\text{avg}} \times \int dr\, r\, G_{-1}^{(e)} \rho_N(r) .$$

For a spin-dependent interaction($\mathcal{O}_N = \vec{\sigma}_N$), a similar chain of reasoning gives

$$\mathscr{M}_{\text{spin}} \propto \phi_{1s}^{\text{avg}} \times \langle \text{g.s.} \,|\, \sum_{i=1}^{A} \vec{\sigma}_N(i)\, \bar{\psi}_e(\vec{r}_i) \,|\, \text{g.s.} \rangle$$

Conversion in the field of a nucleus $(A,Z)\,\mu^- \to (A,Z)\,e^-$

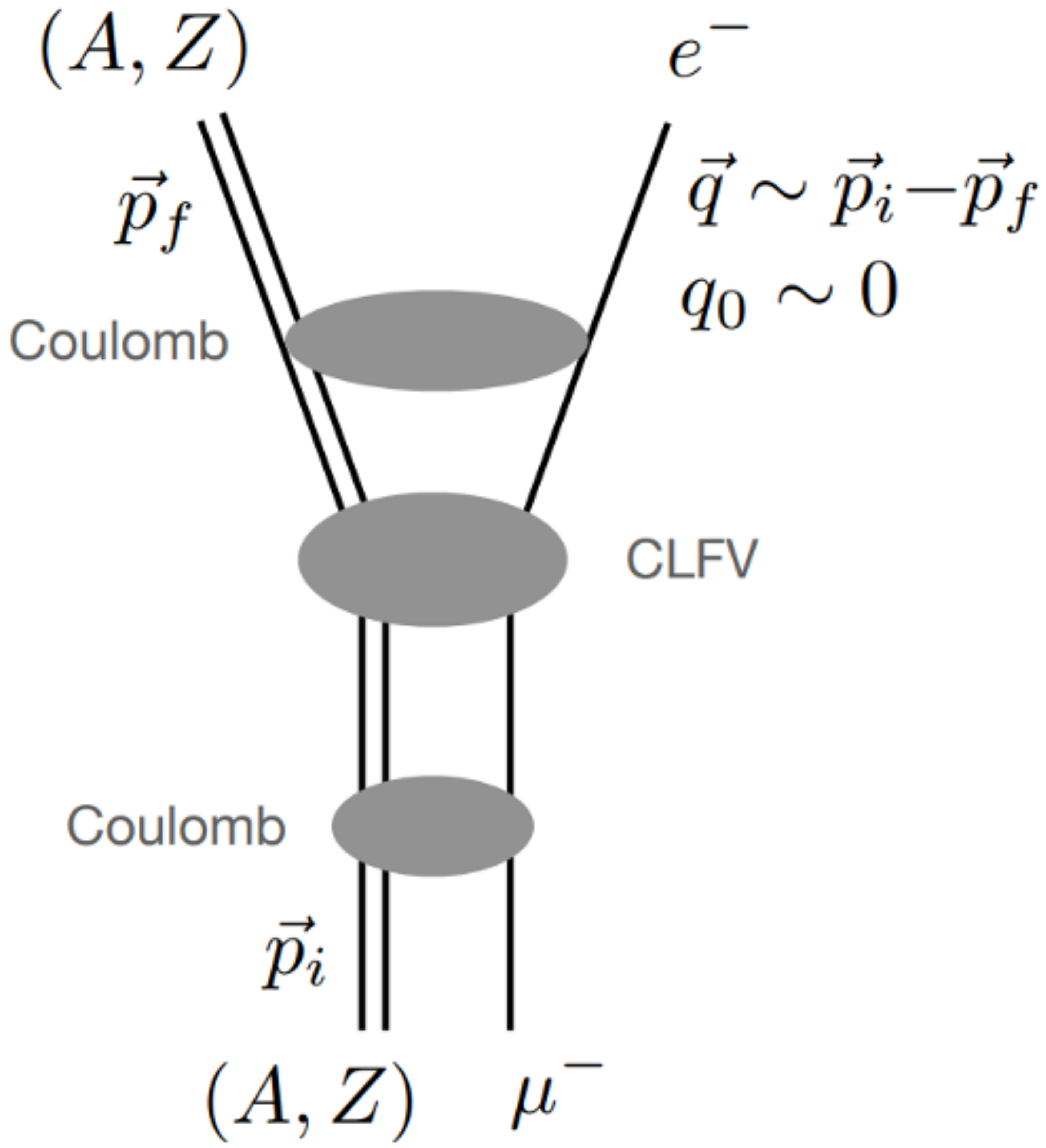


$$\ldots\; \mathscr{M}_{\text{spin}} \propto \phi_{1\text{s}}^{\text{avg}} \times \langle \text{g.s.} \,|\, \sum_{i=1}^{A} \vec{\sigma}_N(i)\, \bar{\psi}_e(\vec{r}_i) \,|\, \text{g.s.} \rangle$$

Paired nucleons contribute with opposite signs (no coherent enhancement). Multiple multipoles J and electron partial waves κ contribute, each requiring a separate numerical radial integral

$$\mathscr{M}_{\text{spin}} \propto \phi_{1\text{s}}^{\text{avg}} \times \sum_{J} \sum_{\kappa} C_{\kappa J} \int dr\, r\, G_{\kappa}^{(e)}(r)\, \mathscr{F}_{J}^{\text{nuc}}(r)\,.$$

Conversion in the field of a nucleus $(A,Z)\,\mu^- \to (A,Z)\,e^-$



If the electron is approximated as a free plane wave, the partial wave expansion can be re-summed

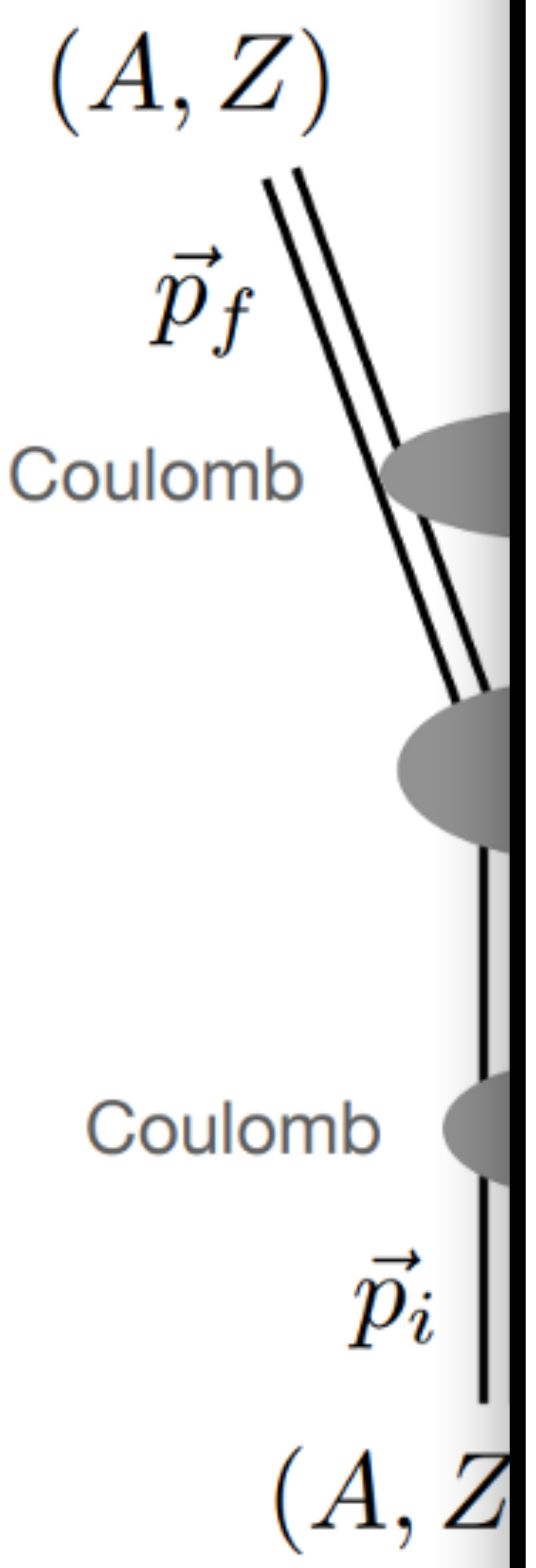
$$\sum_\kappa \begin{pmatrix} G_\kappa(r)/r \;\; \Omega^{\ell}_{jm}(\hat{r}) \\ iF_\kappa(r)/r \;\; \Omega^{\ell'}_{jm}(\hat{r}) \end{pmatrix} \xrightarrow{G_\kappa = q\, r\, j_\ell(qr)} \sqrt{\frac{E_e}{2m_e}} \begin{pmatrix} \xi \\ \vec{\sigma}\cdot\hat{q}\;\xi \end{pmatrix} e^{i\vec{q}\cdot\vec{r}}.$$

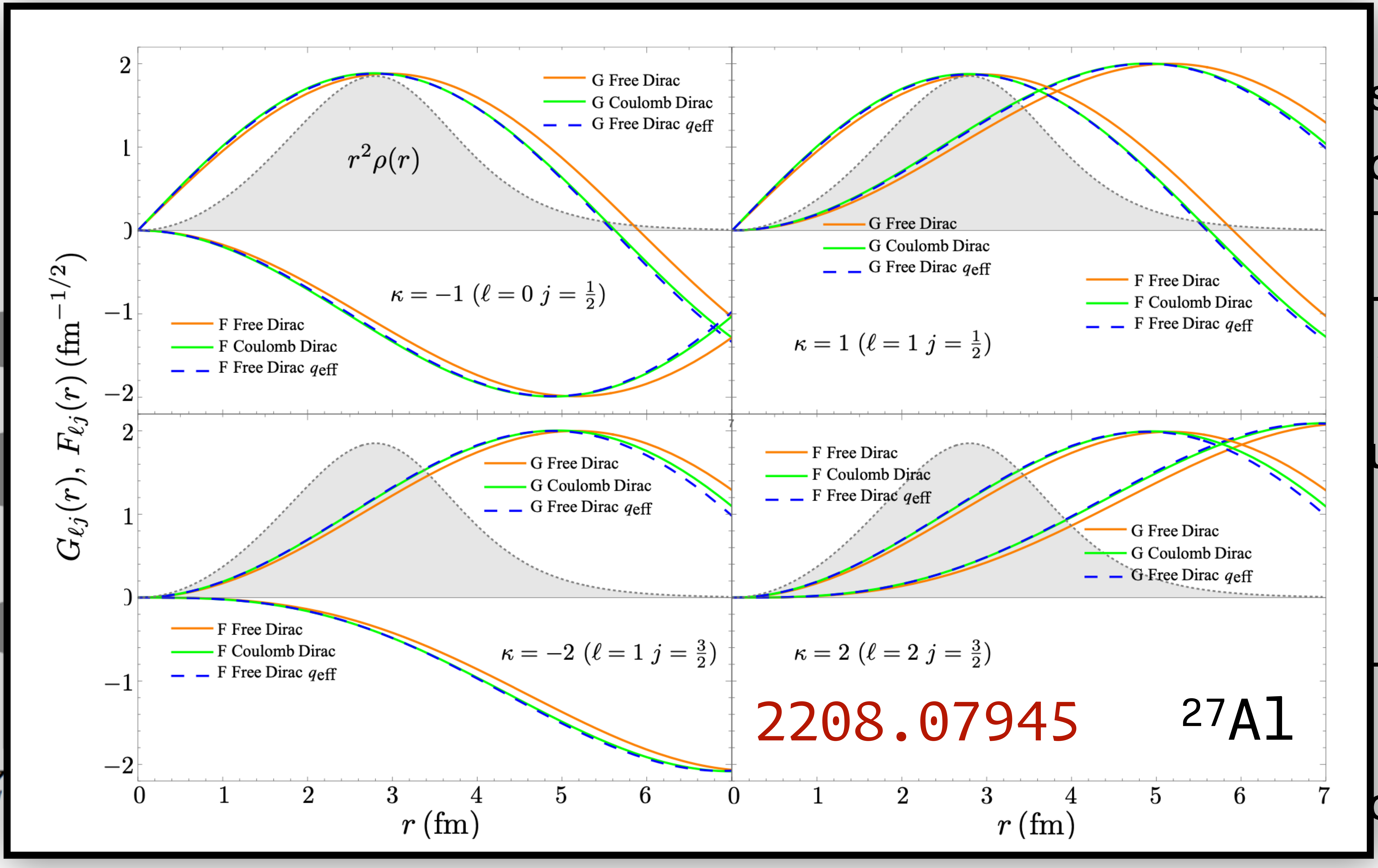
Coulomb distortion ruins this resummation unless a special $\vec{q}$ is chosen

$$q^2_{\text{eff}} = (m_\mu - E^{\text{bind}}_\mu - \bar{V}_C)^2, \quad \bar{V}_C = \frac{\int dr r^2 \rho(r) V_C(r)}{\int dr r^2 \rho(r)}$$

giving solutions with 1-5% accuracy.

Conversion in the field of a nucleus $(A,Z)\,\mu^- \to (A,Z)\,e^-$





s a free plane

can be re-summed

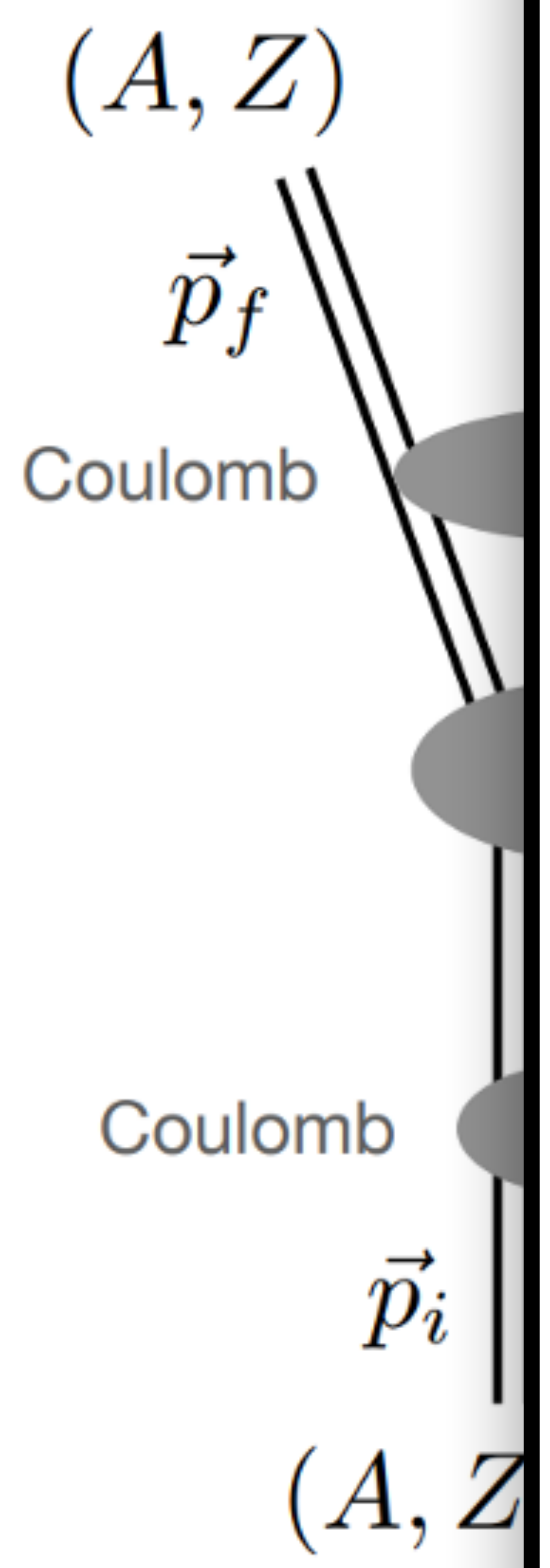
$$\frac{E_e}{2m_e}\begin{pmatrix}\xi\\ \vec{\sigma}\cdot\hat{q}\,\xi\end{pmatrix} e^{i\vec{q}\cdot\vec{r}}.$$

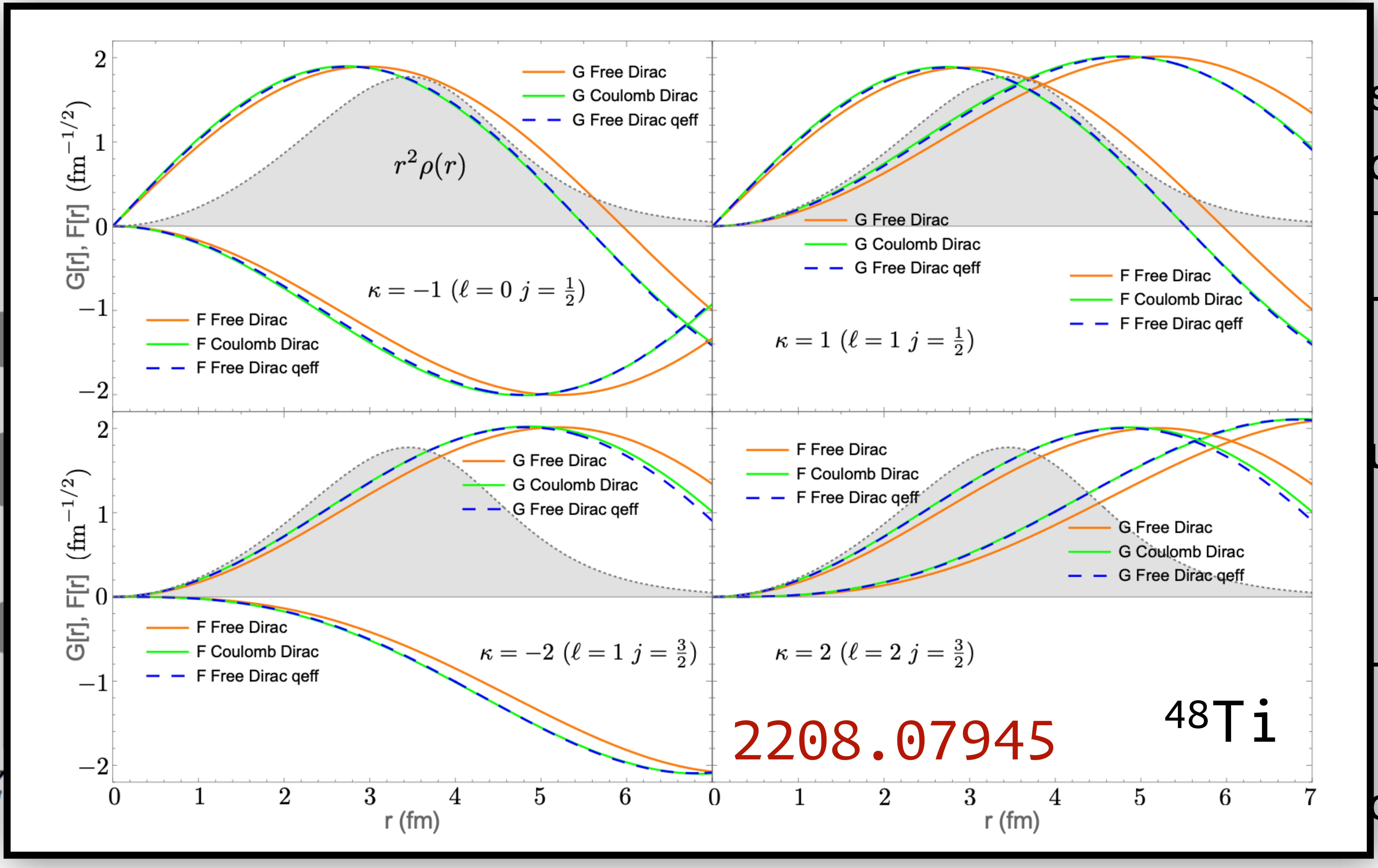
ummation unless a

$$\frac{\int dr r^2\rho(r)V_C(r)}{\int dr r^2\rho(r)}$$

cy.

Conversion in the field of a nucleus $(A,Z)\,\mu^- \to (A,Z)\,e^-$





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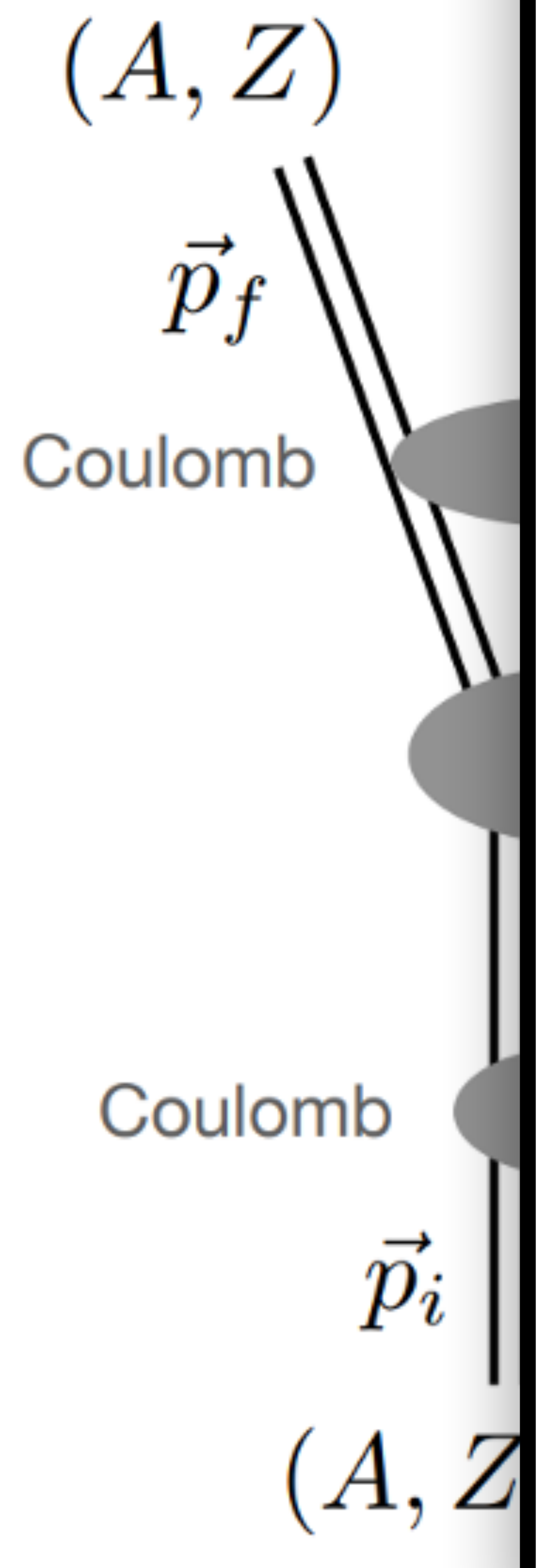

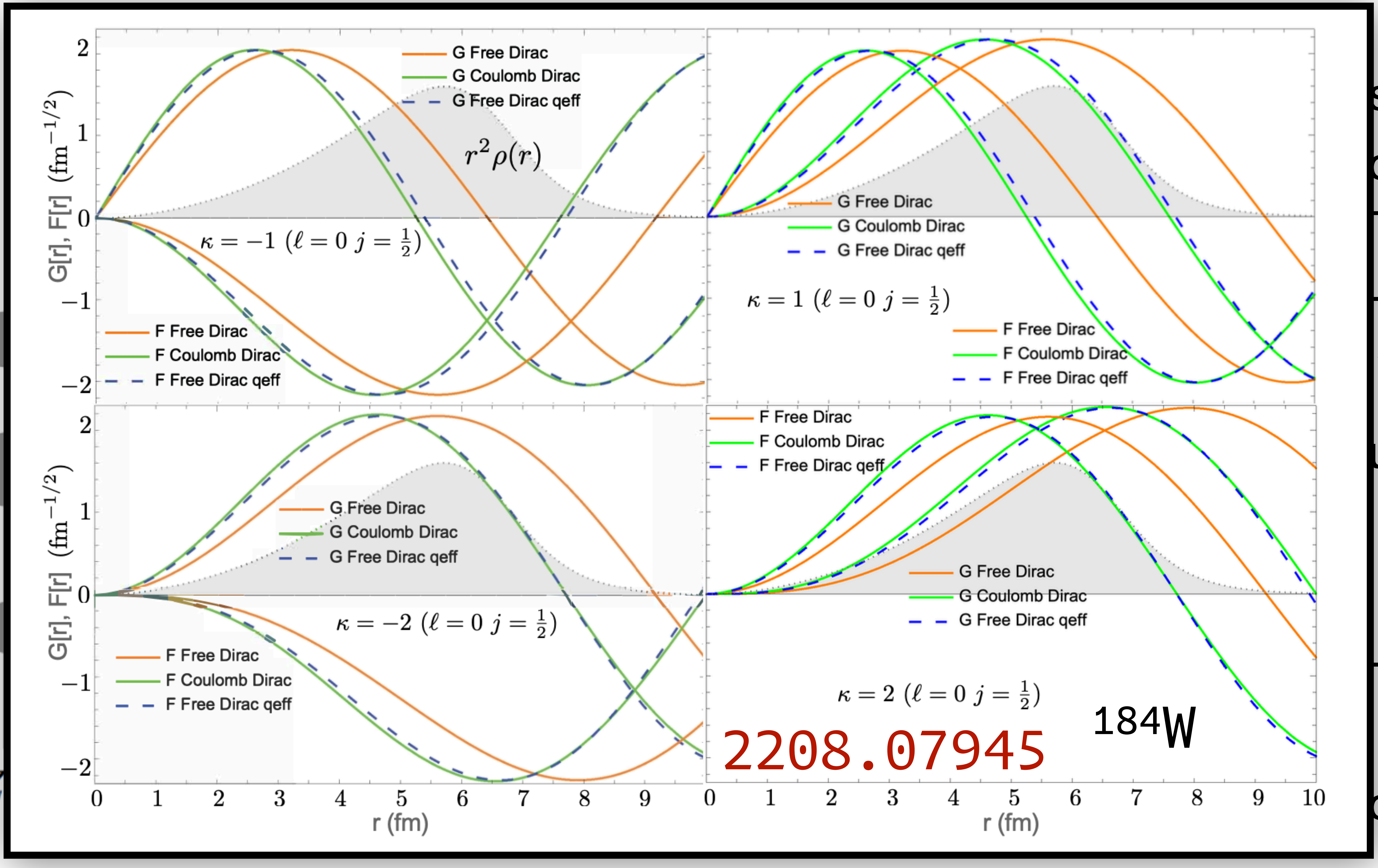

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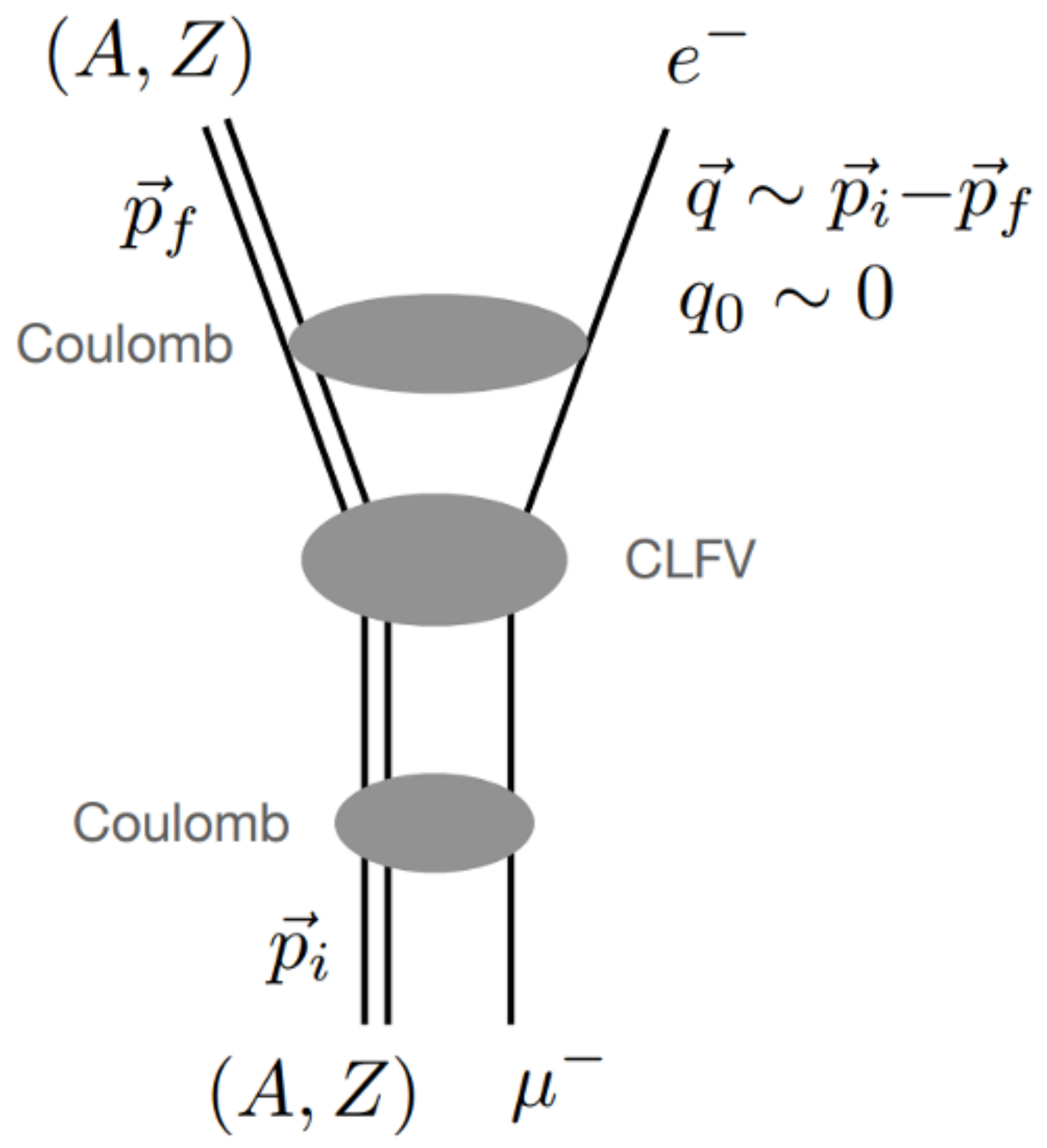
$\frac{E_e}{2m_e} \begin{pmatrix} \xi \\ \vec{\sigma}\cdot\hat{q}\ \xi \end{pmatrix} e^{i\vec{q}\cdot\vec{r}}.$

ummation unless a

$\frac{\int dr r^2 \rho(r) V_C(r)}{\int dr r^2 \rho(r)}$

cy.

The nuclear EFT – Pauli reduction



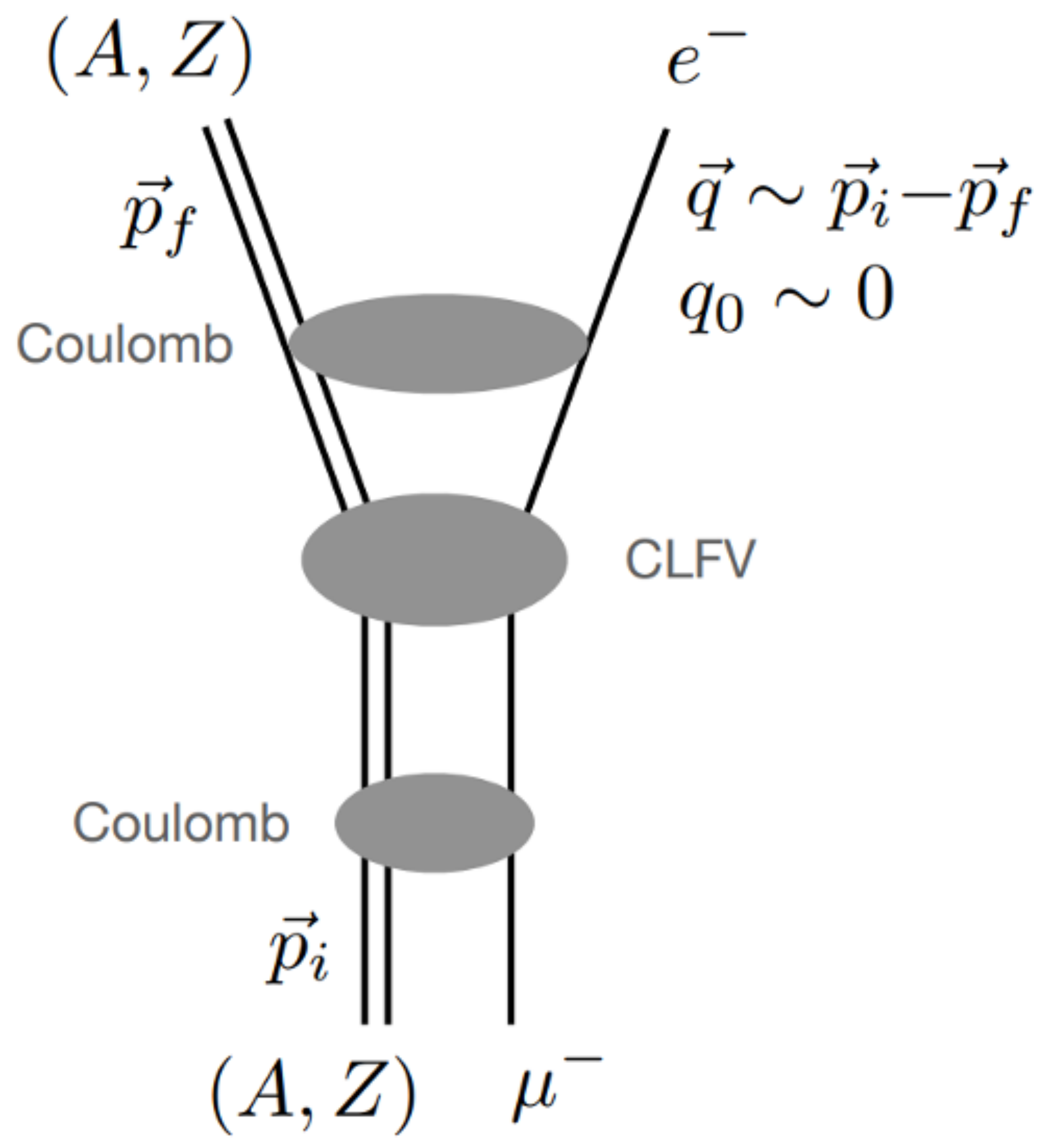
We're left with the following structures

$$\psi_e^{\text{eff}} \sim \sqrt{\frac{E_e}{2m_e}} \begin{pmatrix} \xi_e \\ \vec{\sigma} \cdot \hat{q}\ \xi_e \end{pmatrix} e^{i\vec{q}_{\text{eff}} \cdot \vec{r}}$$

$$\psi_\mu^{\text{upper}} \sim \phi_{1s}^{\text{avg}} \begin{pmatrix} \xi_\mu \\ 0 \end{pmatrix}$$

$$\psi_N \sim \begin{pmatrix} \xi_N \\ \frac{\vec{\sigma} \cdot \vec{p}_N}{2m_N}\ \xi_N \end{pmatrix}$$

The nuclear EFT — Pauli reduction



The Dirac structures decompose as

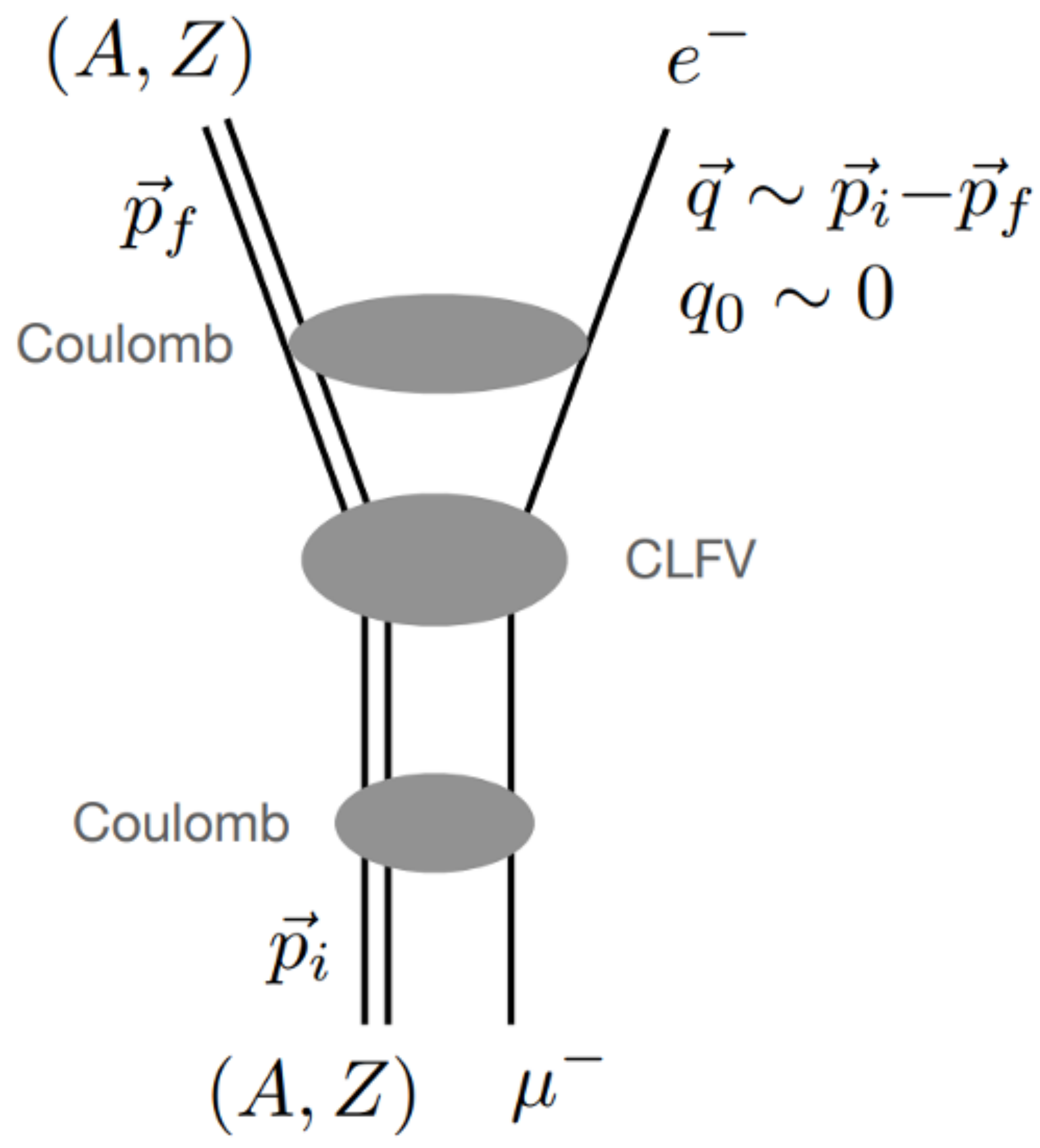
$$\bar{\psi}_e \, \Gamma_{\text{lep}} \, \psi_\mu \longrightarrow \xi_e^\dagger \left(\text{combinations of } \mathbf{1}_L, \vec{\sigma}_L, i\hat{q} \cdot \vec{\sigma}_L\right) \xi_\mu$$

$$\bar{\psi}_N \, \Gamma_N \, \psi_N \longrightarrow \xi_N^\dagger \left(\text{combinations of } \mathbf{1}_N, \vec{\sigma}_N, \vec{v}_N, \vec{v}_N \cdot \vec{\sigma}_N, \vec{v}_N \times \vec{\sigma}_N\right) \xi_N$$

with natural “building blocks”

$$\underbrace{\mathbf{1}_L, \quad \vec{\sigma}_L, \quad i\hat{q}}_{\text{lepton side}} \qquad \times \qquad \underbrace{\mathbf{1}_N, \quad \vec{\sigma}_N, \quad \vec{v}_N, \quad i\hat{q}}_{\text{nucleon side}}.$$

The nuclear EFT – Power counting



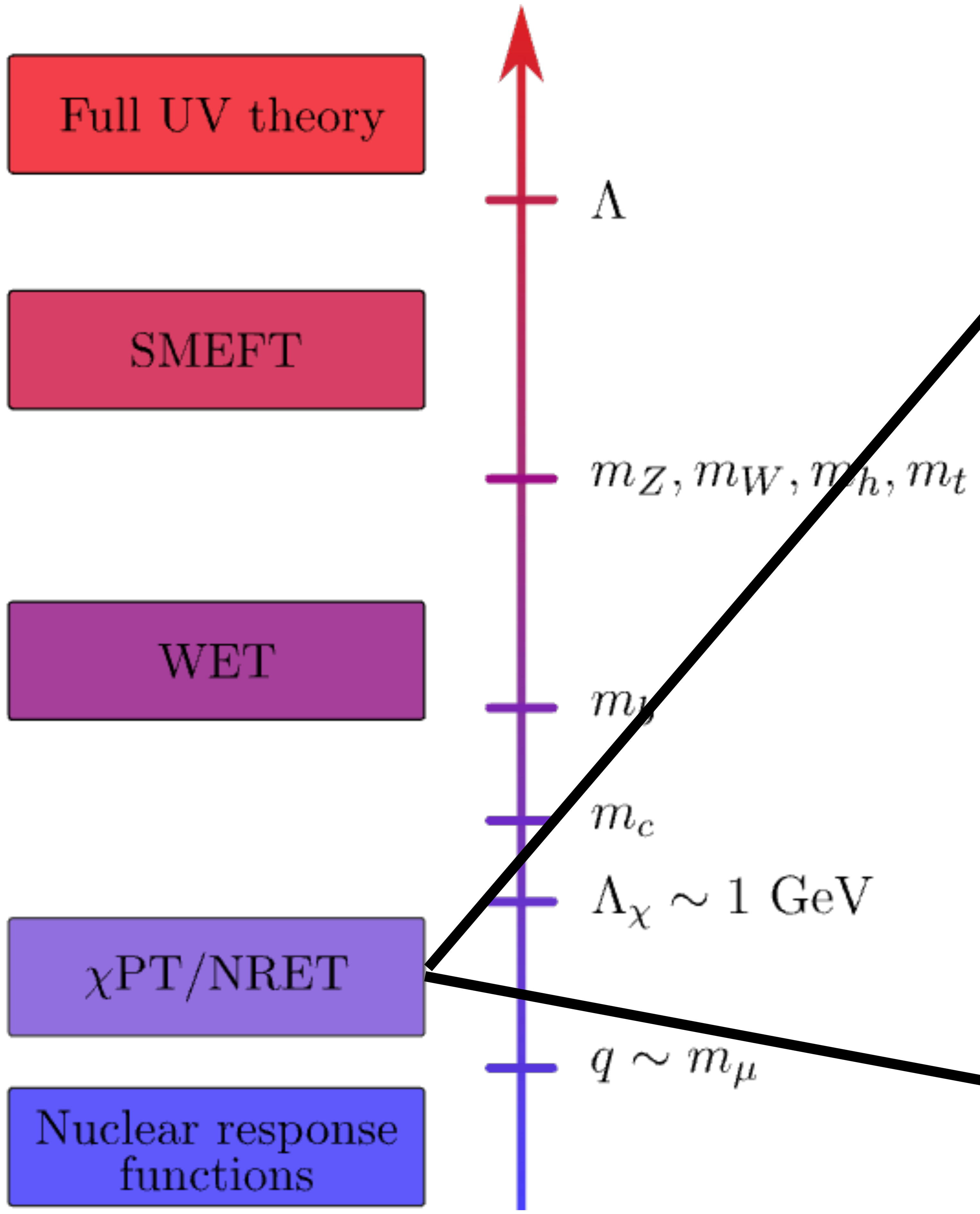
$$\ldots\ \underbrace{\mathbf{1}_L, \quad \vec{\sigma}_L, \quad i\hat{q}}_{\text{lepton side}} \qquad \times \qquad \underbrace{\mathbf{1}_N, \quad \vec{\sigma}_N, \quad \vec{v}_N, \quad i\hat{q}}_{\text{nucleon side}}.$$

In combination with the hierarchical scales in the problem

$$\sqrt{\langle \vec{v}_N^2 \rangle} \approx 0.21 \qquad \gg \qquad v_\mu \sim \frac{Z\alpha}{2} \approx 0.05 \qquad \gg \qquad v_T \sim \frac{q}{M_T} \approx 0.004$$

we can organize the building blocks into a systematic effective theory, truncated at a given order in the expansion. Working through $\mathcal{O}(v_N)$, all scalar combinations give 16 indpendent operators.

The nuclear EFT – Basis



NRET

$$\mathcal{O}_1 = 1_L\, 1_N, \qquad \mathcal{O}_2' = 1_L\, i\hat{q}\cdot\vec{v}_N,$$
$$\mathcal{O}_3 = 1_L\, i\hat{q}\cdot[\vec{v}_N\times\vec{\sigma}_N], \qquad \mathcal{O}_4 = \vec{\sigma}_L\cdot\vec{\sigma}_N,$$
$$\mathcal{O}_5 = \vec{\sigma}_L\cdot(i\hat{q}\times\vec{v}_N), \qquad \mathcal{O}_6 = i\hat{q}\cdot\vec{\sigma}_L\, i\hat{q}\cdot\vec{\sigma}_N,$$
$$\mathcal{O}_7 = 1_L\, \vec{v}_N\cdot\vec{\sigma}_N, \qquad \mathcal{O}_8 = \vec{\sigma}_L\cdot\vec{v}_N,$$
$$\mathcal{O}_9 = \vec{\sigma}_L\cdot(i\hat{q}\times\vec{\sigma}_N), \qquad \mathcal{O}_{10} = 1_L\, i\hat{q}\cdot\vec{\sigma}_N,$$
$$\mathcal{O}_{11} = i\hat{q}\cdot\vec{\sigma}_L\, 1_N, \qquad \mathcal{O}_{12} = \vec{\sigma}_L\cdot[\vec{v}_N\times\vec{\sigma}_N],$$
$$\mathcal{O}_{13}' = \vec{\sigma}_L\cdot(i\hat{q}\times[\vec{v}_N\times\vec{\sigma}_N]), \qquad \mathcal{O}_{14} = i\hat{q}\cdot\vec{\sigma}_L\, \vec{v}_N\cdot\vec{\sigma}_N,$$
$$\mathcal{O}_{15} = i\hat{q}\cdot\vec{\sigma}_L\, i\hat{q}\cdot[\vec{v}_N\times\vec{\sigma}_N], \qquad \mathcal{O}_{16}' = i\hat{q}\cdot\vec{\sigma}_L\, i\hat{q}\cdot\vec{v}_N.$$

The rate

In the NRET basis $\{\mathcal{O}_i\}$, the amplitude factorizes into leptonic and nuclear pieces

$$\mathscr{M} \sim \phi_{1s}^{\text{avg}} \times \sum_i c_i^\tau \, \xi_e^\dagger \, \mathcal{O}_L^{(i)} \, \xi_\mu \times \langle \text{g.s.} | \sum_k \mathcal{O}_N^{(i)}(k) \, e^{-i\vec{q}_{\text{eff}}\cdot\vec{r}_k} \, t^\tau(k) | \text{g.s.} \rangle$$

squaring and summing over lepton spins gives

$$\sum_{\text{spins}} |\mathscr{M}|^2 \propto \sum_{i,j} \underbrace{\text{Tr}\left[\mathcal{O}_L^{(i)} \, \mathcal{O}_L^{(j)\dagger}\right] \times c_i^\tau c_j^{\tau'*}}_{\tilde{R}^{\tau\tau'} \text{ (CLFV coefficients)}} \times \underbrace{\langle \text{g.s.} | \mathcal{O}_N^{(i)} e^{-i\vec{q}_{\text{eff}}\cdot\vec{r}} | \text{g.s.} \rangle \langle \text{g.s.} | \mathcal{O}_N^{(j)} e^{-i\vec{q}_{\text{eff}}\cdot\vec{r}} | \text{g.s.} \rangle^*}_{W^{\tau\tau'} \text{ (nuclear response functions)}}$$

The rate

$$W_{\mathcal{O}}^{\tau\tau'}(q_{\rm eff}) = \frac{4\pi}{2j_N+1}\sum_J \langle j_N || \mathcal{O}_{J;\tau}(q_{\rm eff}) || j_N\rangle \langle j_N || \mathcal{O}_{J;\tau'}(q_{\rm eff}) || j_N\rangle$$

$$\langle j_N || \mathcal{O}_{J;\tau}(q_{\rm eff}) || j_N\rangle = \sum_{a,b} \rho_{ab}^{J,\tau} \langle a || \mathcal{O}_J(q_{\rm eff}) || b\rangle$$

ρ's must be evaluated with your favorite nuclear model

I

nu

so

$$\sum_{\rm spins} |\mathcal{M}|^2 \propto \sum_{i,j} \underbrace{\mathrm{Tr}\left[\mathcal{O}_L^{(i)}\mathcal{O}_L^{(j)\dagger}\right] \times c_i^\tau c_j^{\tau'*}}_{\tilde{R}^{\tau\tau'}\ \text{(CLFV coefficients)}} \times \underbrace{\langle {\rm g.s.} | \mathcal{O}_N^{(i)} e^{-i\vec{q}_{\rm eff}\cdot\vec{r}} | {\rm g.s.}\rangle \langle {\rm g.s.} | \mathcal{O}_N^{(j)} e^{-i\vec{q}_{\rm eff}\cdot\vec{r}} | {\rm g.s.}\rangle^*}_{W^{\tau\tau'}\ \text{(nuclear response functions)}}$$

The rate

After multipole decomposition of $e^{-i\vec{q}_{\text{eff}}\cdot\vec{r}}$ and imposing P and T symmetry of the nuclear ground state, 11 multipole families reduce to 6 allowed responses. At “leading order”

$$\Gamma(\mu \to e) = \frac{1}{2\pi}\frac{q_{\text{eff}}^2}{1+q/M_T}\left|\phi_{1s}^{Z_{\text{eff}}}(\vec{0})\right|^2 \sum_{\tau,\tau'}\Big[R_{MM}^{\tau\tau'}W_{MM}^{\tau\tau'}(q_{\text{eff}}) + R_{\Sigma'\Sigma'}^{\tau\tau'}W_{\Sigma'\Sigma'}^{\tau\tau'}(q_{\text{eff}}) + R_{\Sigma''\Sigma''}^{\tau\tau'}W_{\Sigma''\Sigma''}^{\tau\tau'}(q_{\text{eff}})\Big],$$

$$R_{MM}^{\tau\tau'} = c_1^\tau c_1^{\tau'*} + c_{11}^\tau c_{11}^{\tau'*},$$
$$R_{\Sigma'\Sigma'}^{\tau\tau'} = c_4^\tau c_4^{\tau'*} + c_9^\tau c_9^{\tau'*},$$
$$R_{\Sigma''\Sigma''}^{\tau\tau'} = \left(c_4^\tau - c_6^\tau\right)\left(c_4^{\tau'} - c_6^{\tau'}\right)^* + c_{10}^\tau c_{10}^{\tau'*},$$

The rate

With a consistent truncation to $\mathcal{O}(v_N)$

$$\Gamma(\mu \to e) = \frac{G_F^2}{\pi} \frac{q_{\rm eff}^2}{1+\frac{q}{M_T}} |\phi_{1s}^{Z_{\rm eff}}(\vec{0})|^2 \sum_{\tau=0,1} \sum_{\tau'=0,1} \Bigg\{ \left[\tilde{R}_{MM}^{\tau\tau'} \, W_{MM}^{\tau\tau'}(q_{\rm eff}) + \tilde{R}_{\Sigma''\Sigma''}^{\tau\tau'} \, W_{\Sigma''\Sigma''}^{\tau\tau'}(q_{\rm eff}) + \tilde{R}_{\Sigma'\Sigma'}^{\tau\tau'} \, W_{\Sigma'\Sigma'}^{\tau\tau'}(q_{\rm eff}) \right]$$
$$+ \frac{q_{\rm eff}^2}{m_N^2} \left[\tilde{R}_{\Phi''\Phi''}^{\tau\tau'} \, W_{\Phi''\Phi''}^{\tau\tau'}(q_{\rm eff}) + \tilde{R}_{\tilde{\Phi}'\tilde{\Phi}'}^{\tau\tau'} \, W_{\tilde{\Phi}'\tilde{\Phi}'}^{\tau\tau'}(q_{\rm eff}) + \tilde{R}_{\Delta\Delta}^{\tau\tau'} \, W_{\Delta\Delta}^{\tau\tau'}(q_{\rm eff}) \right]$$
$$- \frac{2q_{\rm eff}}{m_N} \left[\tilde{R}_{\Phi''M}^{\tau\tau'} \, W_{\Phi''M}^{\tau\tau'}(q_{\rm eff}) + \tilde{R}_{\Delta\Sigma'}^{\tau\tau'} \, W_{\Delta\Sigma'}^{\tau\tau'}(q_{\rm eff}) \right] \Bigg\} \quad (59)$$

$$R_{MM}^{\tau\tau'} = c_1^\tau c_1^{\tau'*} + c_{11}^\tau c_{11}^{\tau'*},$$
$$R_{\Sigma'\Sigma'}^{\tau\tau'} = c_4^\tau c_4^{\tau'*} + c_9^\tau c_9^{\tau'*},$$
$$R_{\Sigma''\Sigma''}^{\tau\tau'} = \left(c_4^\tau - c_6^\tau\right)\left(c_4^{\tau'} - c_6^{\tau'}\right)^* + c_{10}^\tau c_{10}^{\tau'*},$$

$$\tilde{R}_{\Phi''\Phi''}^{\tau\tau'} = \tilde{c}_3^\tau \tilde{c}_3^{\tau'*} + (\tilde{c}_{12}^\tau - \tilde{c}_{15}^\tau)(\tilde{c}_{12}^{\tau'*} - \tilde{c}_{15}^{\tau'*})$$
$$\tilde{R}_{\Phi''M}^{\tau\tau'} = {\rm Re}[\tilde{c}_3^\tau \tilde{c}_1^{\tau'*} - (\tilde{c}_{12}^\tau - \tilde{c}_{15}^\tau)\, \tilde{c}_{11}^{\tau'*}]$$
$$\tilde{R}_{\Delta\Delta}^{\tau\tau'} = \tilde{c}_5^\tau \tilde{c}_5^{\tau'*} + \tilde{c}_8^\tau \tilde{c}_8^{\tau'*}$$
$$\tilde{R}_{\Delta\Sigma'}^{\tau\tau'} = {\rm Re}[\tilde{c}_5^\tau \tilde{c}_4^{\tau'*} + \tilde{c}_8^\tau \tilde{c}_9^{\tau'*}]$$
$$\tilde{R}_{\tilde{\Phi}'\tilde{\Phi}'}^{\tau\tau'} = \tilde{c}_{12}^\tau \tilde{c}_{12}^{\tau'*} + \tilde{c}_{13}^\tau \tilde{c}_{13}^{\tau'*}$$

Nuclear effective theory summary

- **Complete:** 16 operators through $\mathcal{O}(v_N)$ with all six nuclear response functions, including three velocity-dependent responses.
- **Consistent:** systematic power counting in $v_N > v_\mu > v_T$.
- **Factorized:** $\Gamma \propto \sum \tilde{R}^{\tau\tau'} \times W^{\tau\tau'}$, CLFV physics cleans separated from nuclear physics. Six independent "nuclear" dials tuned by target selection.
- **Modular:** nuclear structure encoded in ρ's, in principle swappable.

Nuclear effective theory summary

- **Complete:** 16 operators through $\mathcal{O}(v_N)$ with all six nuclear response

$$\Gamma \propto R(c_i) \times W$$

UV physics

Nuclear effective theory summary

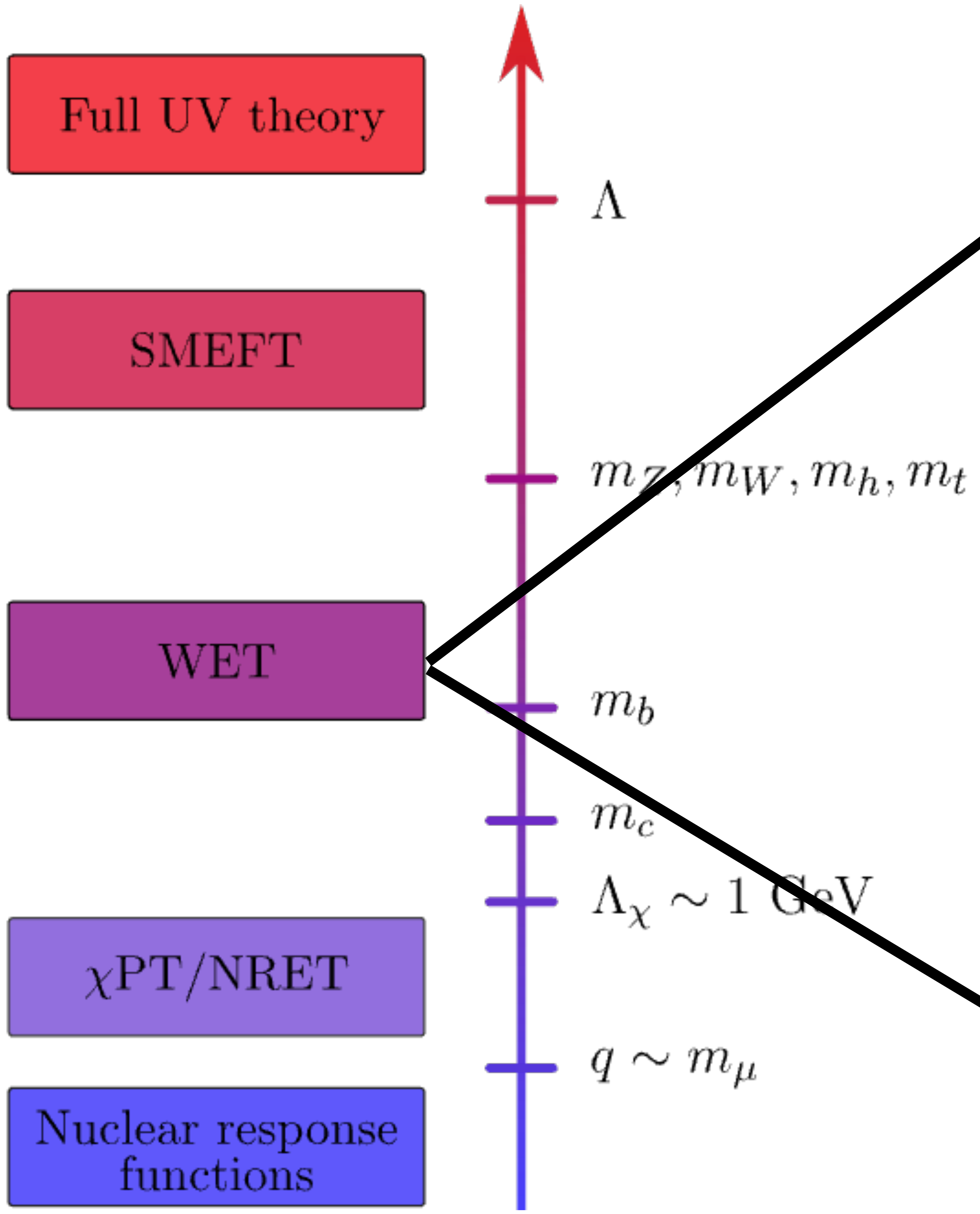
- **Complete:** 16 operators through $\mathcal{O}(v_N)$ with all six nuclear response

First translate to four-spinors e.g.

$\bar{\chi}_e \gamma^\mu \chi_\mu \bar{N} \gamma_\mu N$	$1_L 1_N$ $-\left(\hat{q} 1_L - i\hat{q} \times \vec{\sigma}_L\right) \cdot \left(\vec{v}_N + i\dfrac{\vec{q}}{2m_N} \times \vec{\sigma}_N\right)$	$\mathcal{O}_1$ $+i\mathcal{O}_2' - \mathcal{O}_5 - \dfrac{q}{2m_N}(\mathcal{O}_4 + \mathcal{O}_6)$

going to higher scales, new DOFs…

The tower



WET

$$\mathcal{Q}_1^{(5)} = \frac{e}{8\pi^2}(\bar{e}\sigma^{\alpha\beta}\mu)F_{\alpha\beta}\,, \qquad \mathcal{Q}_2^{(5)} = \frac{e}{8\pi^2}(\bar{e}\sigma^{\alpha\beta}i\gamma_5\mu)F_{\alpha\beta}\,,$$

$$\mathcal{Q}_{1,q}^{(6)} = (\bar{e}\gamma_\alpha\mu)(\bar{q}\gamma^\alpha q)\,, \qquad \mathcal{Q}_{2,q}^{(6)} = (\bar{e}\gamma_\alpha\gamma_5\mu)(\bar{q}\gamma^\alpha q)\,,$$

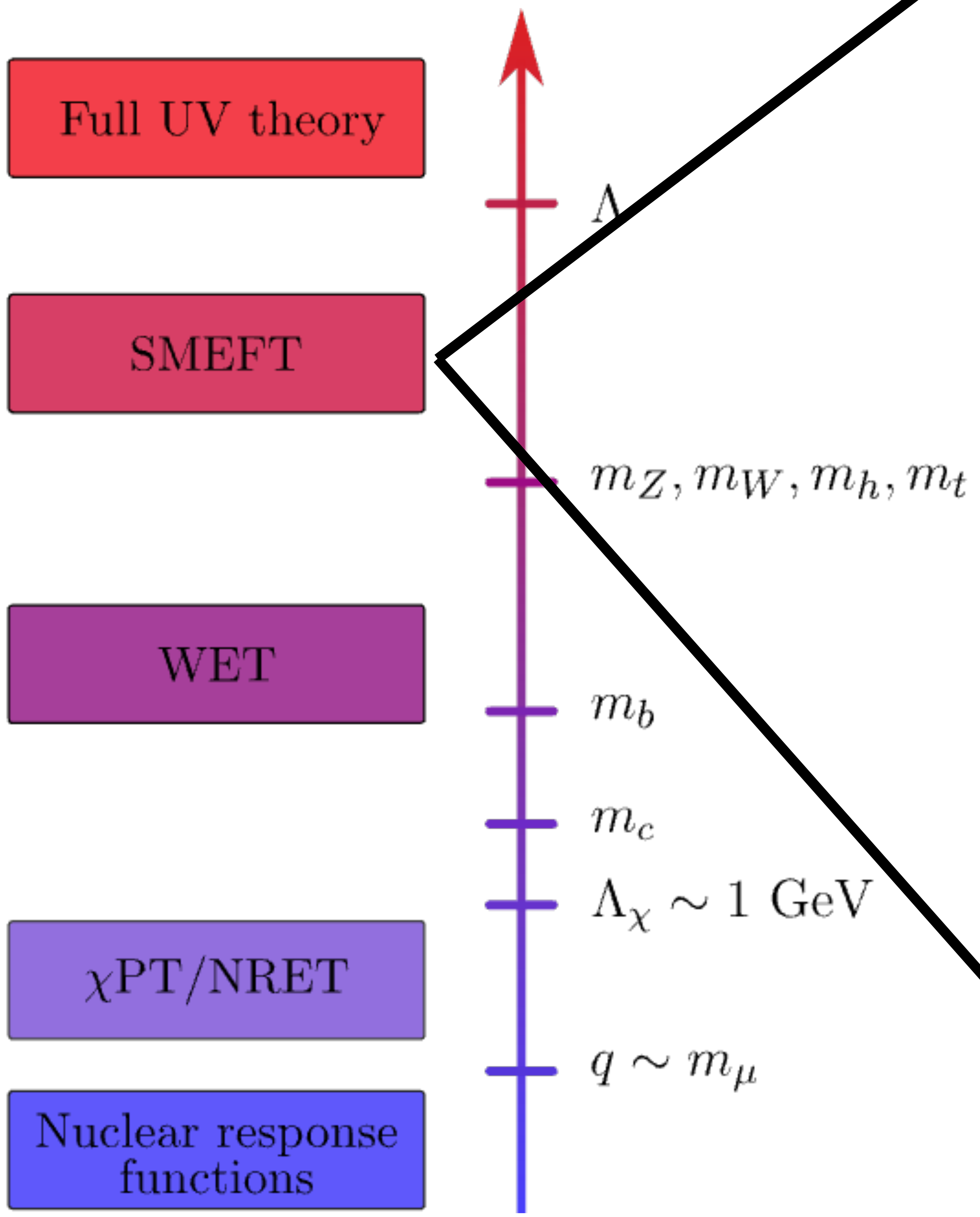
$$\mathcal{Q}_{3,q}^{(6)} = (\bar{e}\gamma_\alpha\mu)(\bar{q}\gamma^\alpha\gamma_5 q)\,, \qquad \mathcal{Q}_{4,q}^{(6)} = (\bar{e}\gamma_\alpha\gamma_5\mu)(\bar{q}\gamma^\alpha\gamma_5 q)\,.$$

$$\mathcal{Q}_{5,q}^{(6)} = (\bar{e}\mu)(\bar{q}q)\,, \qquad \mathcal{Q}_{6,q}^{(6)} = (\bar{e}i\gamma_5\mu)(\bar{q}q)\,,$$

$$\mathcal{Q}_{7,q}^{(6)} = (\bar{e}\mu)(\bar{q}i\gamma_5 q)\,, \qquad \mathcal{Q}_{8,q}^{(6)} = (\bar{e}i\gamma_5\mu)(\bar{q}i\gamma_5 q)\,,$$

$$\mathcal{Q}_{9,q}^{(6)} = (\bar{e}\sigma^{\alpha\beta}\mu)(\bar{q}\sigma_{\alpha\beta}q)\,, \qquad \mathcal{Q}_{10,q}^{(6)} = (\bar{e}i\sigma^{\alpha\beta}\gamma_5\mu)(\bar{q}\sigma_{\alpha\beta}q)\,.$$

The tower



SMEFT

$$
\begin{aligned}
Q_{e\varphi} &: \quad (\varphi^\dagger\varphi)(\bar{L}_2 e_R \varphi), \quad (\varphi^\dagger\varphi)(\bar{\ell}_1 \mu_R \varphi),\\
Q_{eW} &: \quad (\bar{\ell}_2 \sigma^{\mu\nu} e_R)\tau^I \varphi W^I_{\mu\nu}, \quad (\bar{\ell}_1 \sigma^{\mu\nu} \mu_R)\tau^I \varphi W^I_{\mu\nu},\\
Q_{eB} &: \quad (\bar{\ell}_2 \sigma^{\mu\nu} e_R)\varphi B_{\mu\nu}, \quad (\bar{\ell}_1 \sigma^{\mu\nu} \mu_R)\varphi B_{\mu\nu},
\end{aligned}
$$

$$
\begin{aligned}
Q^{(1)}_{\varphi\ell} &: \quad (\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar{\ell}_2 \gamma^\mu \ell_1),\\
Q^{(3)}_{\varphi\ell} &: \quad (\varphi^\dagger i \overleftrightarrow{D}^I_\mu \varphi)(\bar{\ell}_2 \gamma^\mu \tau^I \ell_1),\\
Q_{\varphi e} &: \quad (\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar{\mu}_R \gamma^\mu \tau^I e_R),\\
Q^{(1)}_{\ell q} &: \quad (\bar{\ell}_2 \gamma^\mu \ell_1)(\bar{q}\gamma^\mu q),\\
Q^{(3)}_{\ell q} &: \quad (\bar{\ell}_2 \gamma^\mu \tau^I \ell_1)(\bar{q}\gamma^\mu \tau^I q),\\
Q_{eu} &: \quad (\bar{\mu}_R \gamma^\mu e_R)(\bar{u}_R \gamma^\mu u_R),\\
Q_{ed} &: \quad (\bar{\mu}_R \gamma^\mu e_R)(\bar{d}_R \gamma^\mu d_R),\\
Q_{\ell u} &: \quad (\bar{\ell}_2 \gamma^\mu \ell_1)(\bar{u}_R \gamma^\mu u_R),\\
Q_{\ell d} &: \quad (\bar{\ell}_2 \gamma^\mu \ell_1)(\bar{d}_R \gamma^\mu d_R),\\
Q_{qe} &: \quad (\bar{q}\gamma^\mu q)(\bar{\mu}_R \gamma^\mu e_R),\\
Q_{\ell edq} &: \quad (\bar{\ell}_2 e_R)(\bar{d}_R q), \quad (\bar{\ell}_1 \mu_R)(\bar{d}_R q),\\
Q^{(1)}_{\ell equ} &: \quad (\bar{\ell}^j_2 e_R)\varepsilon_{jk}(\bar{q}^k u), \quad (\bar{\ell}^j_1 \mu_R)\varepsilon_{jk}(\bar{q}^k u),\\
Q^{(3)}_{\ell equ} &: \quad (\bar{\ell}^j_2 \sigma_{\mu\nu} e_R)\varepsilon_{jk}(\bar{q}^k \sigma^{\mu\nu} u), \quad (\bar{\ell}^j_1 \sigma_{\mu\nu} \mu_R)\varepsilon_{jk}(\bar{q}^k \sigma^{\mu\nu} u),
\end{aligned}
$$

Matching

- **Only missing piece is a non-perturbative matching between WET and NRET**
- **Parameterize with nuclear form factors**

$$\langle N'|\bar{q}\gamma^\mu q|N\rangle = \bar{u}'_N\Big[F_1^{q/N}(q^2)\gamma^\mu - \frac{i}{2m_N}F_2^{q/N}(q^2)\sigma^{\mu\nu}q_\nu\Big]u_N\,,$$

$$\langle N'|\bar{q}\gamma^\mu\gamma_5 q|N\rangle = \bar{u}'_N\Big[F_A^{q/N}(q^2)\gamma^\mu\gamma_5 - \frac{1}{2m_N}F_{P'}^{q/N}(q^2)\gamma_5 q^\mu\Big]u_N\,,$$

$$\langle N'|m_q\bar{q}q|N\rangle = F_S^{q/N}(q^2)\,\bar{u}'_N u_N\,,$$

$$\langle N'|m_q\bar{q}i\gamma_5 q|N\rangle = F_P^{q/N}(q^2)\,\bar{u}'_N i\gamma_5 u_N\,,$$

$$\langle N'|\frac{\alpha_s}{12\pi}G^{a\mu\nu}G^a_{\mu\nu}|N\rangle = F_G^N(q^2)\,\bar{u}'_N u_N\,,$$

$$\langle N'|\frac{\alpha_s}{8\pi}G^{a\mu\nu}\tilde{G}^a_{\mu\nu}|N\rangle = F_{\tilde{G}}^N(q^2)\,\bar{u}'_N i\gamma_5 u_N\,,$$

$$\langle N'|\bar{q}\sigma^{\mu\nu}q|N\rangle = \bar{u}'_N\Big[\hat{F}_{T,0}^{q/N}(q^2)\,\sigma^{\mu\nu} - \frac{i}{2m_N}\gamma^{[\mu}q^{\nu]}\hat{F}_{T,1}^{q/N}(q^2) - \frac{i}{m_N^2}q^{[\mu}k_{12}^{\nu]}\hat{F}_{T,2}^{q/N}(q^2)\Big]u_N\,,$$

$$\langle N'|\frac{\alpha}{12\pi}F^{\mu\nu}F_{\mu\nu}|N\rangle = F_\gamma^N(q^2)\,\bar{u}'_N u_N\,,$$

$$\langle N'|\frac{\alpha}{8\pi}F^{\mu\nu}\tilde{F}_{\mu\nu}|N\rangle = F_{\tilde{\gamma}}^N(q^2)\,\bar{u}'_N i\gamma_5 u_N\,.$$

Matching

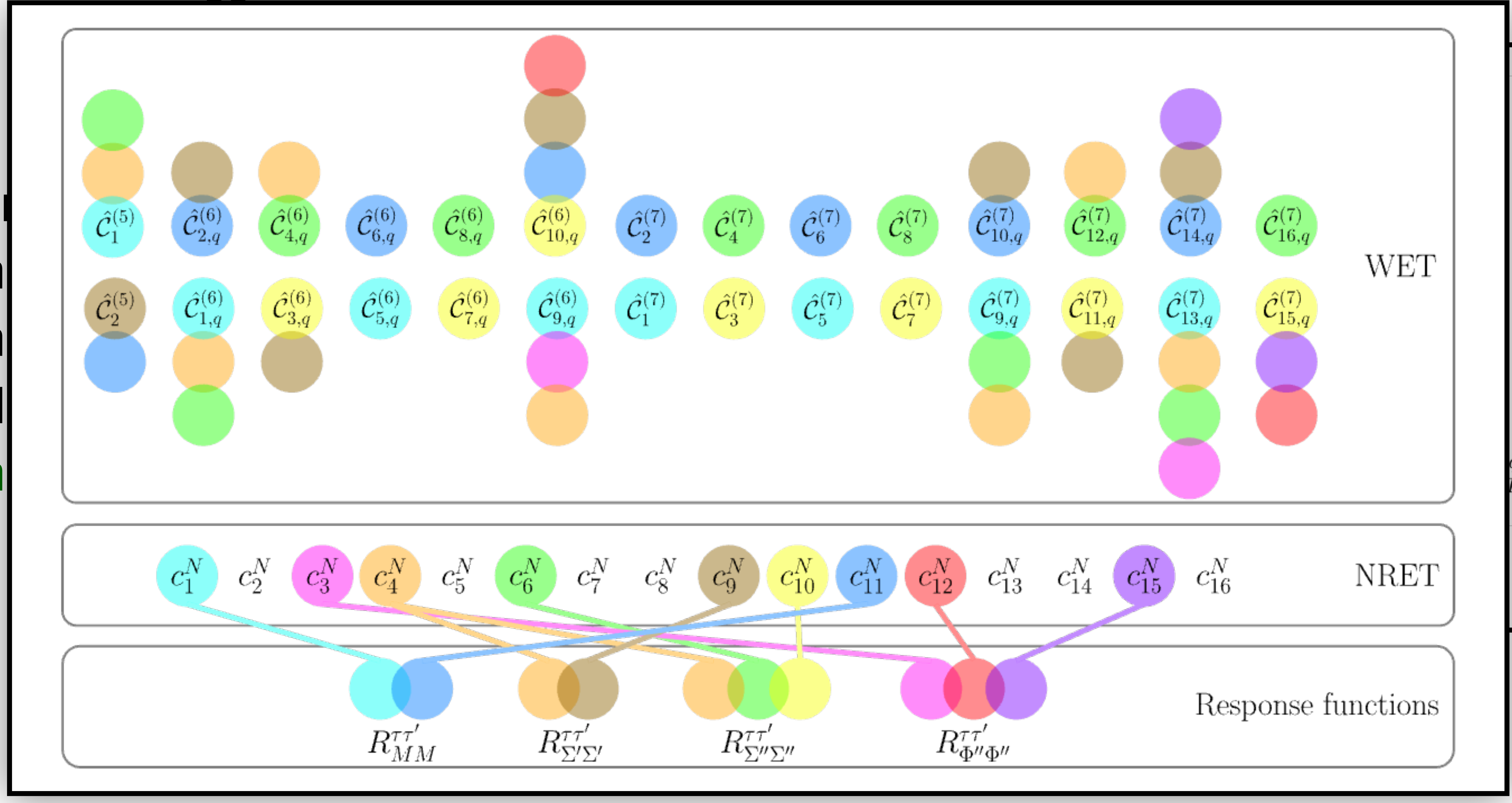
- Only missing piece is a non-perturbative matching between WET and NRET
- Plug-and-chug to obtain matching expressions …

$$
\begin{aligned}
c_1^N &= -\frac{\alpha}{\pi q}\hat{\mathcal{C}}_1^{(5)}\sum_q Q_q F_1^{q/N} + \sum_q \hat{\mathcal{C}}_{1,q}^{(6)} F_1^{q/N} + \sum_q \frac{1}{m_q}\hat{\mathcal{C}}_{5,q}^{(6)} F_S^{q/N} \\
&\quad - \frac{q}{m_N}\sum_q \hat{\mathcal{C}}_{9,q}^{(6)}\left(\hat{F}_{T,0}^{q/N} - \hat{F}_{T,1}^{q/N} + 4\hat{F}_{T,2}^{q/N}\right) \\
&\quad + \hat{\mathcal{C}}_1^{(7)} F_G^N + \hat{\mathcal{C}}_5^{(7)} F_\gamma^N + (q + m_+)\sum_q \hat{\mathcal{C}}_{9,q}^{(7)} F_1^{q/N} \\
&\quad - \frac{q^2}{2m_N}\sum_q \hat{\mathcal{C}}_{13,q}^{(7)}\left[\hat{F}_{T,0}^{q/N} - \hat{F}_{T,1}^{q/N} + \left(4 + \frac{q^2}{m_N^2}\right)\hat{F}_{T,2}^{q/N}\right], \\
c_2^N &= i\left[\sum_q \hat{\mathcal{C}}_{1,q}^{(6)} F_1^{q/N} + m_+ \sum_q \hat{\mathcal{C}}_{9,q}^{(7)} F_1^{q/N} + \frac{q^2}{2m_N}\sum_q \hat{\mathcal{C}}_{13,q}^{(7)}\left(\hat{F}_{T,1}^{q/N} - 4\hat{F}_{T,2}^{q/N}\right)\right],
\end{aligned}
$$

$\vdots$

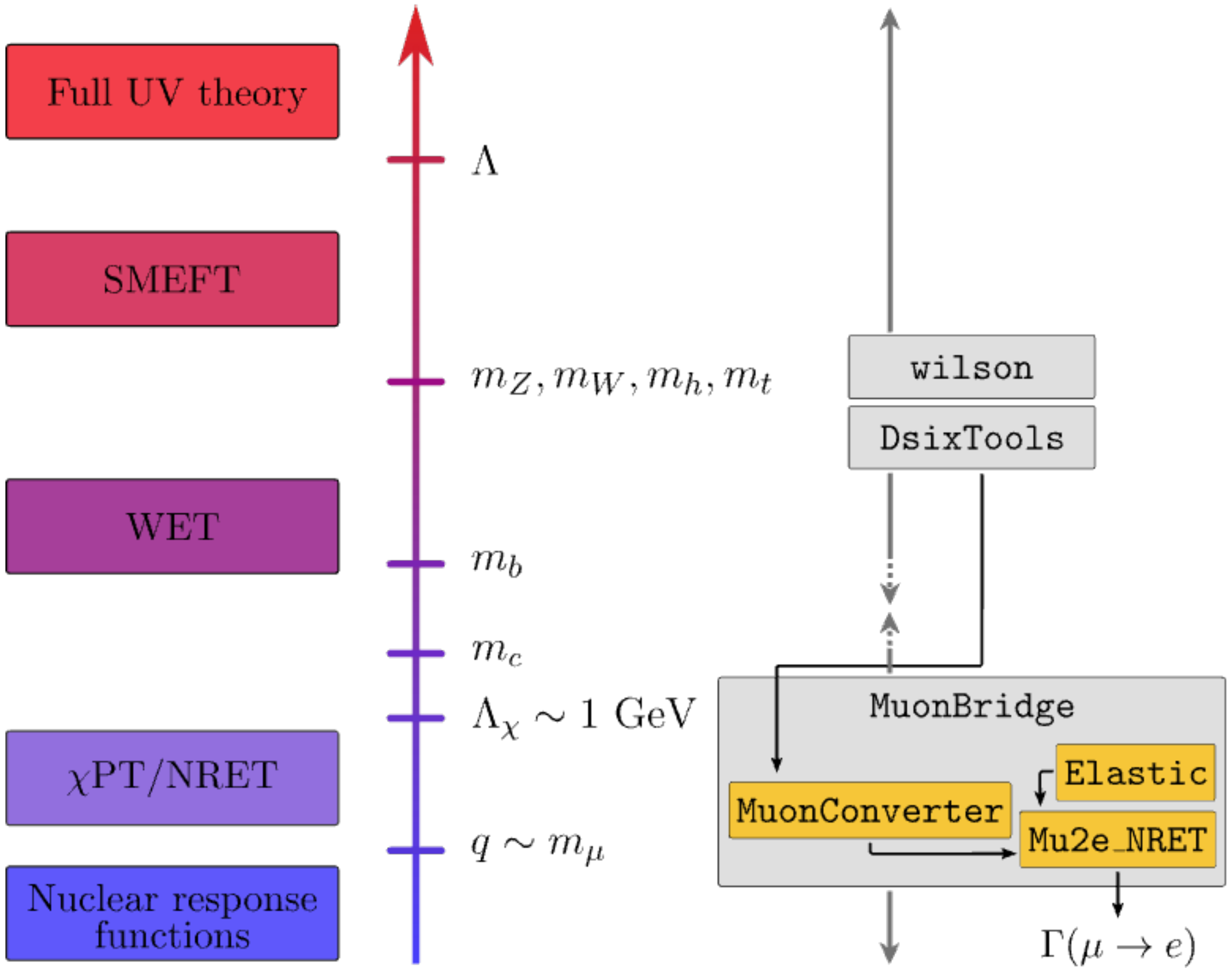
Matching

- Only
  a non
  match
  and N
- Match



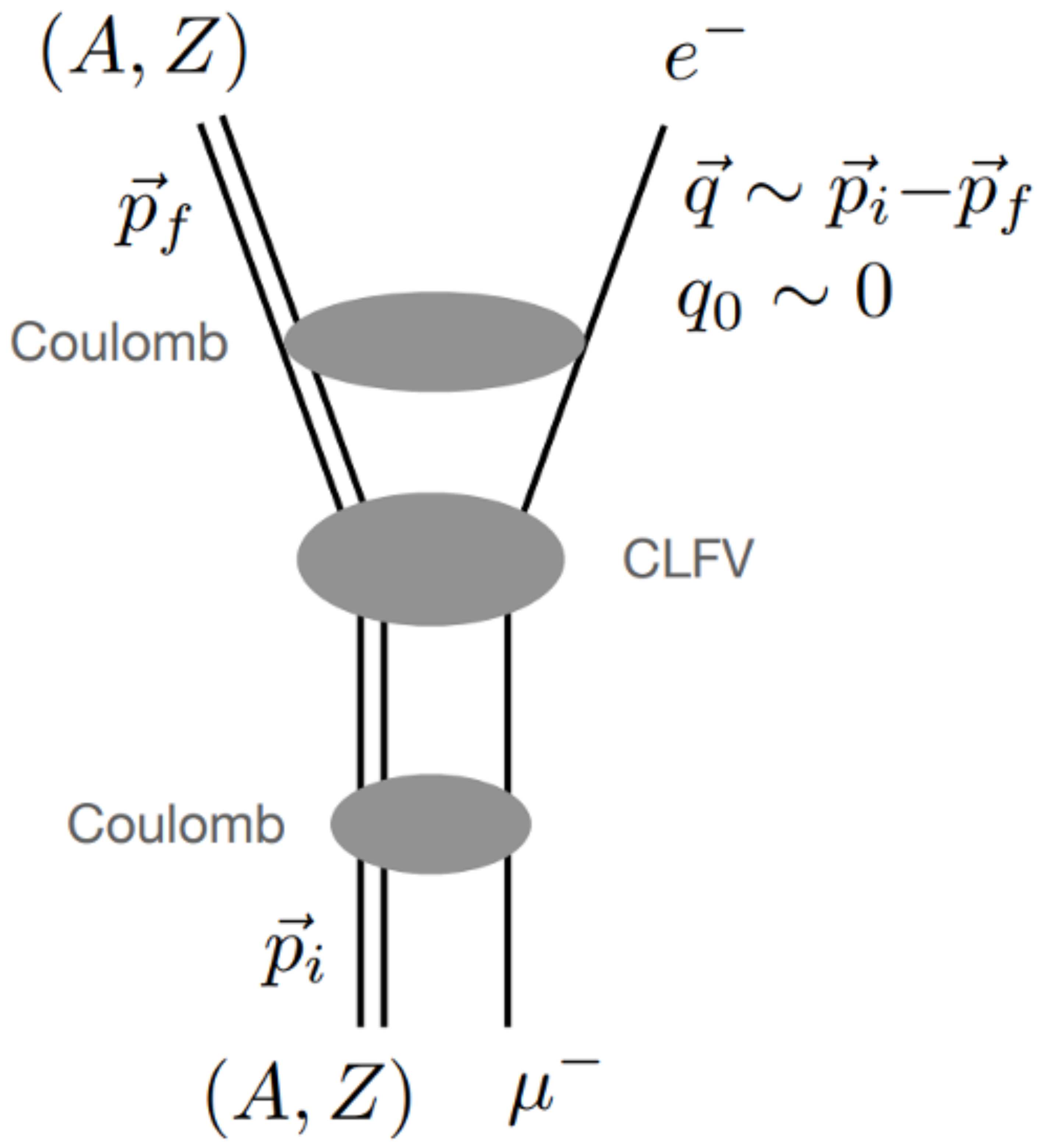
MuonBridge

- **Three components:**
  - Elastic (one-body density matrices, ρ's)
  - Mu2e_NRET (computes conversion rate)
  - MuonConverter (matches WET to NRET and facilitates interface with existing EFT software)



Back-ups

Conversion in the field of a nucleus $(A,Z)\,\mu^- \to (A,Z)\,e^-$



"Fourier trick"

The leptonic and nuclear pieces can be "separated" by inserting a δ-function and Fourier transforming

$$\mathscr{M} \propto \int \frac{d^3q}{(2\pi)^3} \left[\int d^3x\, e^{i\vec{q}\cdot\vec{x}}\, \bar{\psi}_e \mathcal{O}_{\rm lep} \psi_\mu \right] \times \left[\langle {\rm g.s.} | \mathcal{O}_N(i) e^{-i\vec{q}\cdot\vec{r}_i} | {\rm g.s.} \rangle \right].$$

Expanding in spherical harmonics

$$e^{i\vec{q}\cdot\vec{x}} = 4\pi \sum_{L=0}^{\infty} \sum_{M=-L}^{L} i^L j_L(qx)\, Y^*_{LM}(\hat{q})\, Y_{LM}(\hat{x})$$

gives a multipole expansion.