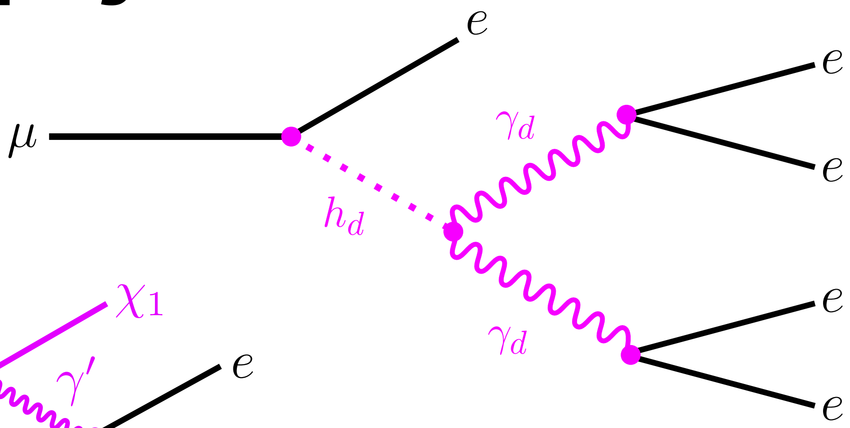
Rare μ decays: high-intensity probes of heavy and light new physics

University of Maryland
EPT Seminar
November 25th, 2024



Tony Menzo

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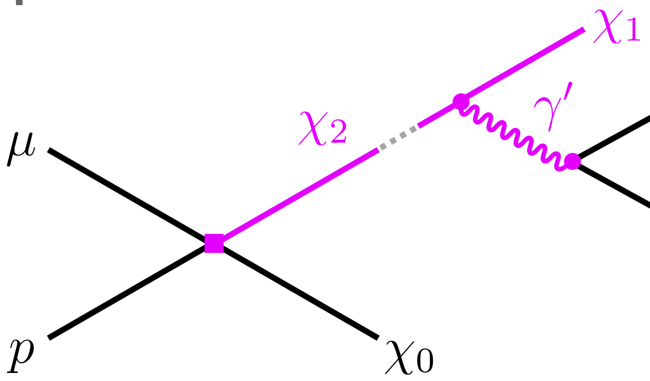
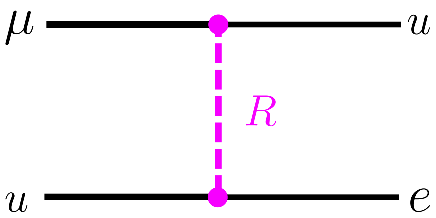




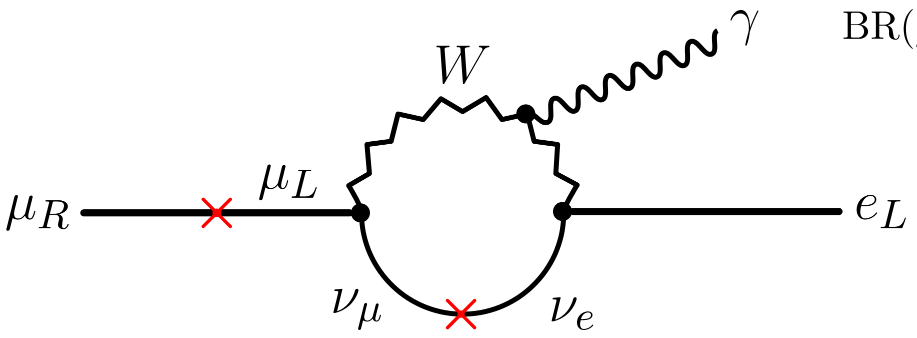
Based on 2306.15631, 2406.13818 and 2407.03450 with Wick Haxton, Ken McElvain, Evan Rule, Patrick Fox, Matheus Hostert, Maxim Pospelov, and Jure Zupan

μ → e

- The Standard Model (SM) has an accidental global flavor symmetry

$$U(1)_e \times U(1)_\mu \times U(1)_\tau$$

- Because $m_\nu \neq 0$ charged-lepton-flavor violation (CLFV) can occur at one-loop



$$\mathrm{BR}(\mu \to e\gamma) \simeq \frac{3\alpha}{32\pi M_W^4} |U_{\mu 3} U^*_{e3} \Delta m^2_{31} + U_{\mu 2} U^*_{e2} \Delta m^2_{21}|^2 \simeq 10^{-54}$$

$$\mathrm{BR}(\mu \to eee) \simeq \frac{\alpha}{3\pi} \left(\log \frac{m_\mu^2}{m_e^2} - 3 \right) \times \mathrm{BR}(\mu \to e\gamma)$$

$$\mathrm{CR}(\mu \mathrm{N} \to e \mathrm{N}) \simeq \alpha \times \mathrm{BR}(\mu \to e\gamma)$$

Bottom line: Observing CLFV = new physics

Experimental status

- **Mu → E Gamma (MEG) @ PSI - μ → eɣ**

Projected: $\mathrm{BR}(\mu^+ \to e^+\gamma) \lesssim 6 \times 10^{-14}$ $(\Gamma(\mu \to e) \lesssim 10^{-10}\ \mathrm{Hz})$

- **Mu3e @ PSI - μ → eee**

Projected: $\mathrm{BR}(\mu^+ \to e^+e^-e^+) \lesssim 10^{-16}$ $(\Gamma(\mu \to e) \lesssim 10^{-12}\ \mathrm{Hz})$

- **Mu2e @ Fermilab, COMET @ J-PARC - Nμ → Ne**

Projected: $\mathrm{CR}(\mu^- \mathrm{Al} \to e^- \mathrm{Al}) \lesssim 10^{-18} - 10^{-17}$ $(\Gamma(\mu \to e) \lesssim 10^{-13}\ \mathrm{Hz})$

Experimental status

- **Mu → E Gar**

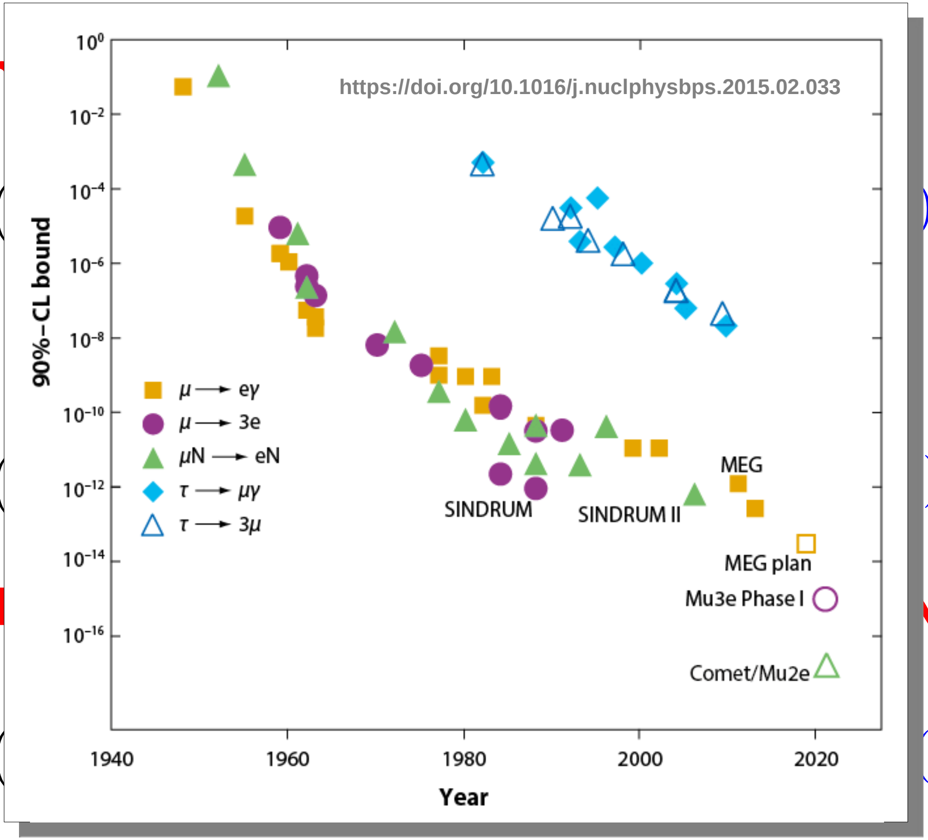
  Projected: $\mathrm{BR}($ … $) \lesssim 10^{-10}\ \mathrm{Hz})$

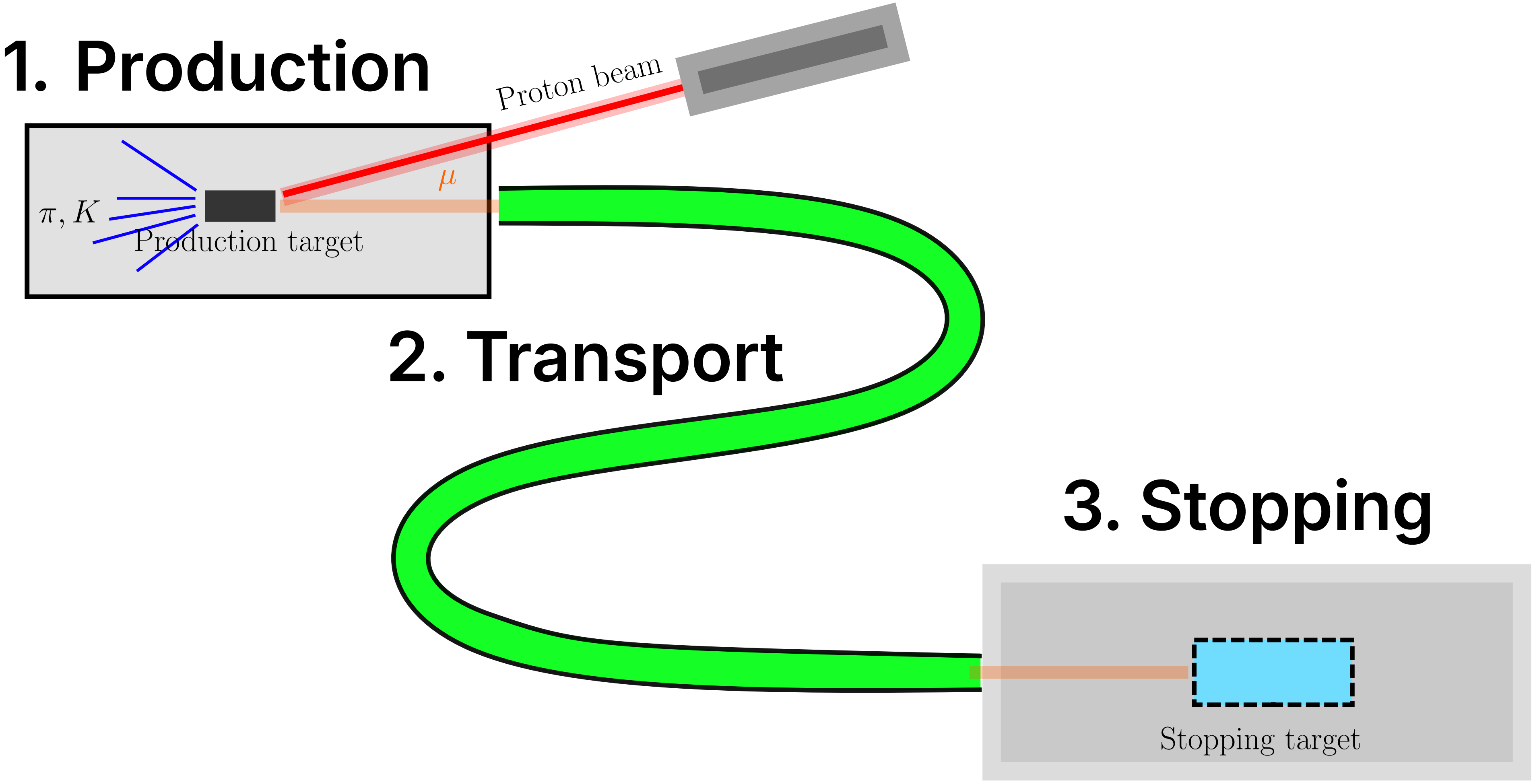
- **Mu3e @ PSI**

  Projected: $\mathrm{BR}($ … $) \lesssim 10^{-12}\ \mathrm{Hz})$

- **Mu2e @ Ferı** … **Nμ → Ne**

  Projected: $\mathrm{CR}($ … $\Gamma(\mu \to e) \lesssim 10^{-13}\ \mathrm{Hz})$

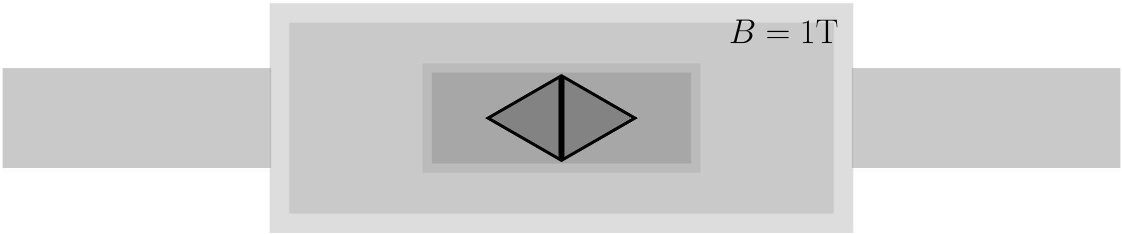



1. Production
Proton beam
μ
π, K
Production target
2. Transport
3. Stopping
Stopping target

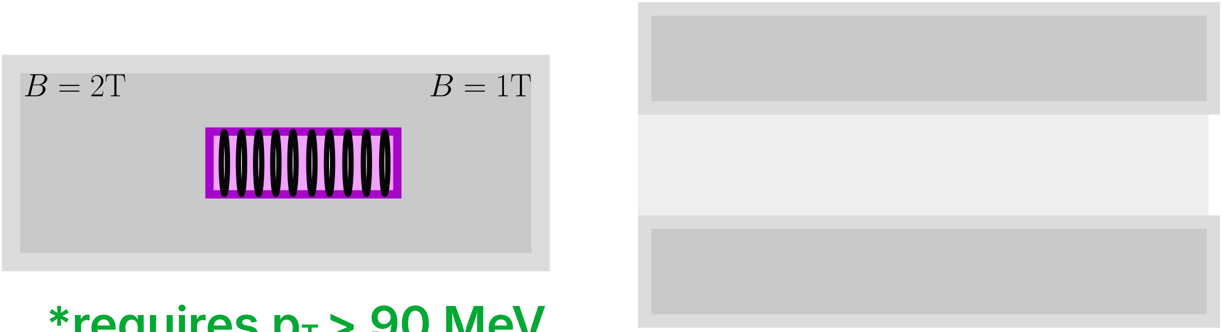
4. Detection (phenomenology is influenced by detector)

Mu3e (μ^+)

*requires $p_T > 10$ MeV



Mu2e (μ^-)



*requires $p_T > 90$ MeV

Exotic μ → e

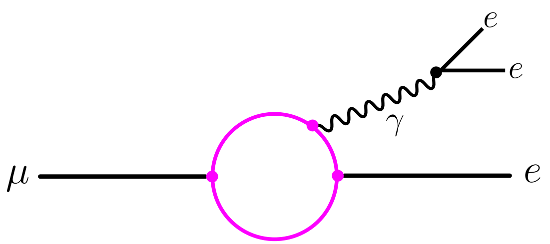
In the *Landscape*, CLFV is common.

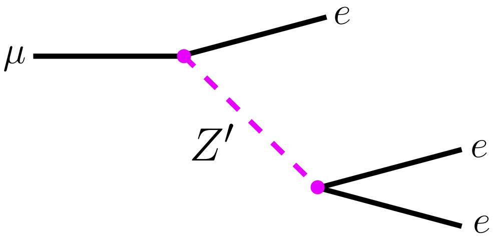
The dichotomy:

- Heavy: SM + "irrelevant" operators

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \sum_{d>4} \frac{1}{\Lambda^{d-4}} O^{(d)}$$

- "Photonic" – e.g. SUSY, massive neutrinos, …
- "Contact" – e.g. Z′, leptoquarks, …



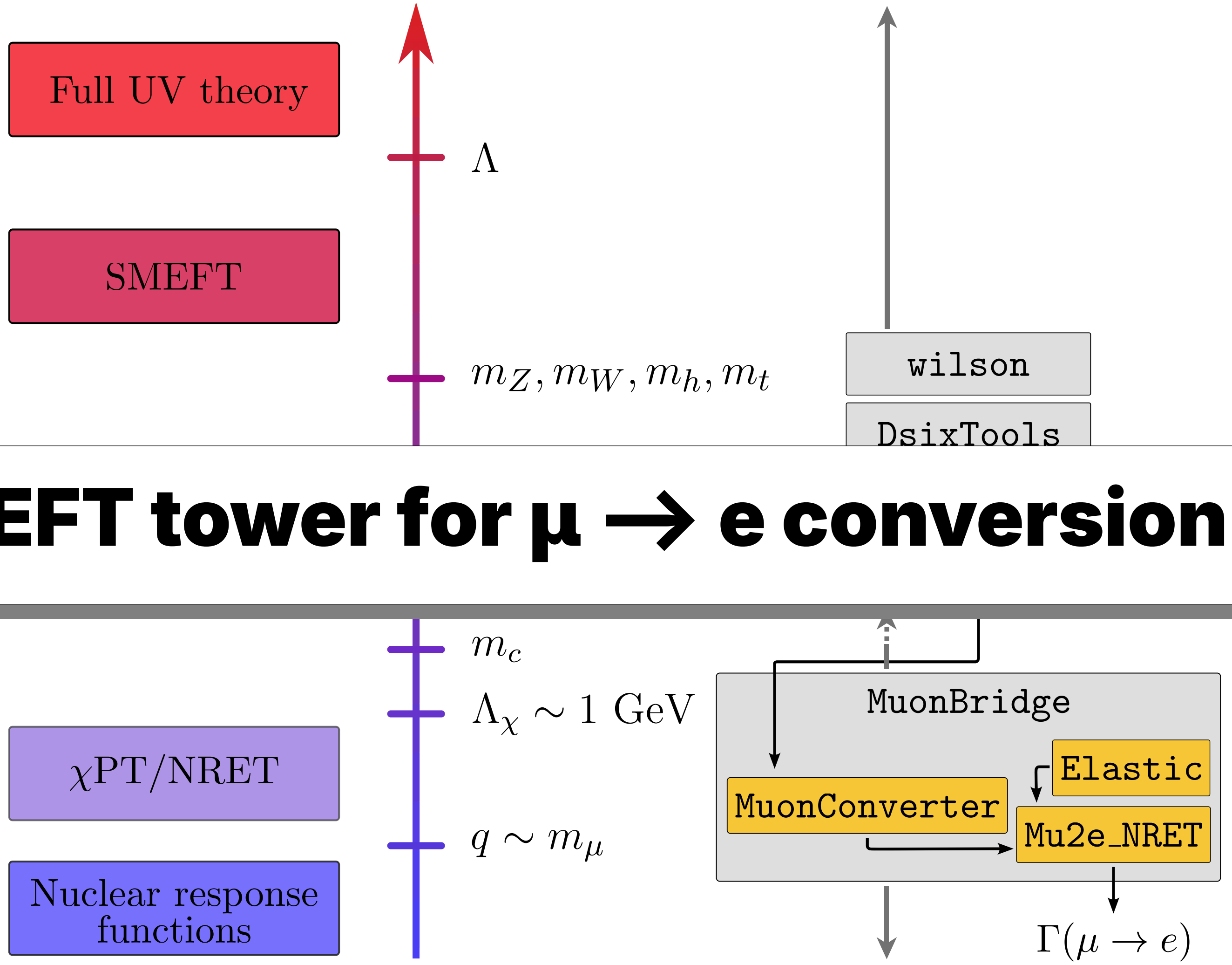


- Light: SM + new fields with weakly coupled interactions

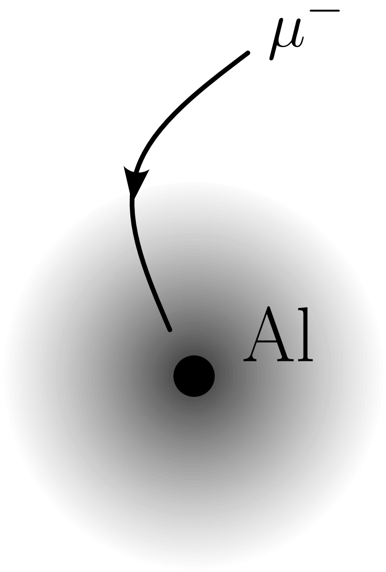
*Naturalness demands an explanation! Appeal to high scale physics to explain smallness

EFT tower for μ → e conversion

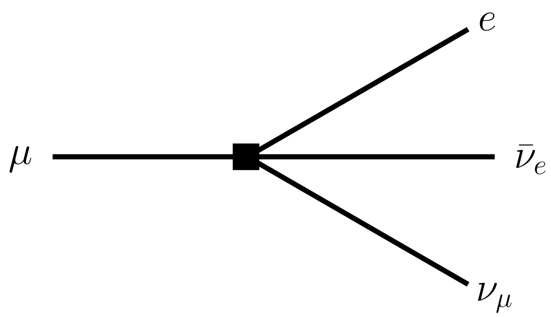
2406.13818



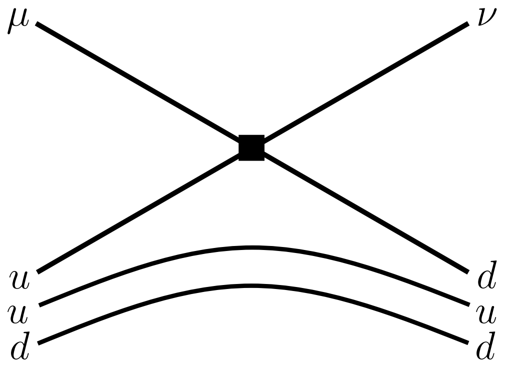
A trapped muon can...



1. Decay in orbit



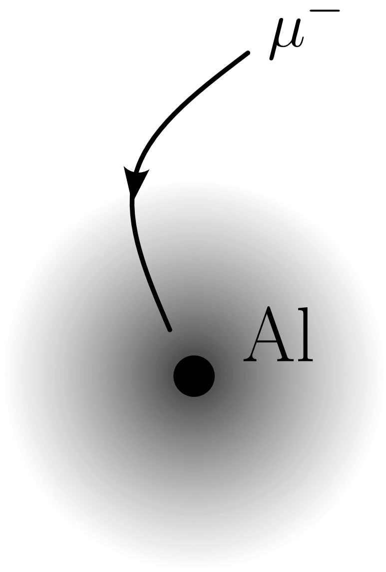
2. Be captured by the nucleus

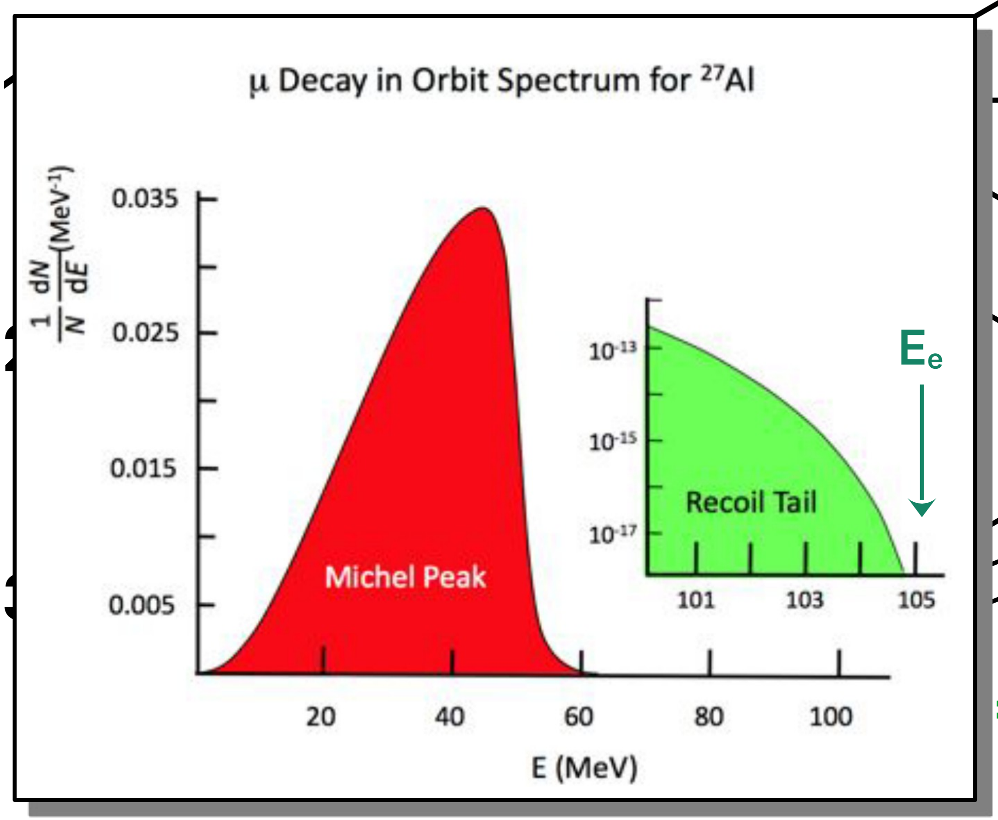


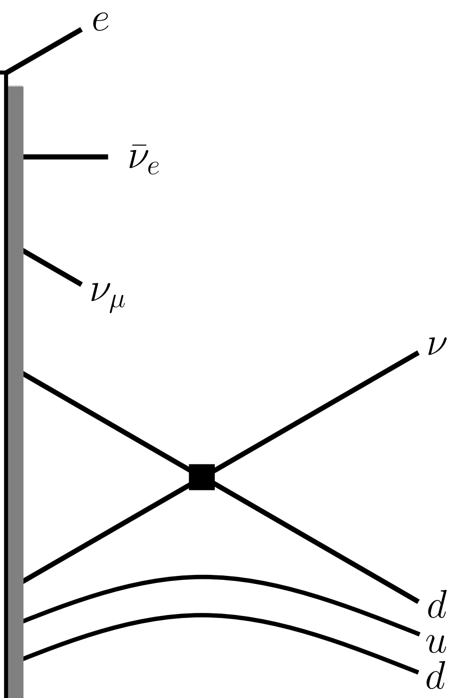
3. Convert to an electron

   a) Mono-energetic electron signal ($E_e = m_\mu$)

A trapped muon can...

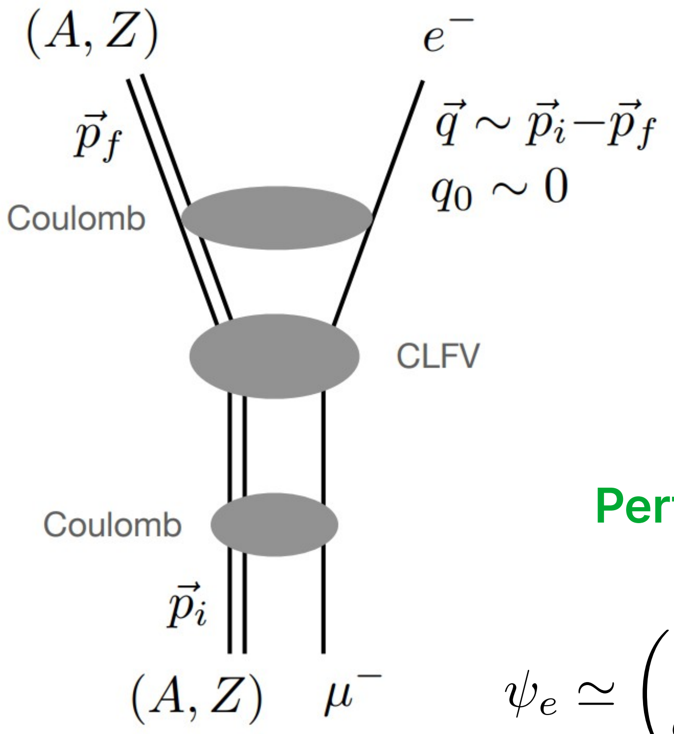






= m_μ)

(A,Z) μ⁻ → (A,Z) e⁻



Schematically...

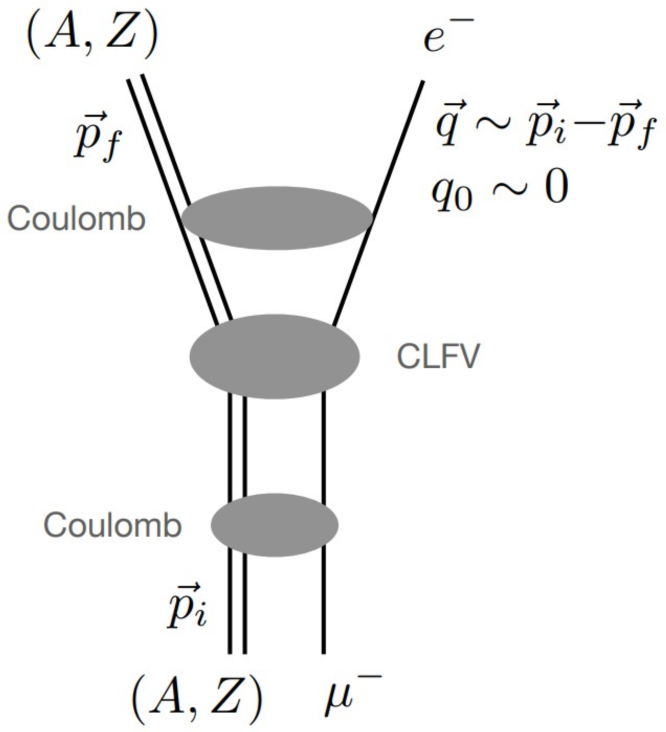
- Effective Hamiltonian:

$$\mathcal{H}_i = c_i \int d^3x \; \bar{\psi}_e(\vec{x}) O_L \psi_\mu(\vec{x}) \bar{\psi}_N(\vec{x}) O_N \psi_N(\vec{x})$$

Perform partial-wave and multipole expansion...

$$\psi_e \simeq \begin{pmatrix} \xi \\ \hat{q} \cdot \vec{\sigma}\xi \end{pmatrix} \quad \psi_\mu \simeq \begin{pmatrix} \xi \\ \frac{1}{2}\vec{v}_\mu \cdot \vec{\sigma}\xi \end{pmatrix} \quad \psi_N \simeq \begin{pmatrix} \xi \\ \frac{1}{2}\vec{v}_N \cdot \vec{\sigma}\xi \end{pmatrix}$$

(A,Z) μ⁻ → (A,Z) e⁻



At the end of the day:

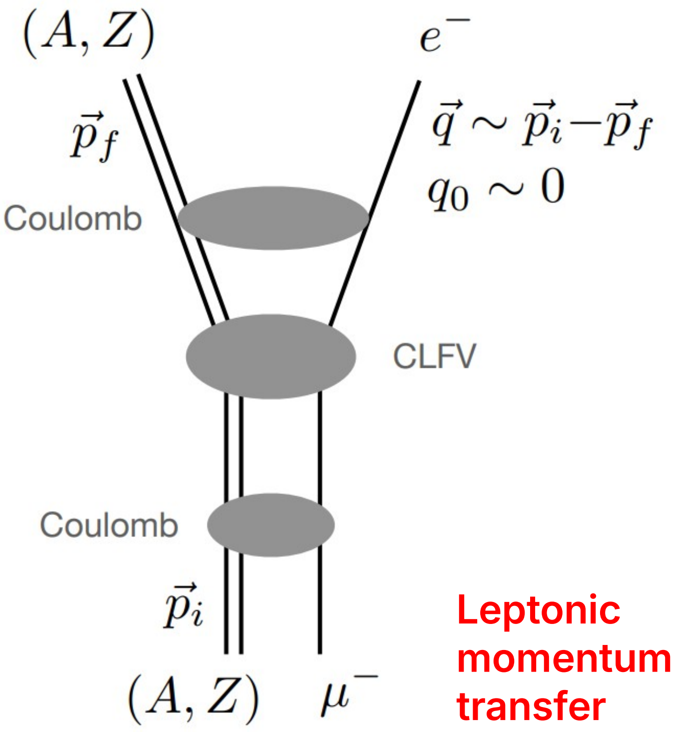
- **Basis of CLFV'ing single-nucleon operators**

$$i\hat{q} = \frac{i\vec{q}}{|\vec{q}|}, \qquad \vec{\sigma}_L, \qquad \vec{\sigma}_N, \qquad \vec{v}_N, \qquad \vec{v}_\mu.$$

- **Natural hierarchy of dimensionless scales**

$$y \equiv \left(\frac{qb}{2}\right)^2 > |\vec{v}_N| > |\vec{v}_\mu| > |\vec{v}_T|$$

(A,Z) μ⁻ → (A,Z) e⁻



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- Natural hierarchy of dimensionless scales

$^{27}\mathrm{Al}$

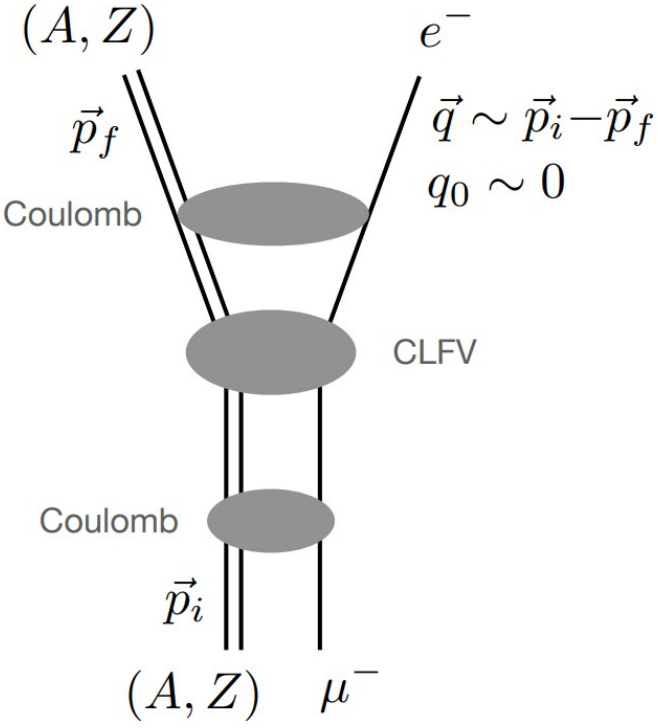
$$y \equiv \left(\frac{qb}{2}\right)^2 > |\vec{v}_N| > |\vec{v}_\mu| > |\vec{v}_T|$$

≈ 0.25 ; ≈ 0.2

Leptonic momentum transfer: $q \approx m_\mu = 1/1.86\ \mathrm{fm}$

Harmonic oscillator parameter: $b = 1.85\ \mathrm{fm}$

(A,Z) μ⁻ → (A,Z) e⁻



2208.07945

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Nuclear-level Effective Theory of $\mu \to e$ Conversion: Formalism and Applications

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(Dated: April 4, 2023)

Over the next decade new $\mu \to e$ conversion searches at Fermilab (Mu2e) and J-PARC (COMET, DeeMe) are expected to advance limits on charged lepton flavor violation (CLFV) by more than four orders of magnitude. By considering the consequence of P and CP on elastic $\mu \to e$ conversion and the structure of possible charge and current densities, we show that rates are governed by six nuclear responses and a single scale, q/m_N, where $q \approx m_\mu$ is the momentum transferred from the leptons to the nucleus. To relate this result to microscopic formulations of CLFV, we construct in nonrelativistic effective theory (NRET) the CLFV nucleon-level interaction, pointing out the relevance of the dimensionless scales $y = (\frac{qb}{2})^2 > |\vec{v}_N| > |\vec{v}_\mu| > |\vec{v}_T|$, where b is the nuclear size, $\vec{v}_N$ and $\vec{v}_\mu$ are the nucleon and muon intrinsic velocities, and $\vec{v}_T$ is the target recoil velocity. We discuss previous work, noting the lack of a systematic treatment of the various small parameters. Because the parameter y is not small, a proper calculation of $\mu \to e$ conversion requires a full multipole expansion of the nuclear response functions, an apparently daunting task with Coulomb-distorted electron partial waves. We demonstrate that the multipole expansion can be carried out to high precision by introducing a simplifying local momentum $q_{\rm eff}$ for the electron. Previous work has been limited to simple charge or spin interactions, thereby treating the nucleus effectively as a point particle. We show that such formulations are not compatible with the general form of the $\mu \to e$ conversion rate, failing to generate three of the six allowed nuclear response functions. The inclusion of the nucleon velocity $\vec{v}_N$ yields an NRET with 16 operators and a rate of the general form. Consequently, in the current discovery era for CLFV, it provides the most sensible starting point for experimental analysis, defining what can and cannot be determined about CLFV from the highly exclusive process of $\mu \to e$ conversion. Finally, we expand the NRET operator basis to account for the effects of $\vec{v}_\mu$, associated with the muon's lower component, generating corrections to the CLFV coefficients of the point-nucleus response functions. Using advanced shell-model methods, we compute $\mu \to e$ conversion rates for a series of experimental targets, deriving bounds on the coefficients of the CLFV operators. These calculations are the first to include a general basis of CLFV operators, full evaluation of the associated nuclear response functions, and an accurate treatment of electron and muon Coulomb effects. We discuss target selection as an experimental "knob" that can be turned to probe the microscopic origins of CLFV. We describe two types of coherence that enhance certain CLFV operators and selection rules that blind elastic $\mu \to e$ conversion to others. We discuss the matching of the NRET onto higher level effective field theories, such as those constructed at the light quark level, noting opportunities to build on existing work in direct detection of dark matter. We discuss the relation of $\mu \to e$ conversion to $\mu \to e + \gamma$ and $\mu \to 3e$, showing how MEG II and Mu3e results will complement those of Mu2e and COMET. Finally we describe a accompanying script – in Mathematica and Python versions – that can be used to compute $\mu \to e$ conversion rates in various nuclear targets for the full set of NRET operators.

I. INTRODUCTION

Muon-to-electron conversion, in which a muon bound to a nucleus converts to a mono-energetic outgoing electron, occurs at an observable level only if there are new sources of flavor violation, beyond those responsible for neutrino mixing [1–4]. It has yond the standard model [5–7]. This has motivated a series of experimental advances that, in sum, have improved limits on $\mu \to e$ conversion rates by ≈12 orders of magnitude over the past 75 years [8].

The experimental attributes of $\mu \to e$ conversion are quite attractive. Intense muon beams exist, with rates on target of $\approx 10^{11}$/s expected in the

The rate

- **The conversion rate:** $\mathcal{O}(y)$

$$\Gamma(\mu \to e) = \frac{1}{2\pi} \frac{q_{\rm eff}^2}{1 + q/M_T} \left|\phi_{1s}^{Z_{\rm eff}}(\vec{0})\right|^2 \sum_{\tau,\tau'} \Big[R_{MM}^{\tau\tau'} W_{MM}^{\tau\tau'}(q_{\rm eff}) + R_{\Sigma'\Sigma'}^{\tau\tau'} W_{\Sigma'\Sigma'}^{\tau\tau'}(q_{\rm eff}) + R_{\Sigma''\Sigma''}^{\tau\tau'} W_{\Sigma''\Sigma''}^{\tau\tau'}(q_{\rm eff}) \Big],$$

$$R_{MM}^{\tau\tau'} = c_1^\tau c_1^{\tau'*} + c_{11}^\tau c_{11}^{\tau'*},$$

$$R_{\Sigma'\Sigma'}^{\tau\tau'} = c_4^\tau c_4^{\tau'*} + c_9^\tau c_9^{\tau'*},$$

$$R_{\Sigma''\Sigma''}^{\tau\tau'} = \left(c_4^\tau - c_6^\tau\right)\left(c_4^{\tau'} - c_6^{\tau'}\right)^* + c_{10}^\tau c_{10}^{\tau'*},$$

The rate

- The conversion rate: $\mathcal{O}(\vec{v}_N)$

***All nuclear responses allowed by symmetries are generated**

$$\omega = \frac{G_F^2}{\pi} \frac{q_{\rm eff}^2}{1+\frac{q}{M_T}} |\phi_{1s}^{Z_{\rm eff}}(\vec{0})|^2 \sum_{\tau=0,1} \sum_{\tau'=0,1} \Bigg\{ \left[\tilde{R}_{MM}^{\tau\tau'} W_{MM}^{\tau\tau'}(q_{\rm eff}) + \tilde{R}_{\Sigma''\Sigma''}^{\tau\tau'} W_{\Sigma''\Sigma''}^{\tau\tau'}(q_{\rm eff}) + \tilde{R}_{\Sigma'\Sigma'}^{\tau\tau'} W_{\Sigma'\Sigma'}^{\tau\tau'}(q_{\rm eff})\right]$$
$$+ \frac{q_{\rm eff}^2}{m_N^2} \left[\tilde{R}_{\Phi''\Phi''}^{\tau\tau'} W_{\Phi''\Phi''}^{\tau\tau'}(q_{\rm eff}) + \tilde{R}_{\tilde{\Phi}'\tilde{\Phi}'}^{\tau\tau'} W_{\tilde{\Phi}'\tilde{\Phi}'}^{\tau\tau'}(q_{\rm eff}) + \tilde{R}_{\Delta\Delta}^{\tau\tau'} W_{\Delta\Delta}^{\tau\tau'}(q_{\rm eff})\right]$$
$$- \frac{2q_{\rm eff}}{m_N} \left[\tilde{R}_{\Phi''M}^{\tau\tau'} W_{\Phi''M}^{\tau\tau'}(q_{\rm eff}) + \tilde{R}_{\Delta\Sigma'}^{\tau\tau'} W_{\Delta\Sigma'}^{\tau\tau'}(q_{\rm eff})\right] \Bigg\} \qquad (59)$$

$$R_{MM}^{\tau\tau'} = c_1^\tau c_1^{\tau'*} + c_{11}^\tau c_{11}^{\tau'*},$$
$$R_{\Sigma'\Sigma'}^{\tau\tau'} = c_4^\tau c_4^{\tau'*} + c_9^\tau c_9^{\tau'*},$$
$$R_{\Sigma''\Sigma''}^{\tau\tau'} = \left(c_4^\tau - c_6^\tau\right)\left(c_4^{\tau'} - c_6^{\tau'}\right)^* + c_{10}^\tau c_{10}^{\tau'*},$$

$$\tilde{R}_{\Phi''\Phi''}^{\tau\tau'} = \tilde{c}_3^\tau \tilde{c}_3^{\tau'*} + (\tilde{c}_{12}^\tau - \tilde{c}_{15}^\tau)(\tilde{c}_{12}^{\tau'*} - \tilde{c}_{15}^{\tau'*})$$
$$\tilde{R}_{\Phi''M}^{\tau\tau'} = \mathrm{Re}[\tilde{c}_3^\tau \tilde{c}_1^{\tau'*} - (\tilde{c}_{12}^\tau - \tilde{c}_{15}^\tau)\, \tilde{c}_{11}^{\tau'*}]$$
$$\tilde{R}_{\Delta\Delta}^{\tau\tau'} = \tilde{c}_5^\tau \tilde{c}_5^{\tau'*} + \tilde{c}_8^\tau \tilde{c}_8^{\tau'*}$$
$$\tilde{R}_{\Delta\Sigma'}^{\tau\tau'} = \mathrm{Re}[\tilde{c}_5^\tau \tilde{c}_4^{\tau'*} + \tilde{c}_8^\tau \tilde{c}_9^{\tau'*}]$$
$$\tilde{R}_{\tilde{\Phi}'\tilde{\Phi}'}^{\tau\tau'} = \tilde{c}_{12}^\tau \tilde{c}_{12}^{\tau'*} + \tilde{c}_{13}^\tau \tilde{c}_{13}^{\tau'*}$$

Nuclear response hierarchy

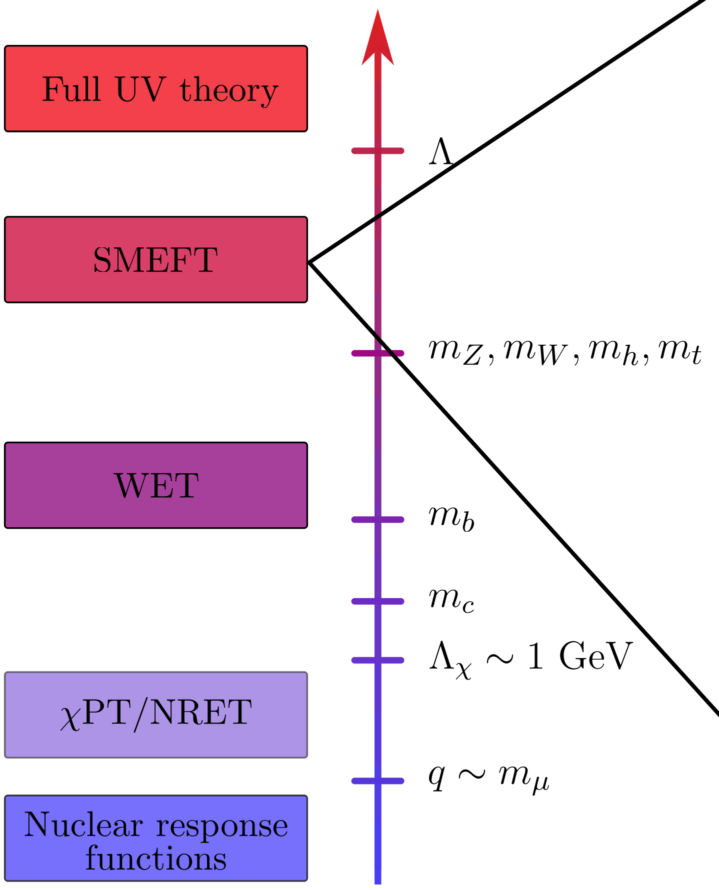
∗Can become semi-coherent in some nuclei with half-filled shells (e.g. Al, Cu)

$$W_{MM}^{00} \sim \mathcal{O}(A^2) \gg \frac{q_{\rm eff}}{m_N} W_{M\Phi''}^{00} \gg \left\{ W_{\Sigma'\Sigma'}^{00}, W_{\Sigma''\Sigma''}^{00}, \frac{q_{\rm eff}^2}{m_N^2} W_{\Phi''\Phi''}^{00} \right\} \gg \left\{ \frac{q_{\rm eff}^2}{m_N^2} W_{\Delta\Delta}^{00}, \frac{q_{\rm eff}^2}{m_N^2} W_{\tilde{\Phi}'\tilde{\Phi}'}^{00} \right\}$$

Where does the UV physics sit?

$$\Gamma \propto \sum R \times W, \qquad R(c_i)$$

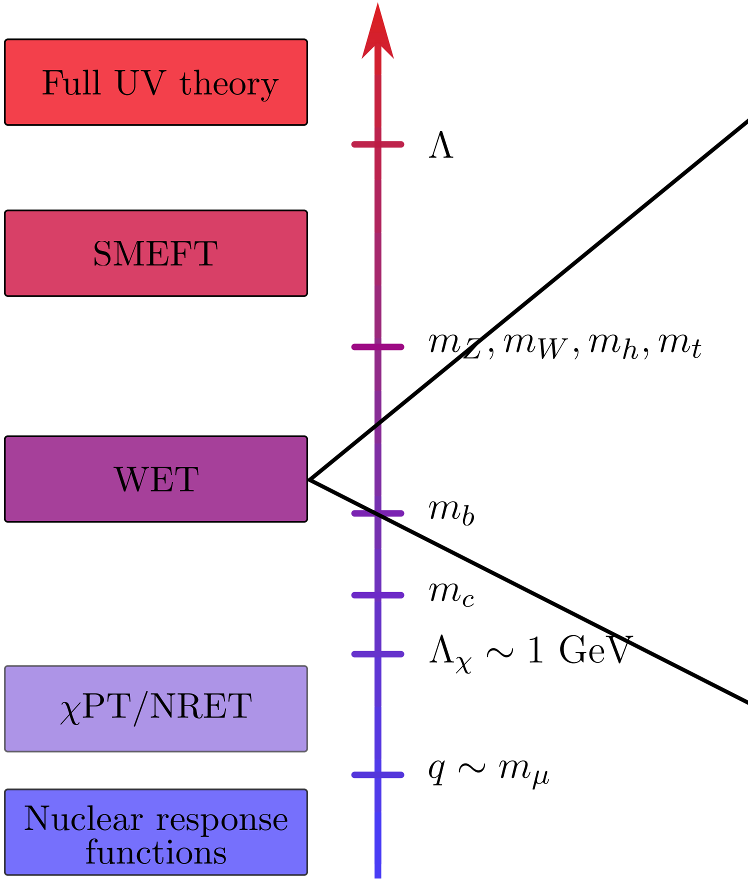
The tower



SMEFT

$$
\begin{array}{ll}
Q_{e\varphi}: & (\varphi^\dagger\varphi)(\bar{L}_2 e_R \varphi), \quad (\varphi^\dagger\varphi)(\bar{\ell}_1 \mu_R \varphi), \\
Q_{eW}: & (\bar{\ell}_2 \sigma^{\mu\nu} e_R)\tau^I \varphi W^I_{\mu\nu}, \quad (\bar{\ell}_1 \sigma^{\mu\nu} \mu_R)\tau^I \varphi W^I_{\mu\nu}, \\
Q_{eB}: & (\bar{\ell}_2 \sigma^{\mu\nu} e_R)\varphi B_{\mu\nu}, \quad (\bar{\ell}_1 \sigma^{\mu\nu} \mu_R)\varphi B_{\mu\nu}, \\
Q^{(1)}_{\varphi\ell}: & (\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar{\ell}_2 \gamma^\mu \ell_1), \\
Q^{(3)}_{\varphi\ell}: & (\varphi^\dagger i \overleftrightarrow{D}^I_\mu \varphi)(\bar{\ell}_2 \gamma^\mu \tau^I \ell_1), \\
Q_{\varphi e}: & (\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar{\mu}_R \gamma^\mu \tau^I e_R), \\
Q^{(1)}_{\ell q}: & (\bar{\ell}_2 \gamma^\mu \ell_1)(\bar{q}\gamma^\mu q), \\
Q^{(3)}_{\ell q}: & (\bar{\ell}_2 \gamma^\mu \tau^I \ell_1)(\bar{q}\gamma^\mu \tau^I q), \\
Q_{eu}: & (\bar{\mu}_R \gamma^\mu e_R)(\bar{u}_R \gamma^\mu u_R), \\
Q_{ed}: & (\bar{\mu}_R \gamma^\mu e_R)(\bar{d}_R \gamma^\mu d_R), \\
Q_{\ell u}: & (\bar{\ell}_2 \gamma^\mu \ell_1)(\bar{u}_R \gamma^\mu u_R), \\
Q_{\ell d}: & (\bar{\ell}_2 \gamma^\mu \ell_1)(\bar{d}_R \gamma^\mu d_R), \\
Q_{qe}: & (\bar{q}\gamma^\mu q)(\bar{\mu}_R \gamma^\mu e_R), \\
Q_{ledq}: & (\bar{\ell}_2 e_R)(\bar{d}_R q), \quad (\bar{\ell}_1 \mu_R)(\bar{d}_R q), \\
Q^{(1)}_{lequ}: & (\bar{\ell}^j_2 e_R)\varepsilon_{jk}(\bar{q}^k u), \quad (\bar{\ell}^j_1 \mu_R)\varepsilon_{jk}(\bar{q}^k u), \\
Q^{(3)}_{lequ}: & (\bar{\ell}^j_2 \sigma_{\mu\nu} e_R)\varepsilon_{jk}(\bar{q}^k \sigma^{\mu\nu} u), \quad (\bar{\ell}^j_1 \sigma_{\mu\nu} \mu_R)\varepsilon_{jk}(\bar{q}^k \sigma^{\mu\nu} u),
\end{array}
$$

The tower



WET

$$\mathcal{Q}_1^{(5)} = \frac{e}{8\pi^2}(\bar{e}\sigma^{\alpha\beta}\mu)F_{\alpha\beta}\,, \qquad \mathcal{Q}_2^{(5)} = \frac{e}{8\pi^2}(\bar{e}\sigma^{\alpha\beta}i\gamma_5\mu)F_{\alpha\beta}\,,$$

$$\mathcal{Q}_{1,q}^{(6)} = (\bar{e}\gamma_\alpha\mu)(\bar{q}\gamma^\alpha q)\,, \qquad \mathcal{Q}_{2,q}^{(6)} = (\bar{e}\gamma_\alpha\gamma_5\mu)(\bar{q}\gamma^\alpha q)\,,$$

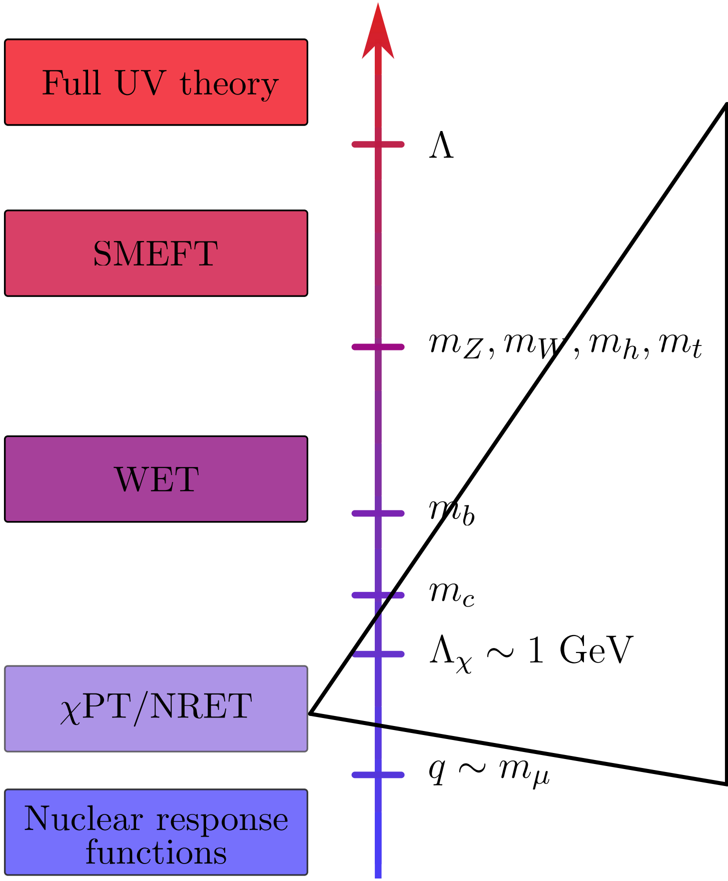
$$\mathcal{Q}_{3,q}^{(6)} = (\bar{e}\gamma_\alpha\mu)(\bar{q}\gamma^\alpha\gamma_5 q)\,, \qquad \mathcal{Q}_{4,q}^{(6)} = (\bar{e}\gamma_\alpha\gamma_5\mu)(\bar{q}\gamma^\alpha\gamma_5 q)\,.$$

$$\mathcal{Q}_{5,q}^{(6)} = (\bar{e}\mu)(\bar{q}q)\,, \qquad \mathcal{Q}_{6,q}^{(6)} = (\bar{e}i\gamma_5\mu)(\bar{q}q)\,,$$

$$\mathcal{Q}_{7,q}^{(6)} = (\bar{e}\mu)(\bar{q}i\gamma_5 q)\,, \qquad \mathcal{Q}_{8,q}^{(6)} = (\bar{e}i\gamma_5\mu)(\bar{q}i\gamma_5 q)\,,$$

$$\mathcal{Q}_{9,q}^{(6)} = (\bar{e}\sigma^{\alpha\beta}\mu)(\bar{q}\sigma_{\alpha\beta}q)\,, \qquad \mathcal{Q}_{10,q}^{(6)} = (\bar{e}i\sigma^{\alpha\beta}\gamma_5\mu)(\bar{q}\sigma_{\alpha\beta}q)\,.$$

The tower



NRET

$$\mathcal{O}_1 = 1_L\, 1_N, \qquad \mathcal{O}_2' = 1_L\, i\hat{q}\cdot\vec{v}_N,$$

$$\mathcal{O}_3 = 1_L\, i\hat{q}\cdot\left[\vec{v}_N\times\vec{\sigma}_N\right], \qquad \mathcal{O}_4 = \vec{\sigma}_L\cdot\vec{\sigma}_N,$$

$$\mathcal{O}_5 = \vec{\sigma}_L\cdot\left(i\hat{q}\times\vec{v}_N\right), \qquad \mathcal{O}_6 = i\hat{q}\cdot\vec{\sigma}_L\, i\hat{q}\cdot\vec{\sigma}_N,$$

$$\mathcal{O}_7 = 1_L\, \vec{v}_N\cdot\vec{\sigma}_N, \qquad \mathcal{O}_8 = \vec{\sigma}_L\cdot\vec{v}_N,$$

$$\mathcal{O}_9 = \vec{\sigma}_L\cdot\left(i\hat{q}\times\vec{\sigma}_N\right), \qquad \mathcal{O}_{10} = 1_L\, i\hat{q}\cdot\vec{\sigma}_N,$$

$$\mathcal{O}_{11} = i\hat{q}\cdot\vec{\sigma}_L\, 1_N, \qquad \mathcal{O}_{12} = \vec{\sigma}_L\cdot\left[\vec{v}_N\times\vec{\sigma}_N\right],$$

$$\mathcal{O}_{13}' = \vec{\sigma}_L\cdot\left(i\hat{q}\times\left[\vec{v}_N\times\vec{\sigma}_N\right]\right), \qquad \mathcal{O}_{14} = i\hat{q}\cdot\vec{\sigma}_L\, \vec{v}_N\cdot\vec{\sigma}_N,$$

$$\mathcal{O}_{15} = i\hat{q}\cdot\vec{\sigma}_L\, i\hat{q}\cdot\left[\vec{v}_N\times\vec{\sigma}_N\right], \qquad \mathcal{O}_{16}' = i\hat{q}\cdot\vec{\sigma}_L\, i\hat{q}\cdot\vec{v}_N.$$

Matching

- Non-perturbative matching between WET and NRET (hadronization)
- Parameterize with nuclear form factors

$$\langle N'|\bar{q}\gamma^\mu q|N\rangle = \bar{u}'_N\Big[F_1^{q/N}(q^2)\gamma^\mu - \frac{i}{2m_N}F_2^{q/N}(q^2)\sigma^{\mu\nu}q_\nu\Big]u_N\,,$$

$$\langle N'|\bar{q}\gamma^\mu\gamma_5 q|N\rangle = \bar{u}'_N\Big[F_A^{q/N}(q^2)\gamma^\mu\gamma_5 - \frac{1}{2m_N}F_{P'}^{q/N}(q^2)\gamma_5 q^\mu\Big]u_N\,,$$

$$\langle N'|m_q\bar{q}q|N\rangle = F_S^{q/N}(q^2)\,\bar{u}'_N u_N\,,$$

$$\langle N'|m_q\bar{q}i\gamma_5 q|N\rangle = F_P^{q/N}(q^2)\,\bar{u}'_N i\gamma_5 u_N\,,$$

$$\langle N'|\frac{\alpha_s}{12\pi}G^{a\mu\nu}G^a_{\mu\nu}|N\rangle = F_G^N(q^2)\,\bar{u}'_N u_N\,,$$

$$\langle N'|\frac{\alpha_s}{8\pi}G^{a\mu\nu}\tilde{G}^a_{\mu\nu}|N\rangle = F_{\tilde{G}}^N(q^2)\,\bar{u}'_N i\gamma_5 u_N\,,$$

$$\langle N'|\bar{q}\sigma^{\mu\nu}q|N\rangle = \bar{u}'_N\Big[\hat{F}_{T,0}^{q/N}(q^2)\,\sigma^{\mu\nu} - \frac{i}{2m_N}\gamma^{[\mu}q^{\nu]}\hat{F}_{T,1}^{q/N}(q^2) - \frac{i}{m_N^2}q^{[\mu}k_{12}^{\nu]}\hat{F}_{T,2}^{q/N}(q^2)\Big]u_N\,,$$

$$\langle N'|\frac{\alpha}{12\pi}F^{\mu\nu}F_{\mu\nu}|N\rangle = F_\gamma^N(q^2)\,\bar{u}'_N u_N\,,$$

$$\langle N'|\frac{\alpha}{8\pi}F^{\mu\nu}\tilde{F}_{\mu\nu}|N\rangle = F_{\tilde{\gamma}}^N(q^2)\,\bar{u}'_N i\gamma_5 u_N\,.$$

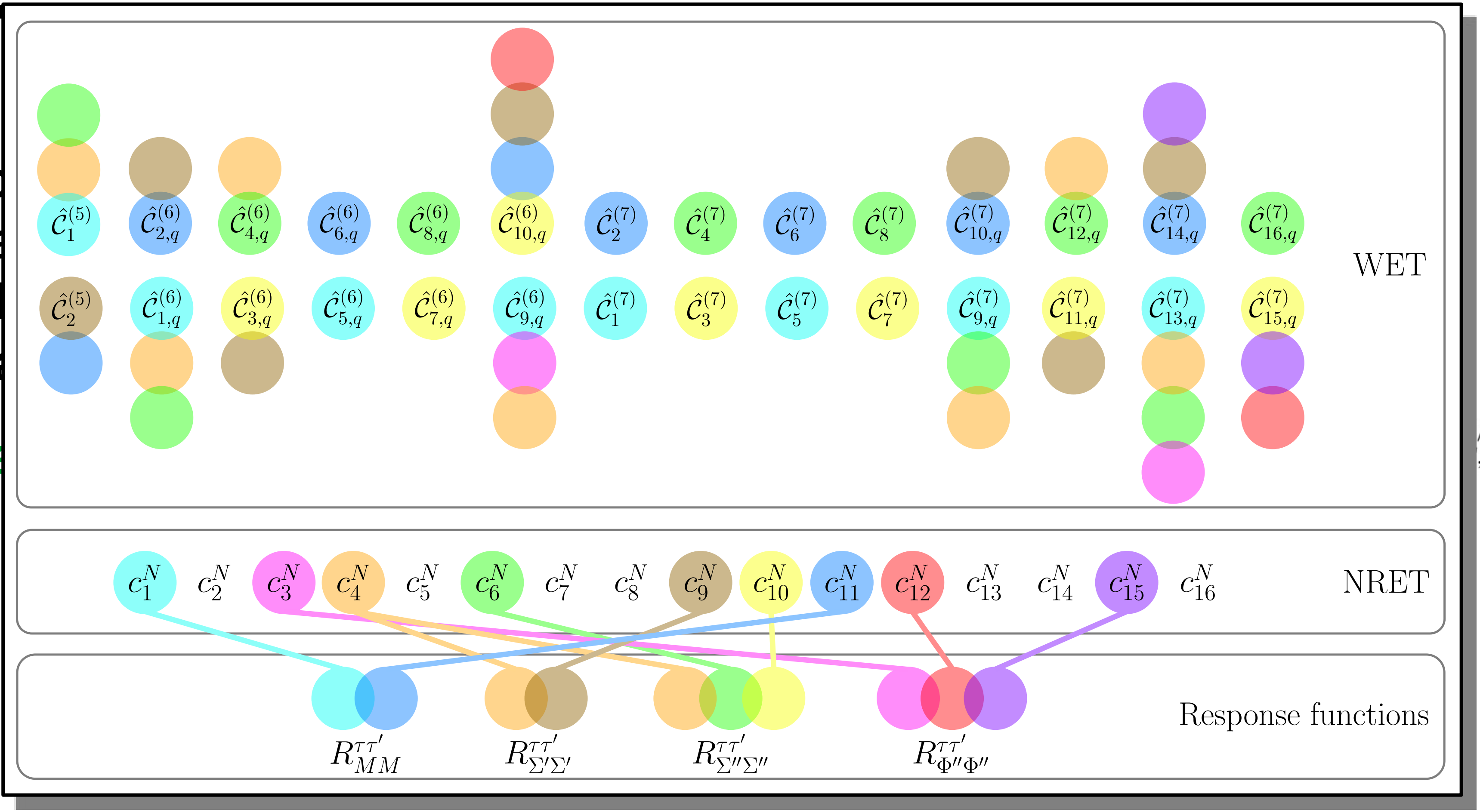
EFT tower

- Non-perturbative matching between WET and NRET (hadronization)
- Matching expressions

$$c_1^N = -\frac{\alpha}{\pi q}\hat{\mathcal{C}}_1^{(5)}\sum_q Q_q F_1^{q/N} + \sum_q \hat{\mathcal{C}}_{1,q}^{(6)} F_1^{q/N} + \sum_q \frac{1}{m_q}\hat{\mathcal{C}}_{5,q}^{(6)} F_S^{q/N}$$
$$- \frac{q}{m_N}\sum_q \hat{\mathcal{C}}_{9,q}^{(6)}\left(\hat{F}_{T,0}^{q/N} - \hat{F}_{T,1}^{q/N} + 4\hat{F}_{T,2}^{q/N}\right)$$
$$+ \hat{\mathcal{C}}_1^{(7)} F_G^N + \hat{\mathcal{C}}_5^{(7)} F_\gamma^N + (q + m_+)\sum_q \hat{\mathcal{C}}_{9,q}^{(7)} F_1^{q/N}$$
$$- \frac{q^2}{2m_N}\sum_q \hat{\mathcal{C}}_{13,q}^{(7)}\left[\hat{F}_{T,0}^{q/N} - \hat{F}_{T,1}^{q/N} + \left(4 + \frac{q^2}{m_N^2}\right)\hat{F}_{T,2}^{q/N}\right],$$
$$c_2^N = i\left[\sum_q \hat{\mathcal{C}}_{1,q}^{(6)} F_1^{q/N} + m_+ \sum_q \hat{\mathcal{C}}_{9,q}^{(7)} F_1^{q/N} + \frac{q^2}{2m_N}\sum_q \hat{\mathcal{C}}_{13,q}^{(7)}\left(\hat{F}_{T,1}^{q/N} - 4\hat{F}_{T,2}^{q/N}\right)\right],$$
$$\vdots$$

EFT

- No ma Wl (ha
- Ma ex



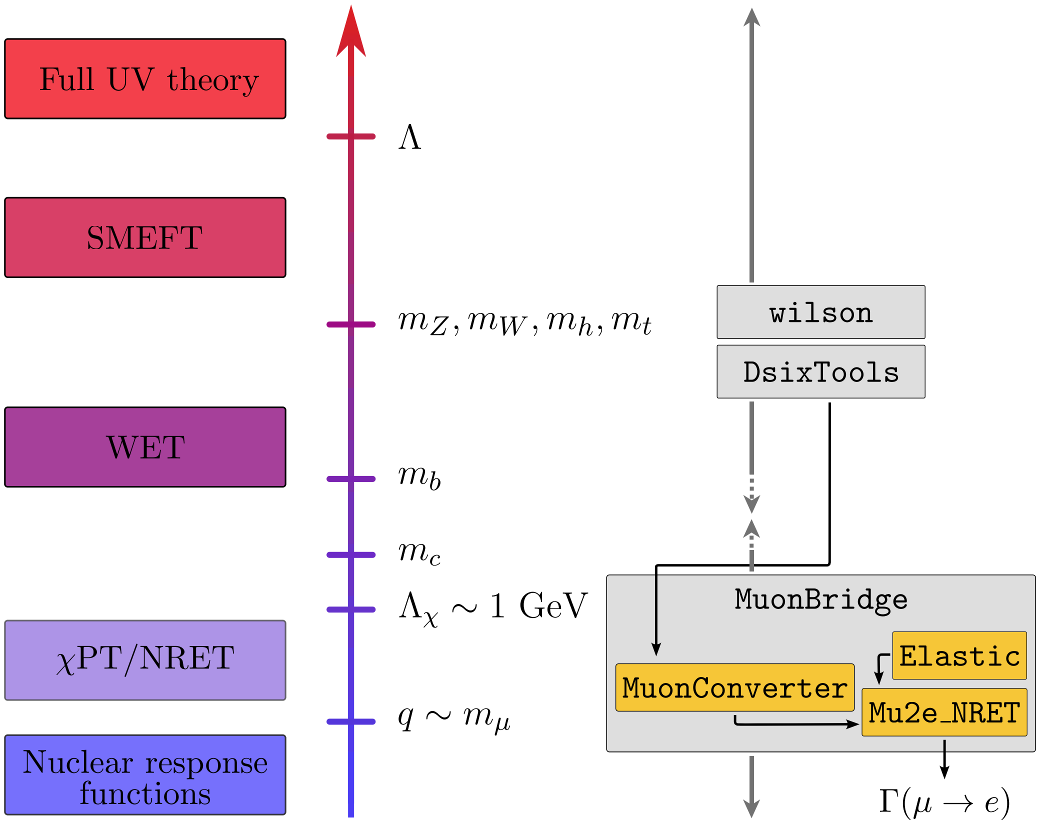
$/N \ ,2)],$

MuonBridge

https://github.com/Berkeley-Electroweak-Physics/MuonBridge

Three components:

1. Elastic - one body density matrices
2. Mu2e_NRET – Computes μ⟶e conversion rate
3. MuonConverter – matches WET to NRET and facilitates interface with existing EFT software



M

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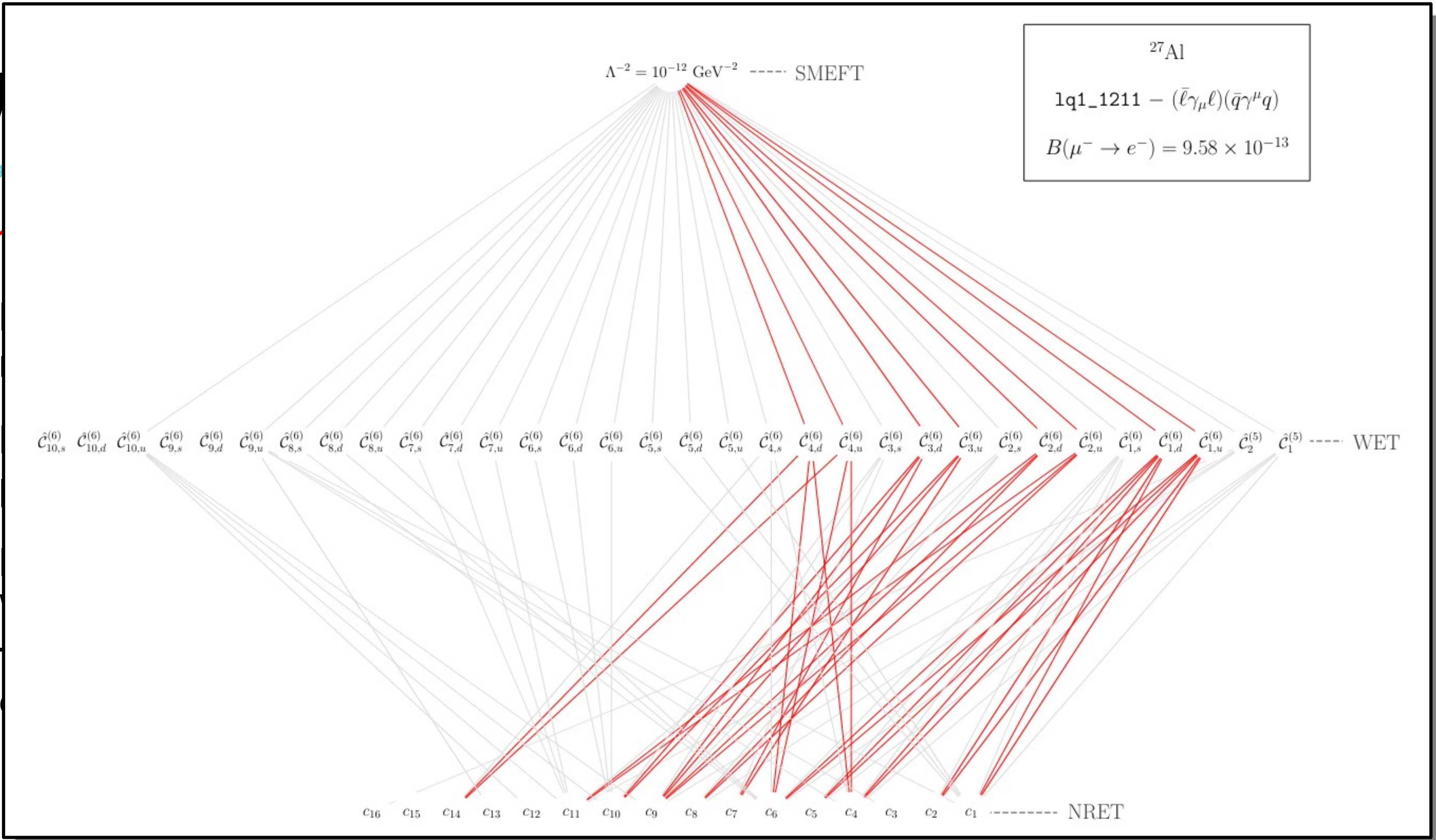
Th

1.

2.

3.



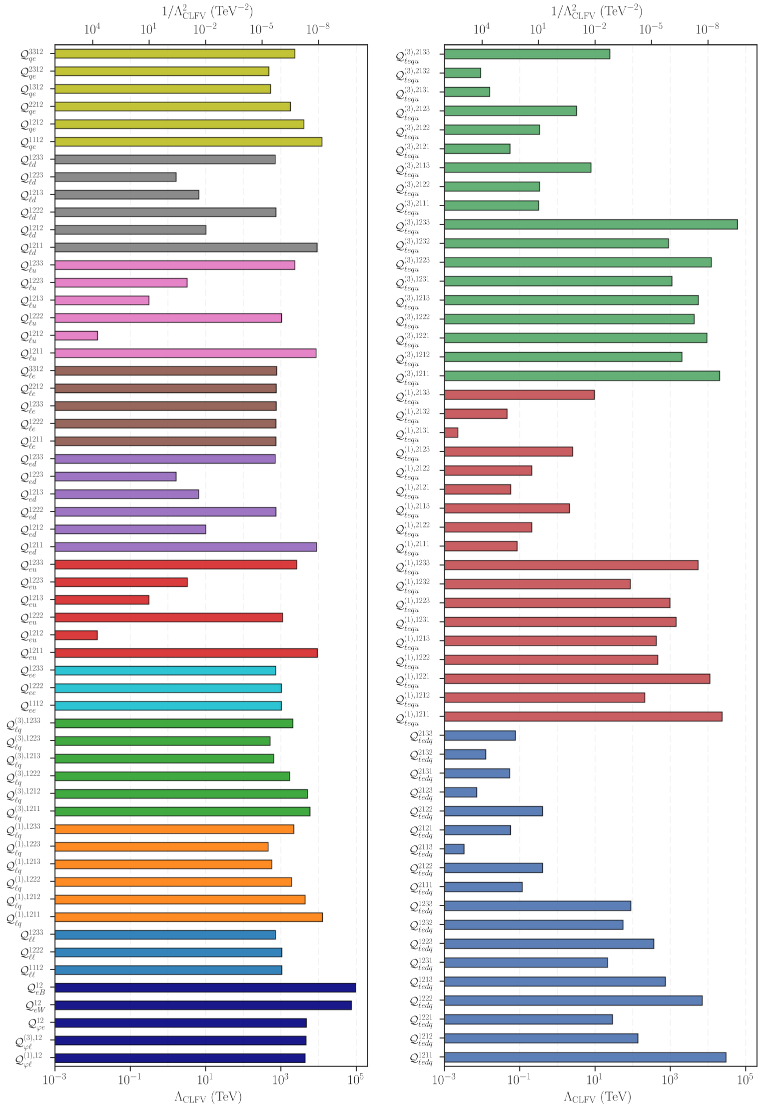
tic

RET

$e)$

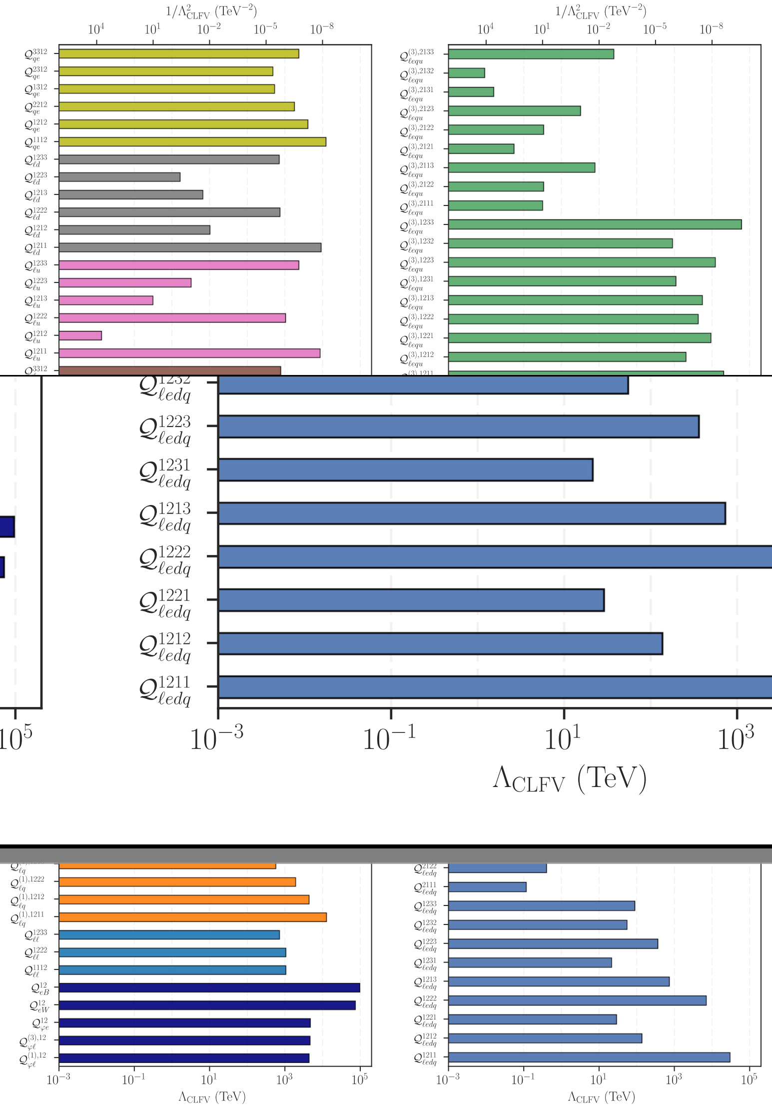
Bottom-up

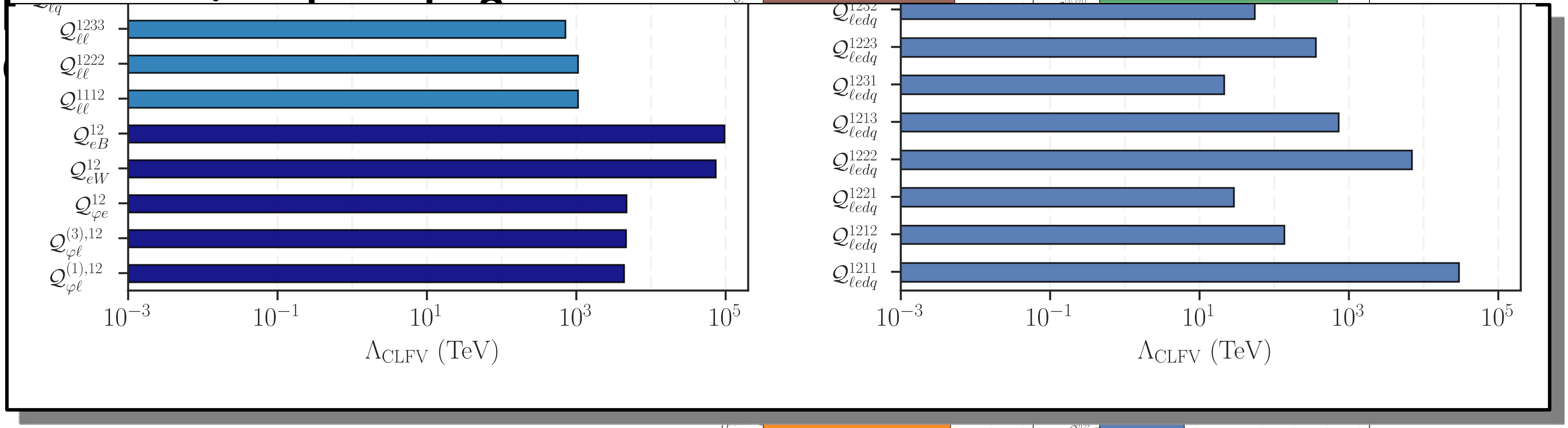
- Single dim-6 SMEFT operator bounds (with one-loop running down to 2 GeV)



Bottom-up

- Single dim-6 SMEFT operator bounds (with one-

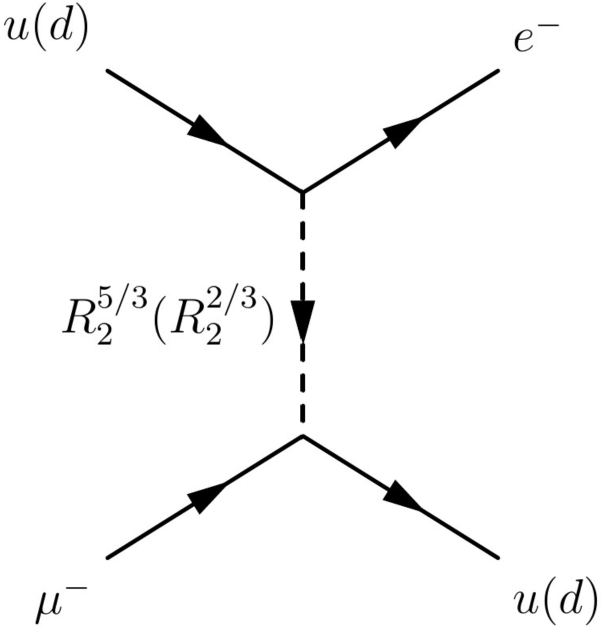




Top-down

- Consider the following leptoquark model

$$\mathcal{L} \supset -y_{2\,ij}^{RL}\,\bar{u}_R^i R_2^a \epsilon^{ab} L_L^{j,b} + y_{2\,ij}^{LR}\,\bar{e}_R^i R_2^{a\,*} Q_L^{j,a} + \text{h.c.},$$



Match onto SMEFT @ Λ = m_{LQ}:

$$C_{\substack{\ell u\\12ii}} = -\frac{1}{2m_{LQ}^2} y_{2\,i2}^{RL} y_{2\,i1}^{RL*},$$

$$C_{\substack{qe\\ii12}} = -\frac{1}{2m_{LQ}^2} y_{2\,2i}^{LR*} y_{2\,1i}^{LR},$$

$$C_{\substack{\ell equ\\12ii}}^{(1)} = 2C_{\substack{\ell equ\\12ii}}^{(3)} = -\frac{1}{2m_{LQ}^2} y_{2\,2i}^{LR*} y_{2\,i1}^{RL*},$$

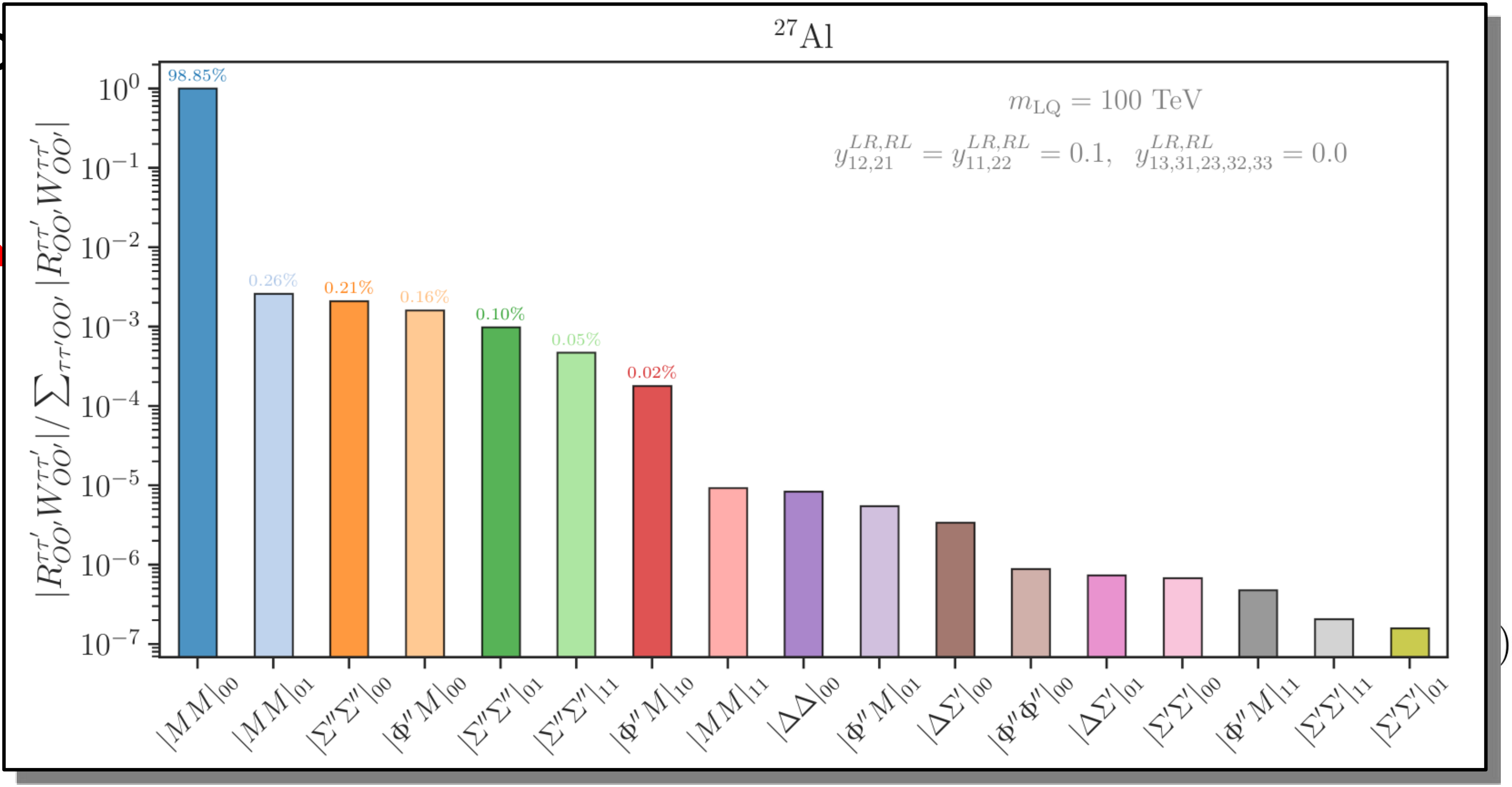
$$C_{\substack{\ell equ\\21ii}}^{(1)*} = 2C_{\substack{\ell equ\\21ii}}^{(3)*} = -\frac{1}{2m_{LQ}^2} y_{2\,i2}^{LR} y_{2\,1i}^{RL},$$

Top-down

• C

$\mathcal{L} \supset$

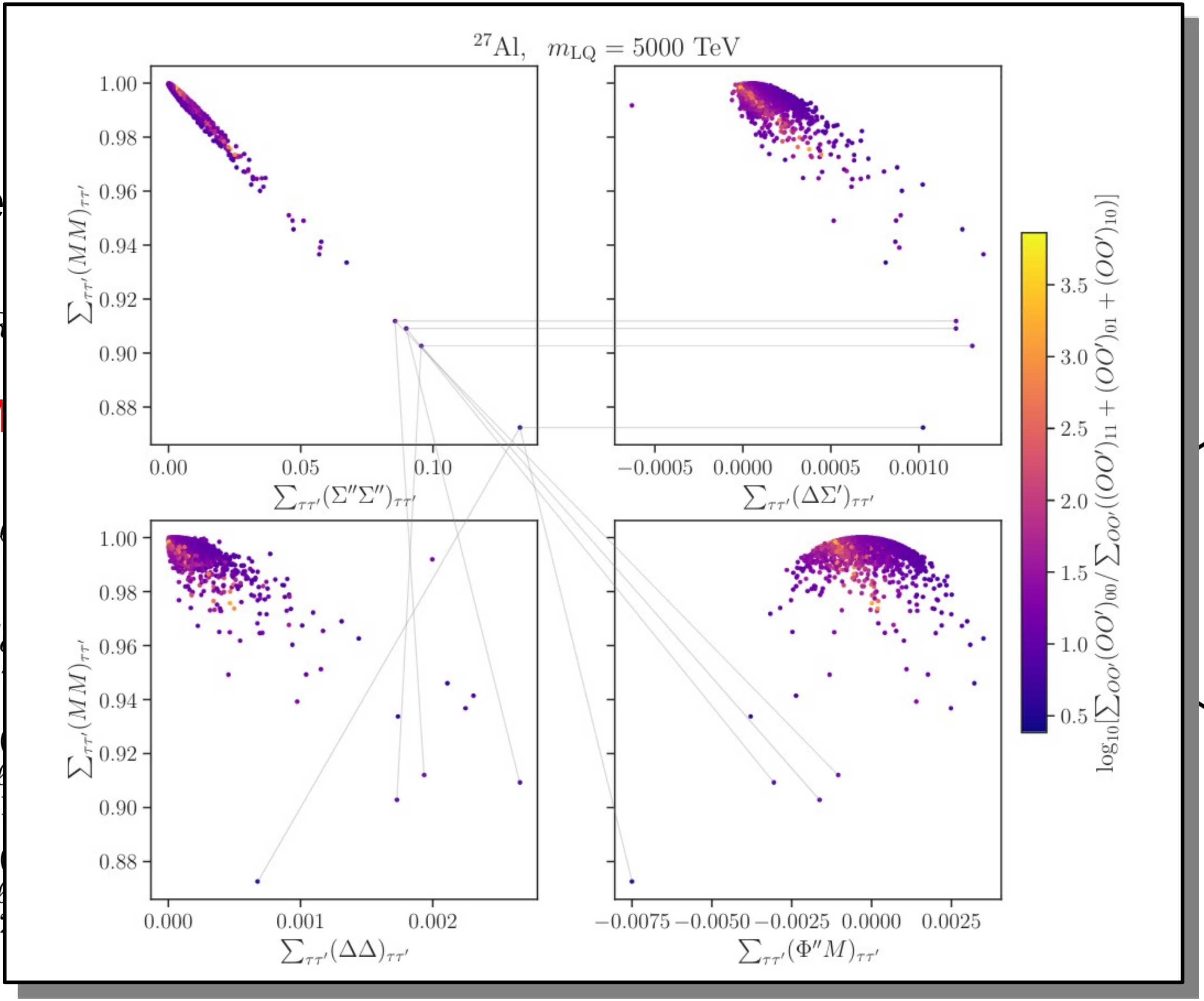
Match

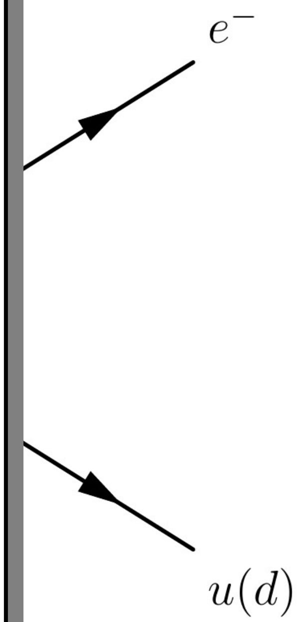


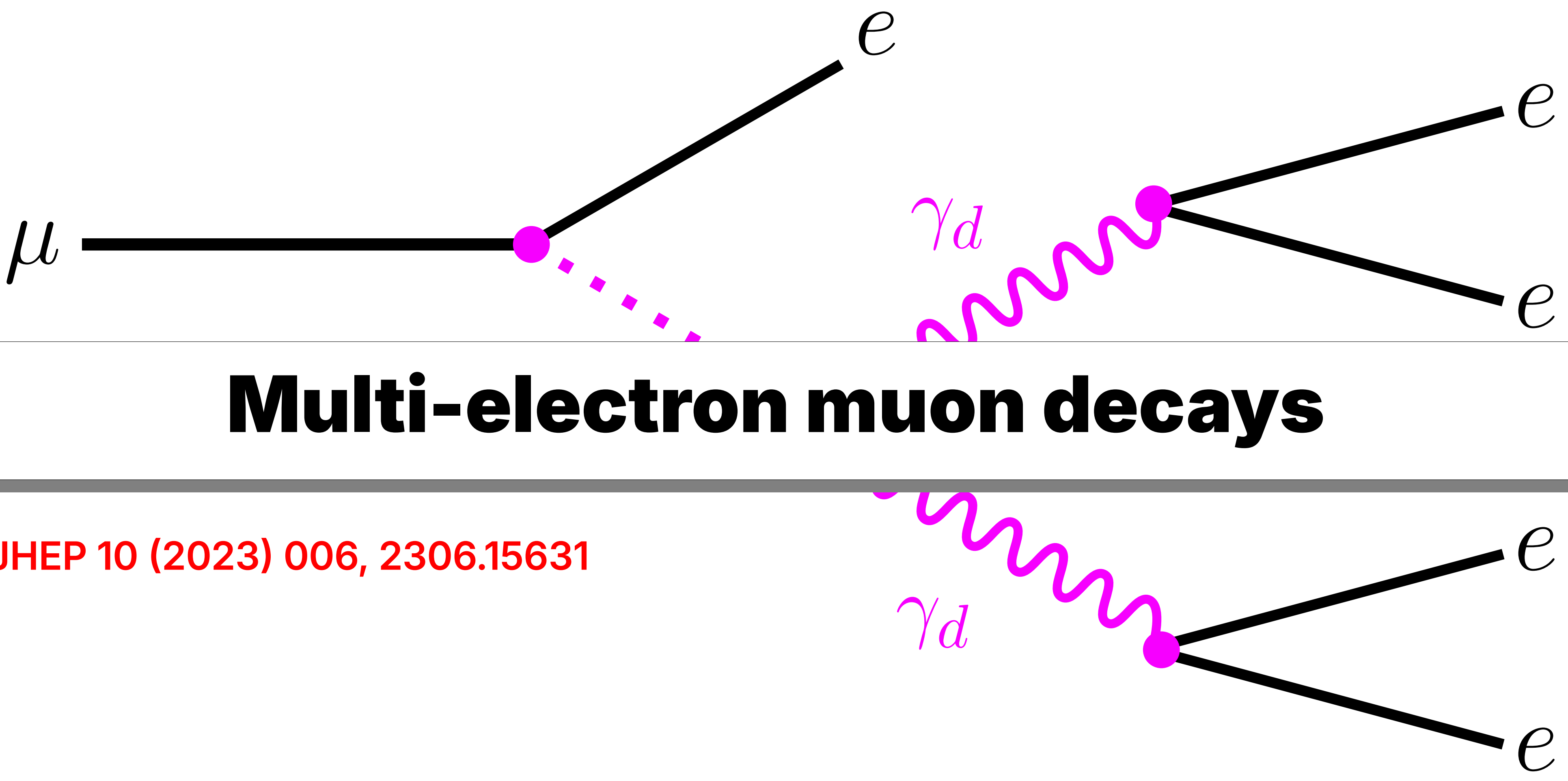
- **Conside**

$$\mathcal{L} \supset -y^{RL}_{2\,ij}\,\bar{u}^{i}_{R}$$

Match onto SM







Multi-electron muon decays

JHEP 10 (2023) 006, 2306.15631

Mu3e → Mu5e

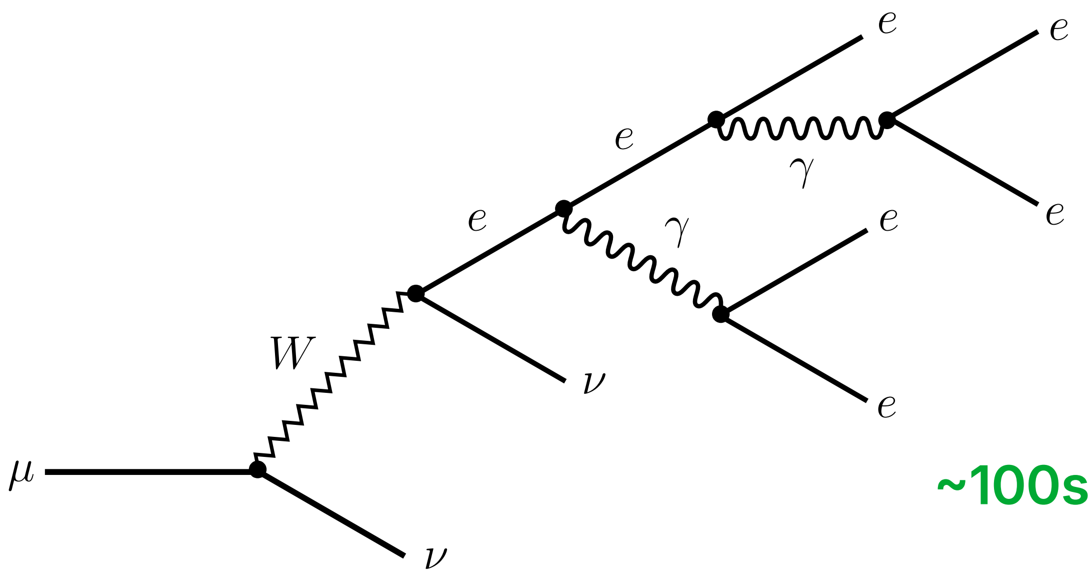
- Mu3e will see ~10^{15} total muon decays from the stopping target.
- With these statistics are there any other interesting channels?
  - What about $\mu \to 5e$?

μ → eeeeeνν

- **SM background for Mu5e**

From MG5: $\mathcal{B}(\mu^+ \to e^+e^-e^+e^-e^+\nu_e\bar{\nu}_\mu) = (3.929 \pm 0.001) \times 10^{-10}$

$$\mathcal{B}\left(\mu^+ \to e^+e^-e^+e^-e^+\nu_e\bar{\nu}_\mu \,|\, \text{all } p_{e^\pm}^{T,\text{true}} > 10 \text{ MeV}\right) = (1.4 \pm 0.1) \times 10^{-14}$$



$\propto G_F^2 \alpha^4$

~100s of μ → 5eνν events after cuts!

$\mu \to eeeee$

- Higgsed $U(1)_D$ + SM portal via kinetic mixing

$$\mathscr{L}_{\rm DS} = (D_\mu\phi)^\dagger D^\mu\phi - \frac{1}{4}F_d^{\mu\nu}F_{d\,\mu\nu} - \frac{\varepsilon}{2}F_d^{\mu\nu}F_{\mu\nu} - \mu^2(\phi^\dagger\phi) - \lambda(\phi^\dagger\phi)^2$$

$$\mathscr{L}_{\rm LFV} = -\frac{C_{ij}}{\Lambda}\phi\left(\bar{L}_i H\right)\ell_j + \text{h.c.}$$

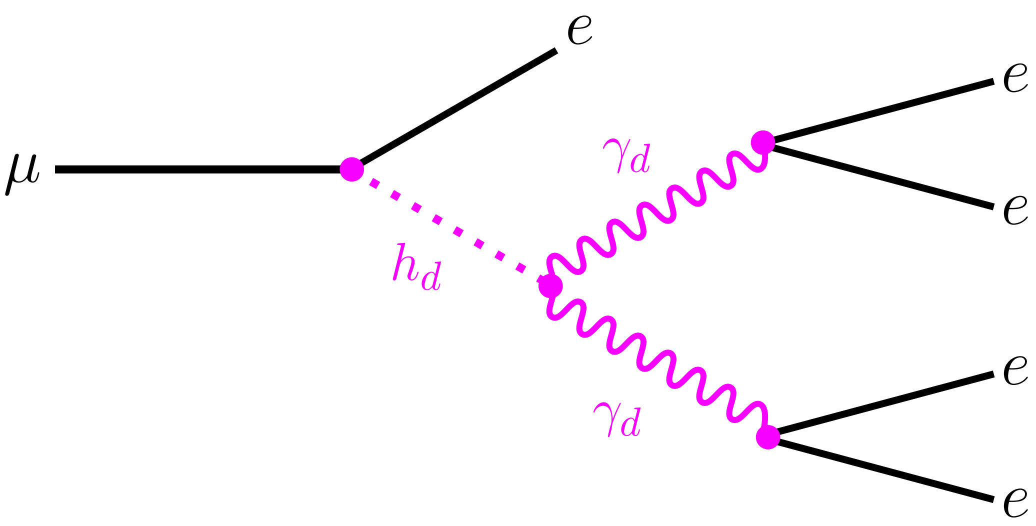
$$\downarrow$$

$$y_{ij} \simeq \frac{Cv}{\Lambda}$$

$$\mathscr{L} \supset -m_{\ell_i}\bar{\ell}_{Li}\ell_{Ri}\left(1+\frac{h}{v}\right) - y_{ij}\bar{\ell}_{Li}\ell_{Rj}h_d\left(1+\frac{h}{v}\right) + \text{h.c.},$$

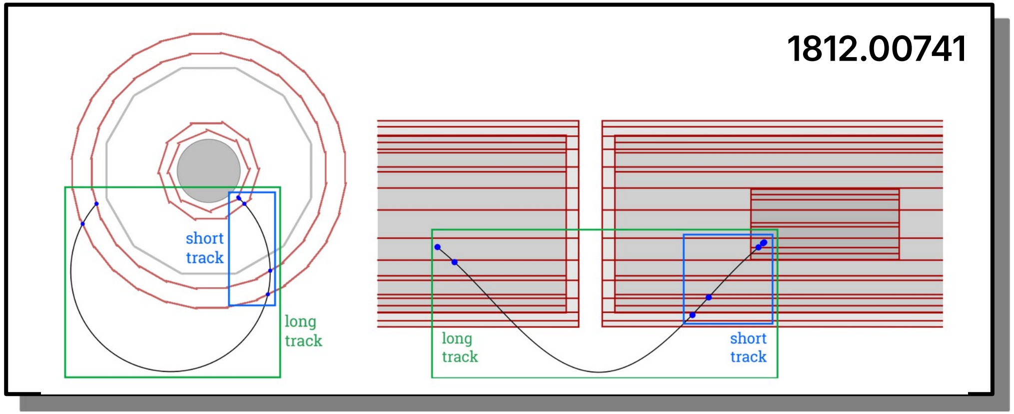
μ → eeeee

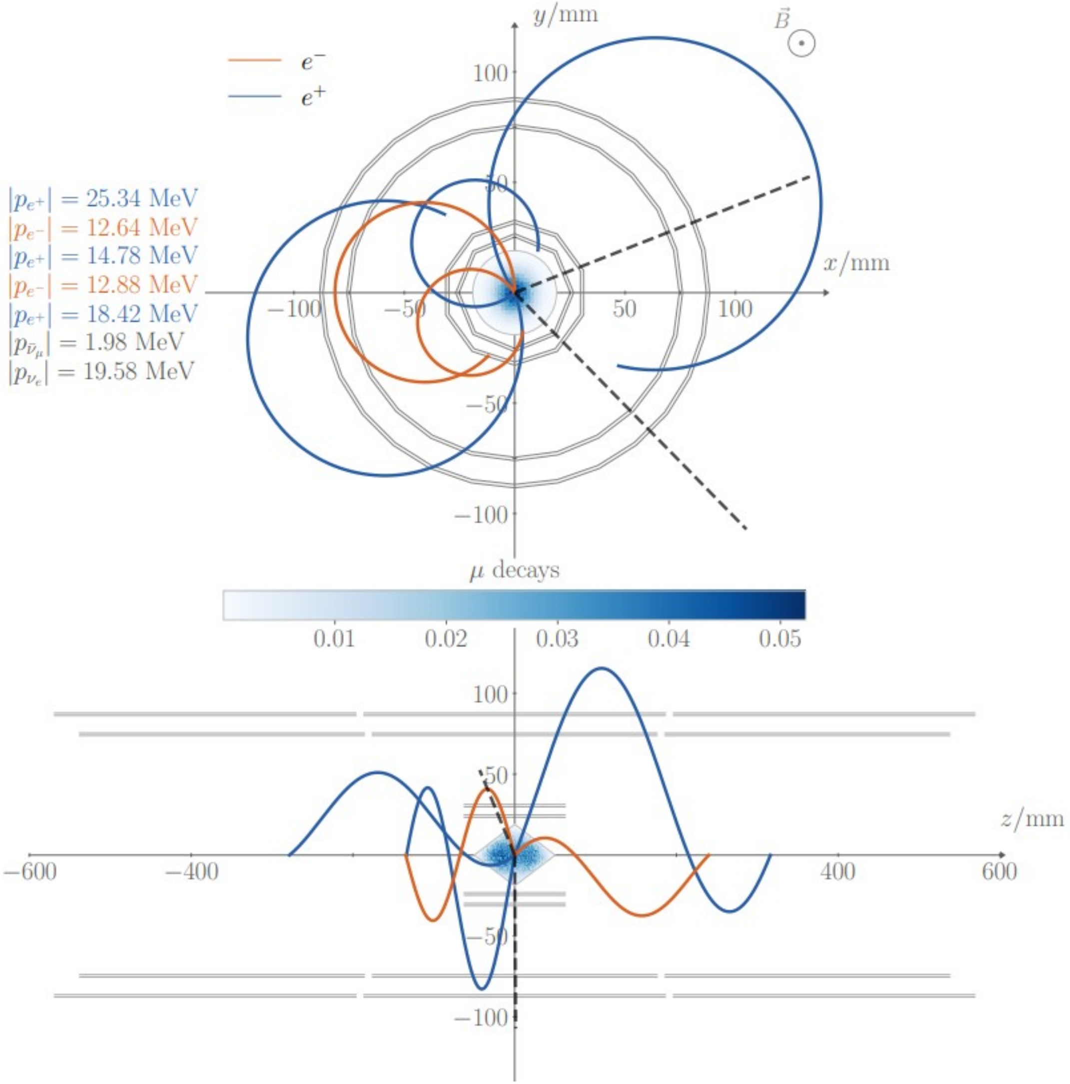
- **Leads to cascade decay**



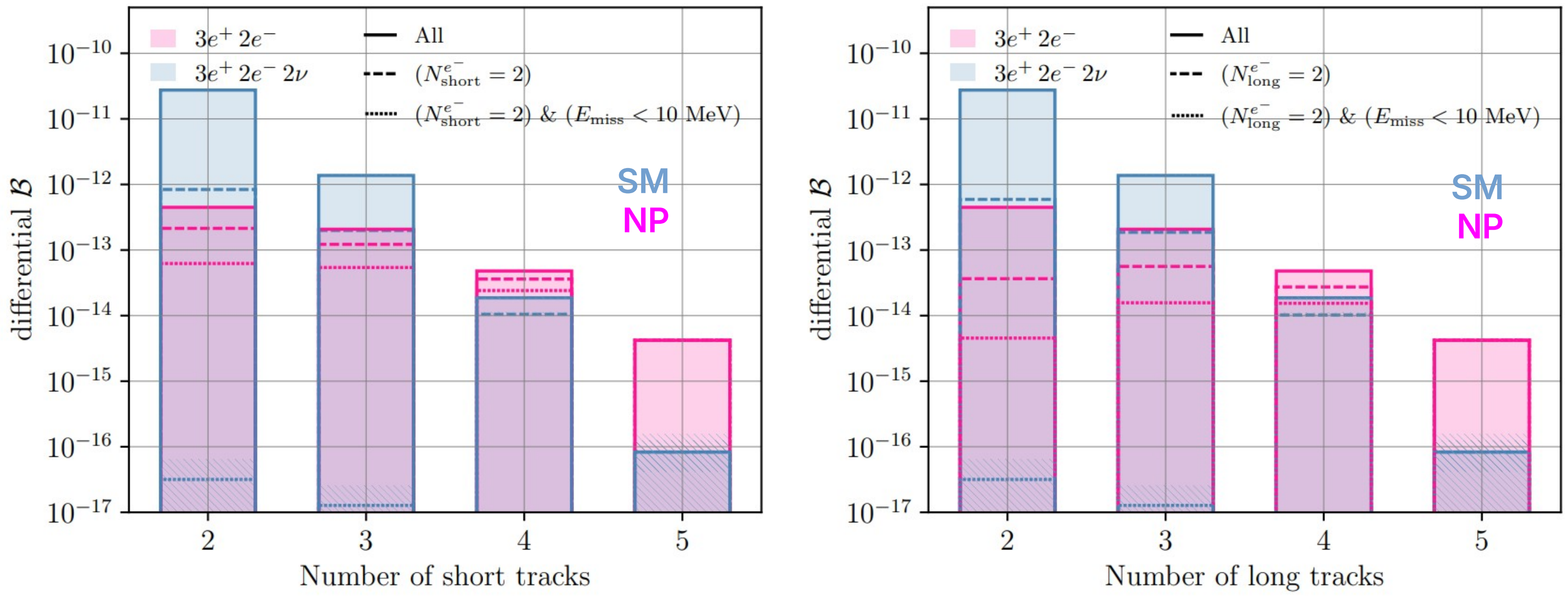
Signatures

- Momentum of tracks must be reconstructed from energy deposits or ‘hits’ in the layers of the detectors.
- 4 hits = short track
- 6+ hits = long track

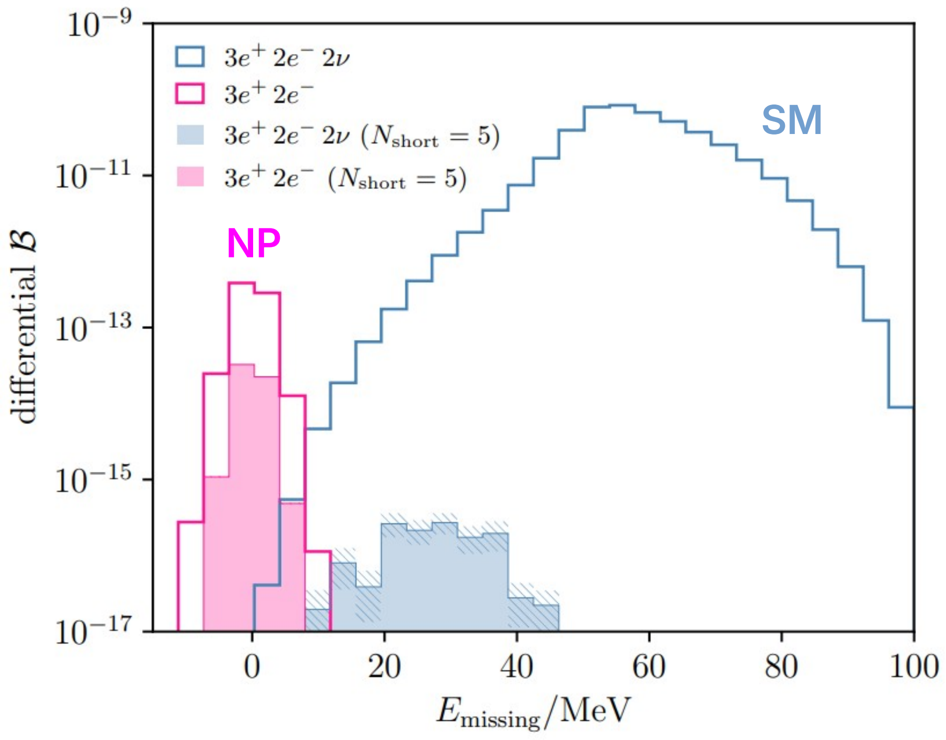


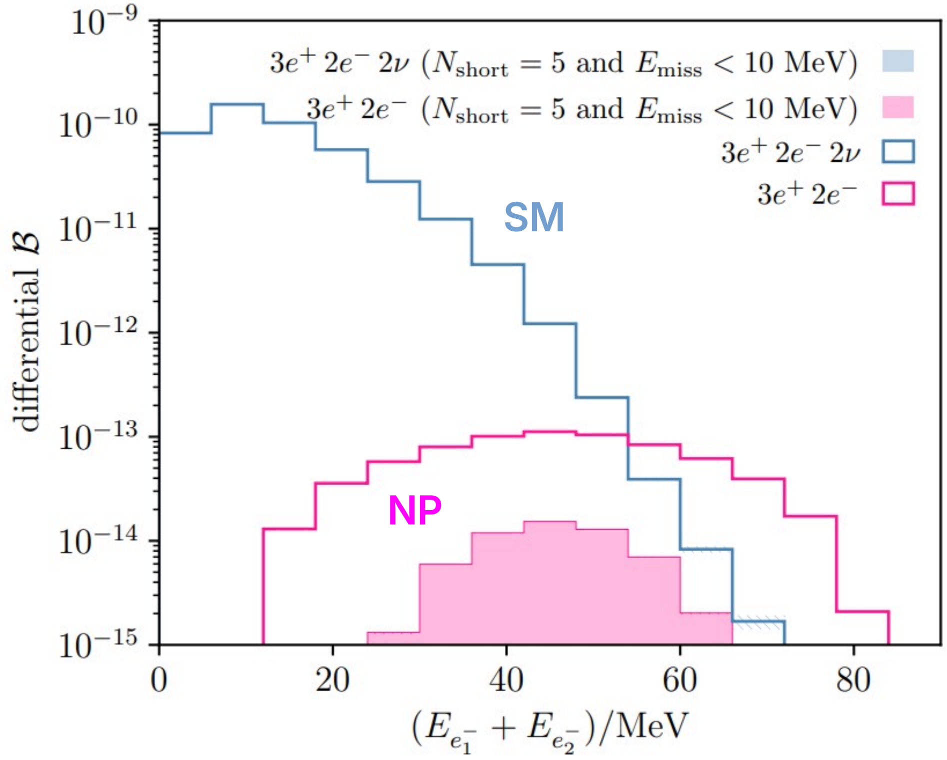


Signatures

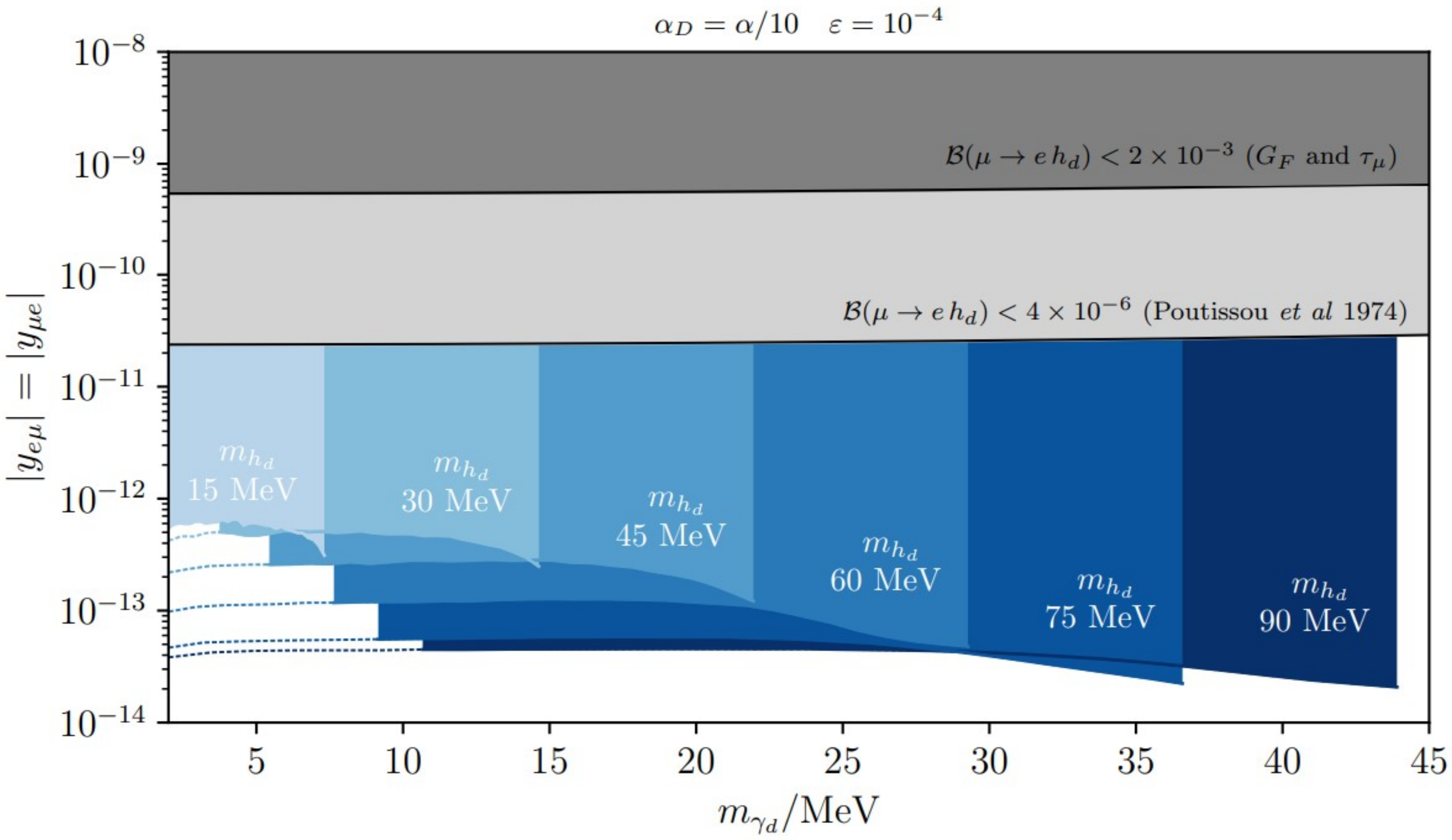

$3e^+ 2e^-$
$3e^+ 2e^- 2\nu$
All
$(N^{e^-}_{\text{short}} = 2)$
$(N^{e^-}_{\text{short}} = 2)$ & $(E_{\text{miss}} < 10$ MeV$)$
SM
NP
differential $\mathcal{B}$
10^{-10}
10^{-11}
10^{-12}
10^{-13}
10^{-14}
10^{-15}
10^{-16}
10^{-17}
2
3
4
5
Number of short tracks
$3e^+ 2e^-$
$3e^+ 2e^- 2\nu$
All
$(N^{e^-}_{\text{long}} = 2)$
$(N^{e^-}_{\text{long}} = 2)$ & $(E_{\text{miss}} < 10$ MeV$)$
SM
NP
differential $\mathcal{B}$
10^{-10}
10^{-11}
10^{-12}
10^{-13}
10^{-14}
10^{-15}
10^{-16}
10^{-17}
2
3
4
5
Number of long tracks

Signatures


$3e^+ 2e^- 2\nu$
$3e^+ 2e^-$
$3e^+ 2e^- 2\nu$ ($N_{\text{short}} = 5$)
$3e^+ 2e^-$ ($N_{\text{short}} = 5$)
SM
NP
differential $\mathcal{B}$
E_{missing}/MeV

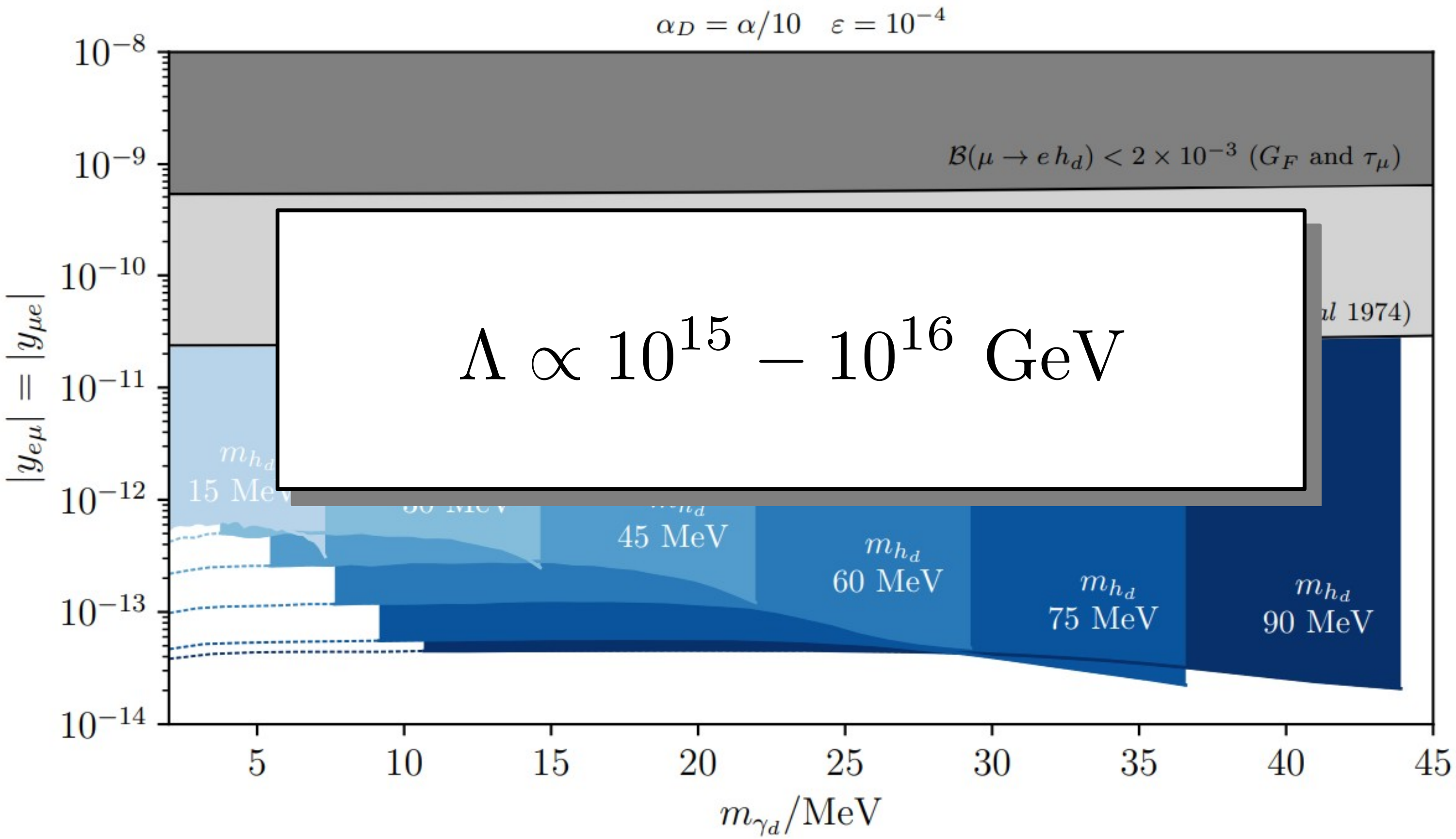

$3e^+ 2e^- 2\nu$ ($N_{\text{short}} = 5$ and $E_{\text{miss}} < 10$ MeV)
$3e^+ 2e^-$ ($N_{\text{short}} = 5$ and $E_{\text{miss}} < 10$ MeV)
$3e^+ 2e^- 2\nu$
$3e^+ 2e^-$
SM
NP
differential $\mathcal{B}$
$(E_{e_1^-} + E_{e_2^-})$/MeV

Reach

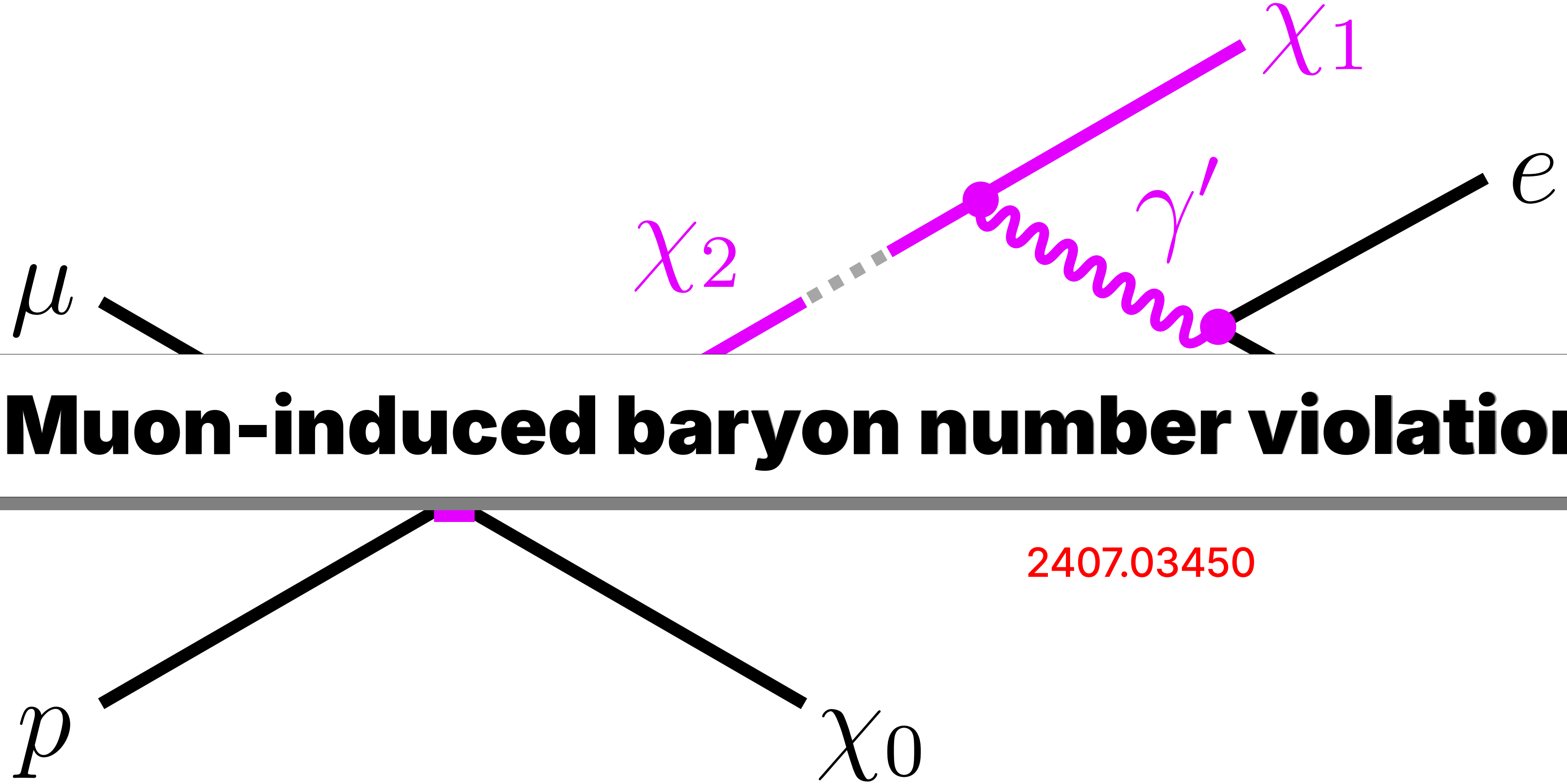


$\alpha_D = \alpha/10 \quad \varepsilon = 10^{-4}$
$\mathcal{B}(\mu \to e\, h_d) < 2 \times 10^{-3}$ (G_F and τ_μ)
$\mathcal{B}(\mu \to e\, h_d) < 4 \times 10^{-6}$ (Poutissou *et al* 1974)
m_{h_d} 15 MeV
m_{h_d} 30 MeV
m_{h_d} 45 MeV
m_{h_d} 60 MeV
m_{h_d} 75 MeV
m_{h_d} 90 MeV
$|y_{e\mu}| = |y_{\mu e}|$
m_{γ_d}/MeV
10^{-8}
10^{-9}
10^{-10}
10^{-11}
10^{-12}
10^{-13}
10^{-14}
5
10
15
20
25
30
35
40
45

Reach



$$\Lambda \propto 10^{15} - 10^{16}\ \text{GeV}$$



Muon-induced baryon number violation

2407.03450

Widening the view

- Can the CLFV signal @ Mu2e stem from non-CLFV physics?
- NP scenarios where E_e>105 MeV?
  - Can we take advantage of the huge energy reservoir that is the nucleus?

$\mu\,^{27}\text{Al} \to\,^{26}\text{Mg}^{(*)}\chi_0\chi_1 ee$

- Three dark states + $U(1)_D$ + SM portal via kinetic mixing

$$\mathcal{L}_{\mu p} = G_{\mu p}(\bar{p}\chi_2)(\bar{\mu}\chi_0) + \text{h.c.}$$

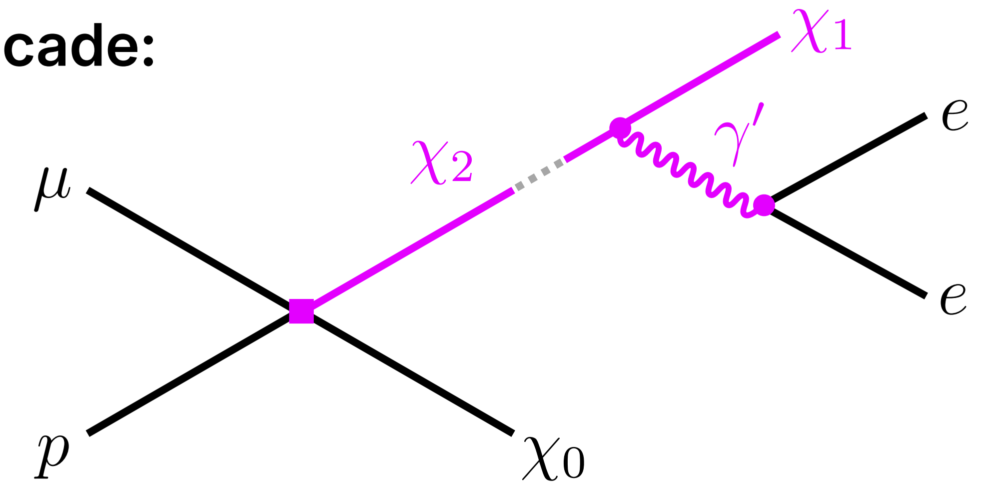
$$\mathcal{L}_{eA'} = e\varepsilon A'_\mu J^\mu_{\text{EM}}$$

$$\mathcal{L}_\chi = g_D\big(\bar{\chi}_2\gamma^\mu\chi_1\big)A'_\mu + \text{h.c.}$$

$\mu\,{}^{27}\mathrm{Al} \to {}^{26}\mathrm{Mg}^{(*)}\chi_0\chi_1 ee$

Benchmark: $m_2 = 1030$ MeV, $m_1 = 900$ MeV, $m_0 = 0$, $m_{A'} = 20$ MeV, $G_{\mu p} = (300\ \mathrm{TeV})^{-2}$, $\varepsilon = 10^{-4}$, $\alpha_D = 10^{-3}$

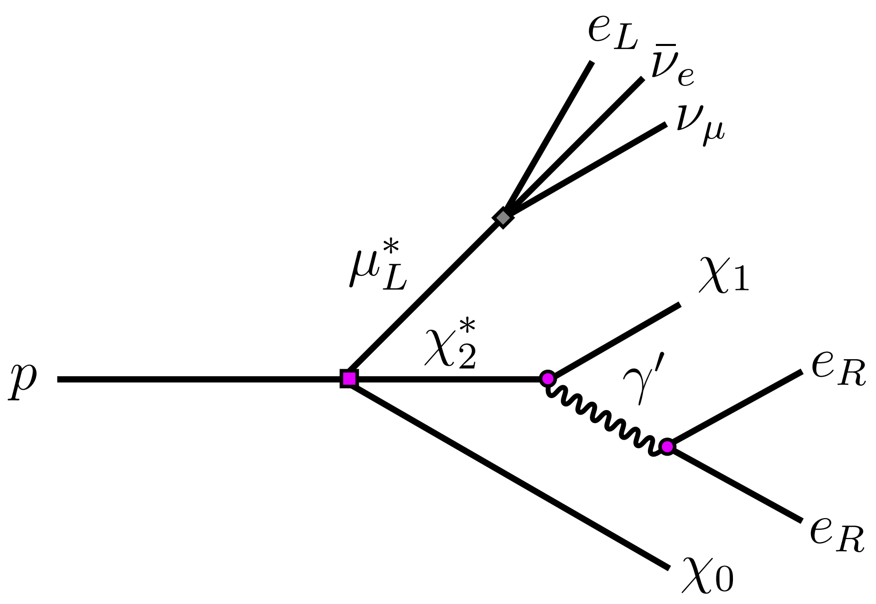
- **Leads to the cascade:**



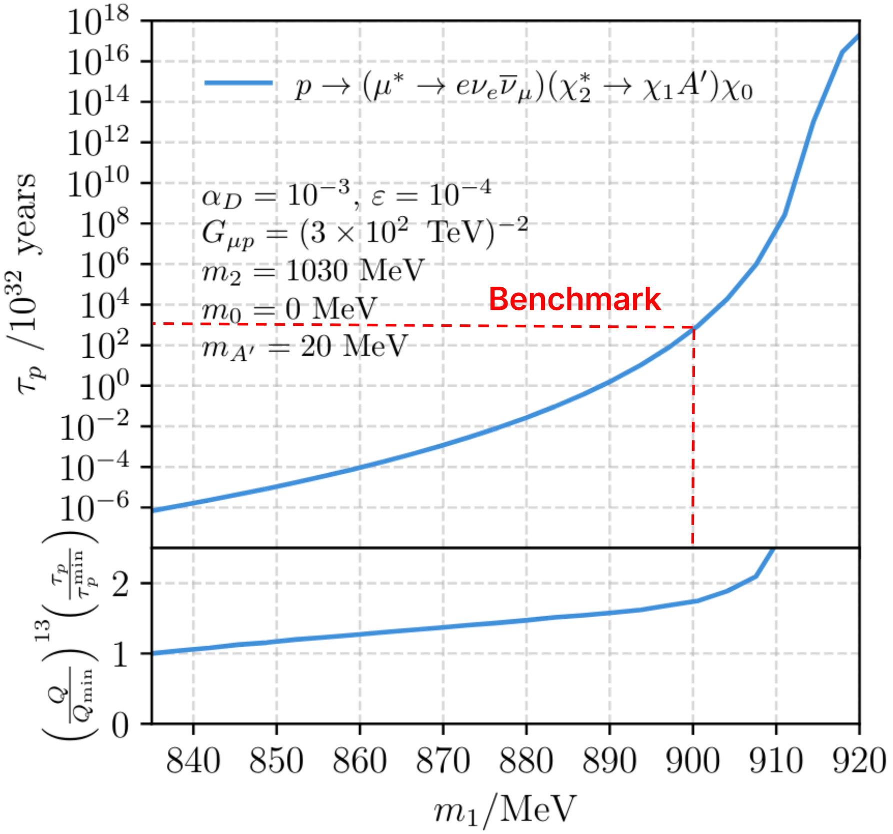
~4 × 10^-3^

$$\Gamma(\mu\,{}^{27}\mathrm{Al} \to \chi_0\chi_2\,{}^{26}\mathrm{Mg}) \simeq r_{\mathrm{p.s.}}\frac{G_{\mu p}^2}{G_F^2}\Gamma(\mu\,{}^{27}\mathrm{Al} \to \nu_\mu n\,{}^{26}\mathrm{Mg}) \longrightarrow R \equiv \frac{\Gamma_{\mathrm{exotic}}}{\Gamma_{\mu\mathrm{Al}}} \sim 10^{-15}$$

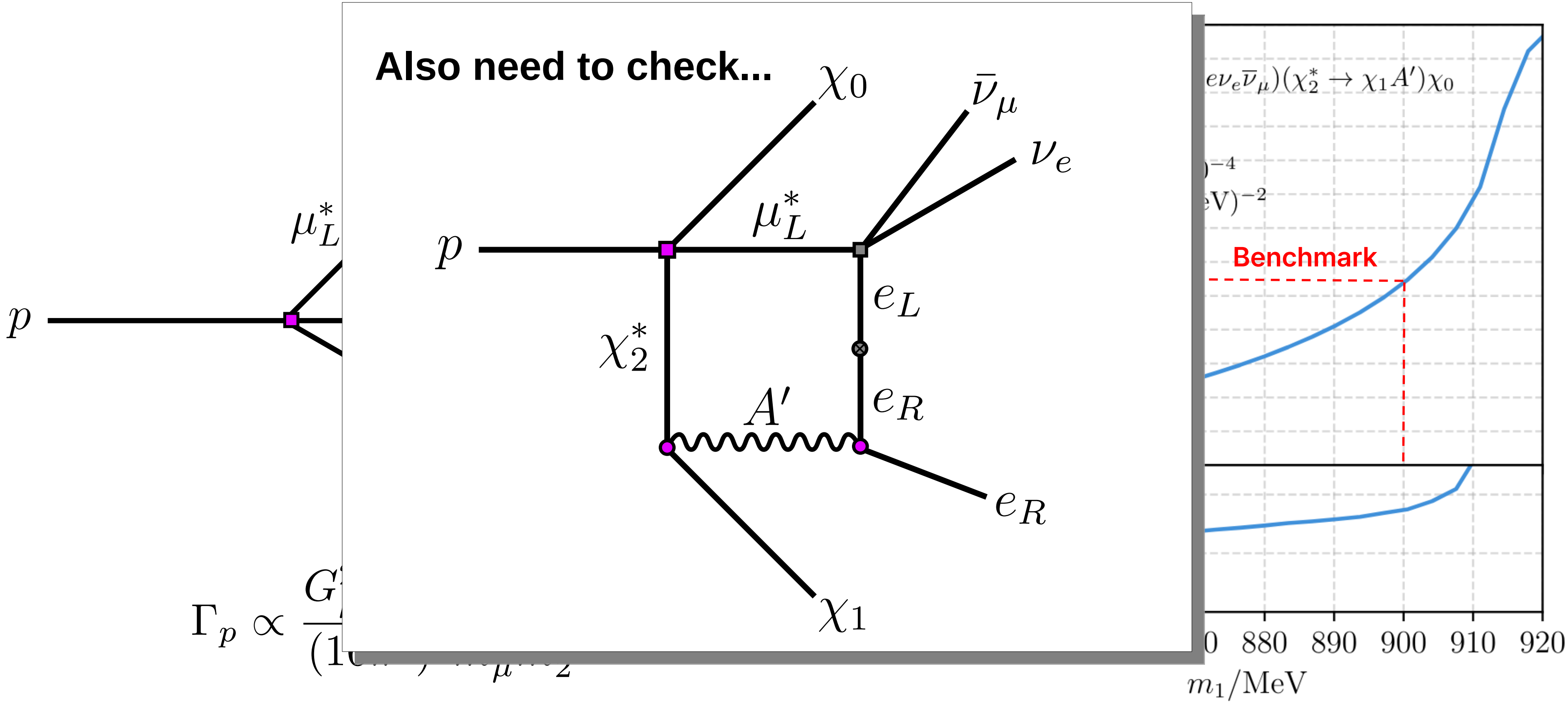
Tug-of-war with nucleon decay



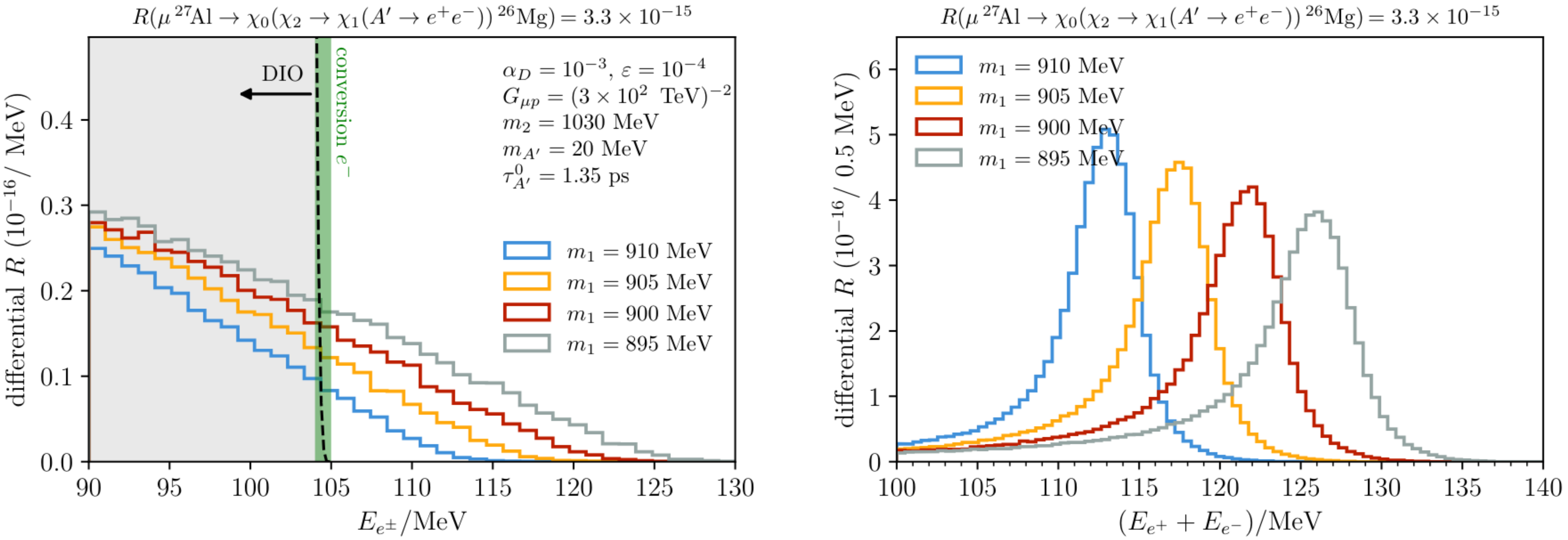
$$\Gamma_p \propto \frac{G_{\mu p}^2 G_F^2 \alpha_D Q^{13}}{(16\pi^2)^4 m_\mu^2 m_2^2}$$



Tug-of-war with nucleon decay



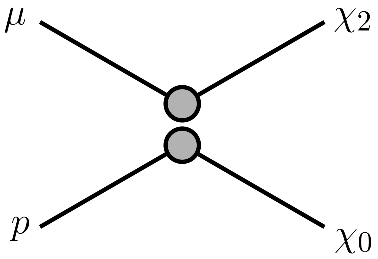
Signature

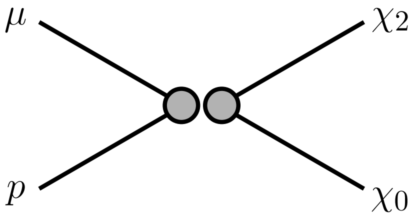

$R(\mu\,^{27}\mathrm{Al} \to \chi_0(\chi_2 \to \chi_1(A' \to e^+e^-))\,^{26}\mathrm{Mg}) = 3.3 \times 10^{-15}$
DIO
conversion e^-
$\alpha_D = 10^{-3}$, $\varepsilon = 10^{-4}$
$G_{\mu p} = (3 \times 10^2\ \mathrm{TeV})^{-2}$
$m_2 = 1030$ MeV
$m_{A'} = 20$ MeV
$\tau^0_{A'} = 1.35$ ps
$m_1 = 910$ MeV
$m_1 = 905$ MeV
$m_1 = 900$ MeV
$m_1 = 895$ MeV
differential R (10^{-16}/ MeV)
$E_{e^\pm}$/MeV
$R(\mu\,^{27}\mathrm{Al} \to \chi_0(\chi_2 \to \chi_1(A' \to e^+e^-))\,^{26}\mathrm{Mg}) = 3.3 \times 10^{-15}$
$m_1 = 910$ MeV
$m_1 = 905$ MeV
$m_1 = 900$ MeV
$m_1 = 895$ MeV
differential R (10^{-16}/ 0.5 MeV)
$(E_{e^+} + E_{e^-})$/MeV

A UV completion

$$\mathcal{L}_{\mu p} = G_{\mu p}(\bar{p}\chi_2)(\bar{\mu}\chi_0) + \text{h.c.}$$

- **What's the new physics reach? Naively:**

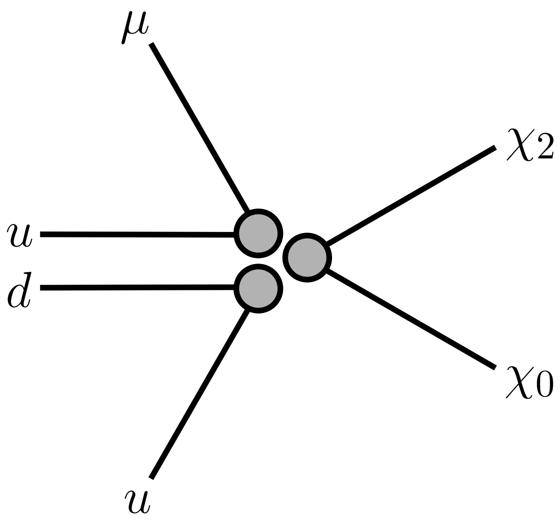




$$G_{\mu p} \simeq 10^{-8} G_F \to \Lambda \simeq 10^3 \text{ TeV}$$

but Λ isn't fundamental...

$$\frac{1}{\Lambda^2} \simeq \frac{\Lambda_{\text{QCD}}^3}{\Lambda_{\text{col.}}^{d_{\text{col.}}} \Lambda_{\text{sing.}}^{5-d_{\text{col.}}}}$$

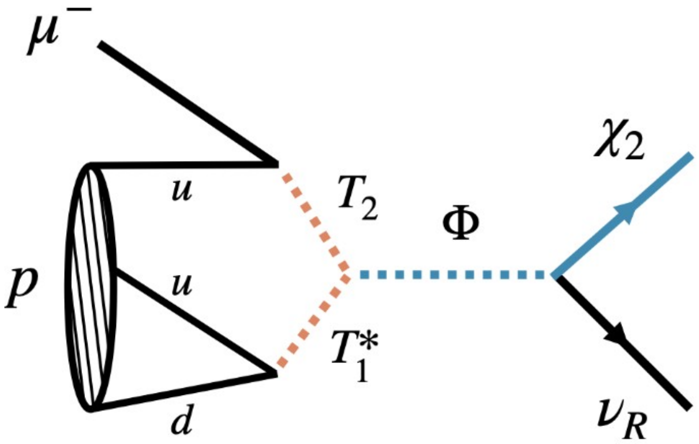


A UV completion

Introduce two colored scalars - a diquark T_1 and a leptoquark T_2 as well as a complex scalar Φ.

$$\mathscr{L} \supset -y_{ud}(\overline{u_R^{iC}} d_R^j)\epsilon_{ijk} T_1^k - y_{\mu u}(\overline{u_R^{iC}} \mu_R)(T_2^*)_i + \text{h.c.}$$

$$\mathscr{L} \supset \rho T_1^{k*} T_{2k} \Phi^* + y_\chi \Phi(\overline{\nu_R} \chi_{2R}^C) + \text{h.c.}$$



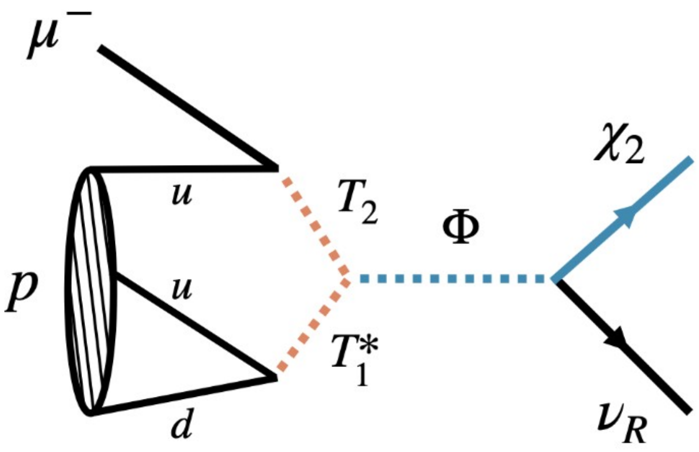
A UV completion

Introduce two colored scalars - a diquark T_1 and a leptoquark T_2 as well as a complex scalar Φ.

Integrate out T_1 and T_2, for off-shell Φ:

$$\mathscr{L}_{\text{eff}} = \frac{1}{\Lambda_{\text{col}}^3}\frac{1}{\Lambda_{\text{sin}}^2}\left(\overline{u_R^{iC}}\mu_R\right)\epsilon_{ijk}\left(\overline{u_R^{jC}}d_R^k\right)\left(\overline{\nu_R}\chi_{2R}^C\right) + \text{h.c.}$$

$$\frac{1}{\Lambda_{\text{col}}^3} = y_{ud}y_{\mu u}\frac{\rho}{m_{T_1}^2 m_{T_2}^2} \qquad \frac{1}{\Lambda_{\text{sin}}^2} = \frac{y_\chi}{m_\Phi^2}$$



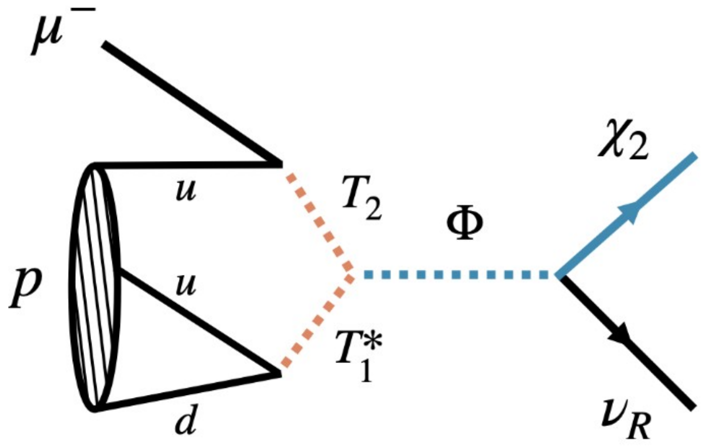
A UV completion

Introduce two colored scalars - a diquark T_1 and a leptoquark T_2 as well as a complex scalar Φ.

Integrate out T_1 and T_2, for off-shell Φ:

$$\widetilde{G}_{\mu p} \simeq 10^{-6} G_F$$

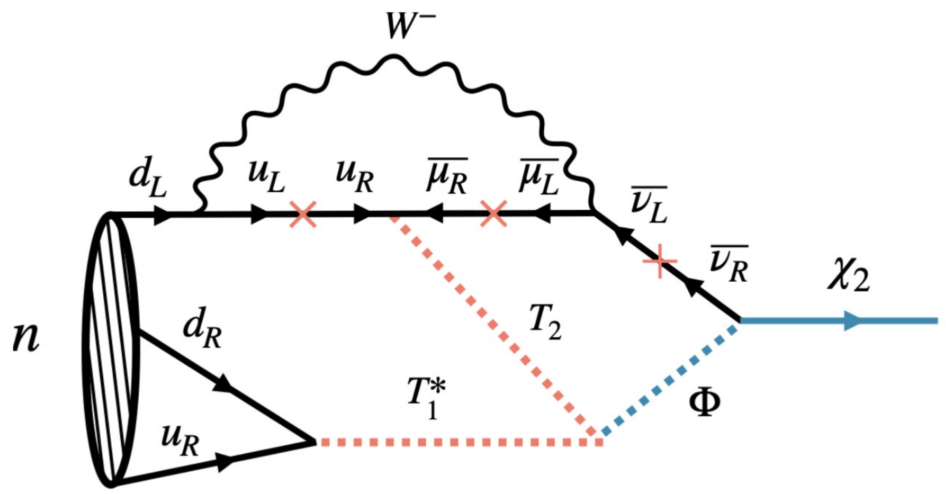
$$\sim 10^{-6} G_F \, y_{ud} y_{\mu u} y_{\chi} \left(\frac{1 \text{ TeV}}{\sqrt{m_{T_1} m_{T_2}}} \right)^4 \left(\frac{\rho}{4 \text{ TeV}} \right) \left(\frac{2 \text{ GeV}}{m_\Phi} \right)^2$$



A UV completion

Introduce tw ... uark T_2
as well as a

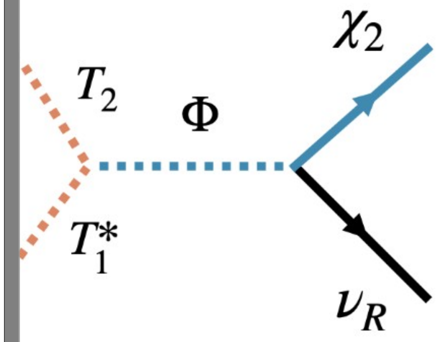
Also need to check neutron decay, neutron mixing...



Integrate out T_1 and

$\widetilde{G}_{\mu p} \simeq$

$\sim 10^{-6} G_F\, y_{ud} y_{\mu u} y_\chi \Big($



Conclusions

- New EFT tower for model building in $N\mu \to Ne$
  - Consistent nuclear EFT with all allowed nuclear responses
  - Rate can be computed easily using the MuonBridge software suite
- Mu3e should be able to see ~100s of $\mu \to eeeee\nu\nu$ events and can be very constraining on NP models (for the described model).
- We hope Mu3e adds the $\mu \to eeeee$ channel to their analysis pipeline
- Interesting non-CLFV physics at Mu2e (and COMET)
- We hope Mu2e + COMET add highly energetic electrons (above the conversion energy) to their analysis pipeline

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Thank you :)