Building bridges: an effective field theory for muon-to-electron conversion

Berkeley Nuclear Theory Seminar
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Tony Menzo

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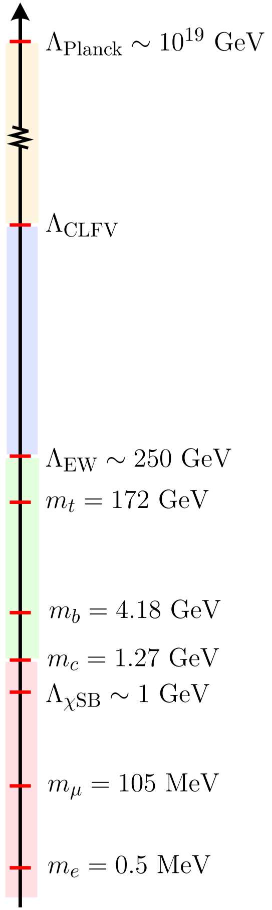


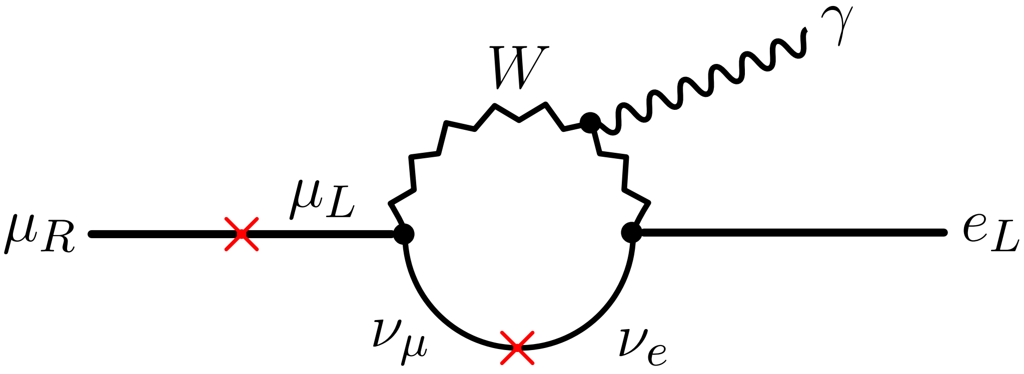
Based on 2405.xxxxx with Wick Haxton, Evan Rule, Ken McElvain, and Jure Zupan

μ → e

- **The Standard Model (SM) has an accidental global flavor symmetry**

$$U(1)_e \times U(1)_\mu \times U(1)_\tau$$

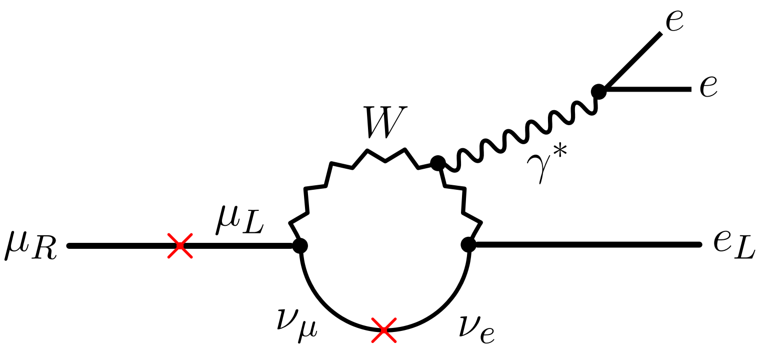
- **Because $m_\nu \neq 0$ charged-lepton-flavor violation (CLFV) can occur at one-loop**



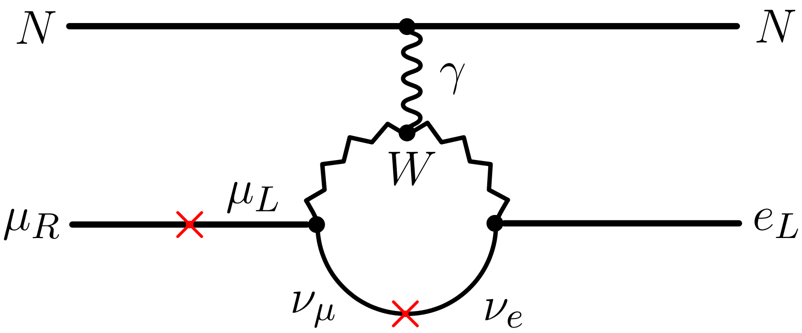
$$\mathrm{BR}(\mu \to e\gamma) \simeq \frac{3\alpha}{32\pi M_W^4} |U_{\mu 3} U_{e3}^* \Delta m_{31}^2 + U_{\mu 2} U_{e2}^* \Delta m_{21}^2|^2$$
$$\simeq 10^{-54}$$

μ → e

- μ → eee



- Nμ → Ne



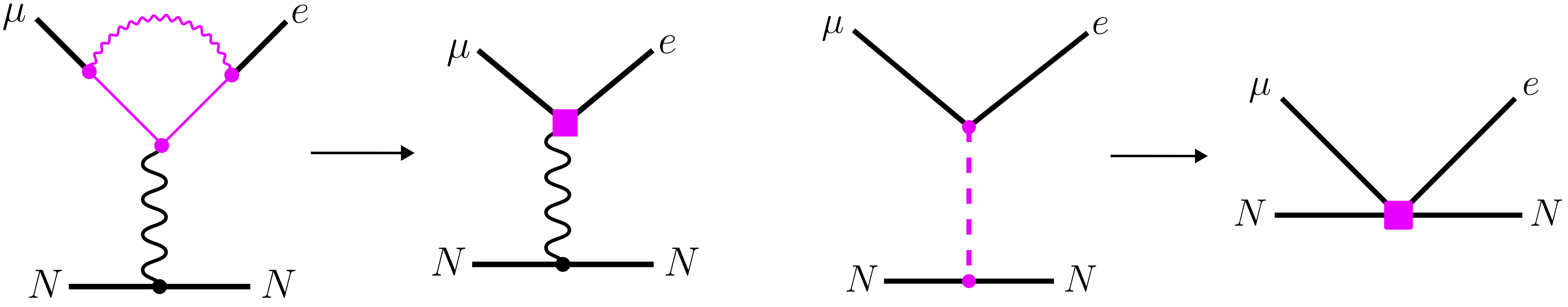
Bottom line: Observing CLFV = new physics

Exotic μ → e

In the space of all UV models, CLFV is common. For heavy new physics:

- “Photonic” – e.g. SUSY, heavy steriles, …
- “Contact” – e.g. Z’, leptoquarks, …

New physics

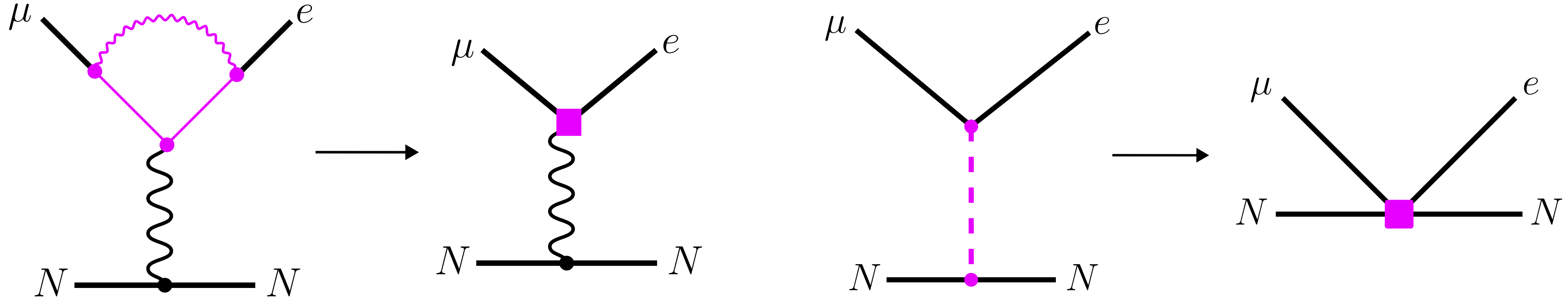


Exotic μ → e

In the space of all UV models, CLFV is common. For heavy new physics:

- "Photonic" – e.g. SUSY, heavy steriles, …
- "Contact" – e.g. Z', leptoquarks, …

New physics



Motivates an extensive experimental program

Upcoming experiments

- Mu → E Gamma (MEG) @ PSI - μ → eɣ

Projected: $\mathrm{BR}(\mu^+ \to e^+\gamma) \lesssim 6 \times 10^{-14}$

- Mu3e @ PSI - μ → eee

Projected: $\mathrm{BR}(\mu^+ \to e^+ e^- e^+) \lesssim 10^{-12}$

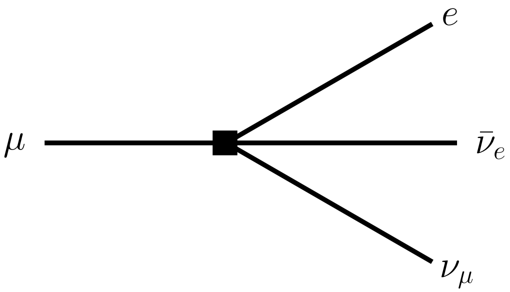
- Mu2e @ Fermilab, COMET @ J-PARC - Nμ → Ne

Projected: $\mathrm{CR}(\mu^- \mathrm{Al} \to e^- \mathrm{Al}) \lesssim 10^{-18} - 10^{-17}$

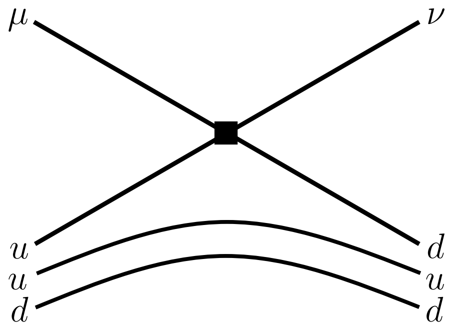
Upcoming experiments

Note that the muon width is extremely narrow:

$$\mathrm{CR} \equiv \frac{\Gamma(\mu \to e)}{\Gamma(\mu p \to e\nu)}, \mathrm{BR} \simeq \frac{\Gamma(\mu \to e)}{\Gamma(\mu \to e\nu\nu)}, \quad \Gamma(\mu \to e\nu\nu) = \frac{G_F^2 m_\mu^5}{192\pi^3} \simeq 10^{-16}\ \mathrm{MeV}$$



$^{27}\mathrm{Al}$
$\approx$



Projected: $\mathrm{CR}(\mu^- \mathrm{Al} \to e^- \mathrm{Al}) \lesssim 10^{-18} - 10^{-17}$

Upcoming experiments

- **Mu → E Gamma (MEG) @ PSI - μ → eɣ**

Projected: $\mathrm{BR}(\mu^+ \to e^+\gamma) \lesssim 6 \times 10^{-14}$ $(\Gamma(\mu \to e) \lesssim 10^{-33}\ \mathrm{GeV})$

- **Mu3e @ PSI - μ → eee**

Projected: $\mathrm{BR}(\mu^+ \to e^+e^-e^+) \lesssim 10^{-12}$ $(\Gamma(\mu \to e) \lesssim 10^{-31}\ \mathrm{GeV})$

- **Mu2e @ Fermilab, COMET @ J-PARC - Nμ → Ne**

Projected: $\mathrm{CR}(\mu^- \mathrm{Al} \to e^- \mathrm{Al}) \lesssim 10^{-18} - 10^{-17}$ $(\Gamma(\mu \to e) \lesssim 10^{-36}\ \mathrm{GeV})$

Upcoming experiments

- **Mu → E Gamma (MEG) @ PSI - μ → eɣ**

Projected: $\mathrm{BR}(\mu^+ \to e^+\gamma) \lesssim 6 \times 10^{-14}$ $(\Gamma(\mu \to e) \lesssim 10^{-10}\ \mathrm{Hz})$

- **Mu3e @ PSI - μ → eee**

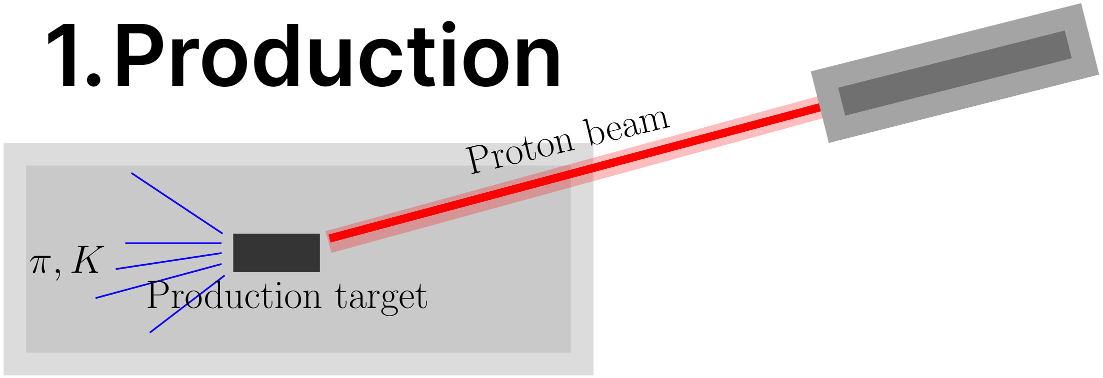
Projected: $\mathrm{BR}(\mu^+ \to e^+e^-e^+) \lesssim 10^{-12}$ $(\Gamma(\mu \to e) \lesssim 10^{-8}\ \mathrm{Hz})$

- **Mu2e @ Fermilab, COMET @ J-PARC - Nμ → Ne**

Projected: $\mathrm{CR}(\mu^- \mathrm{Al} \to e^- \mathrm{Al}) \lesssim 10^{-18} - 10^{-17}$ $(\Gamma(\mu \to e) \lesssim 10^{-13}\ \mathrm{Hz})$

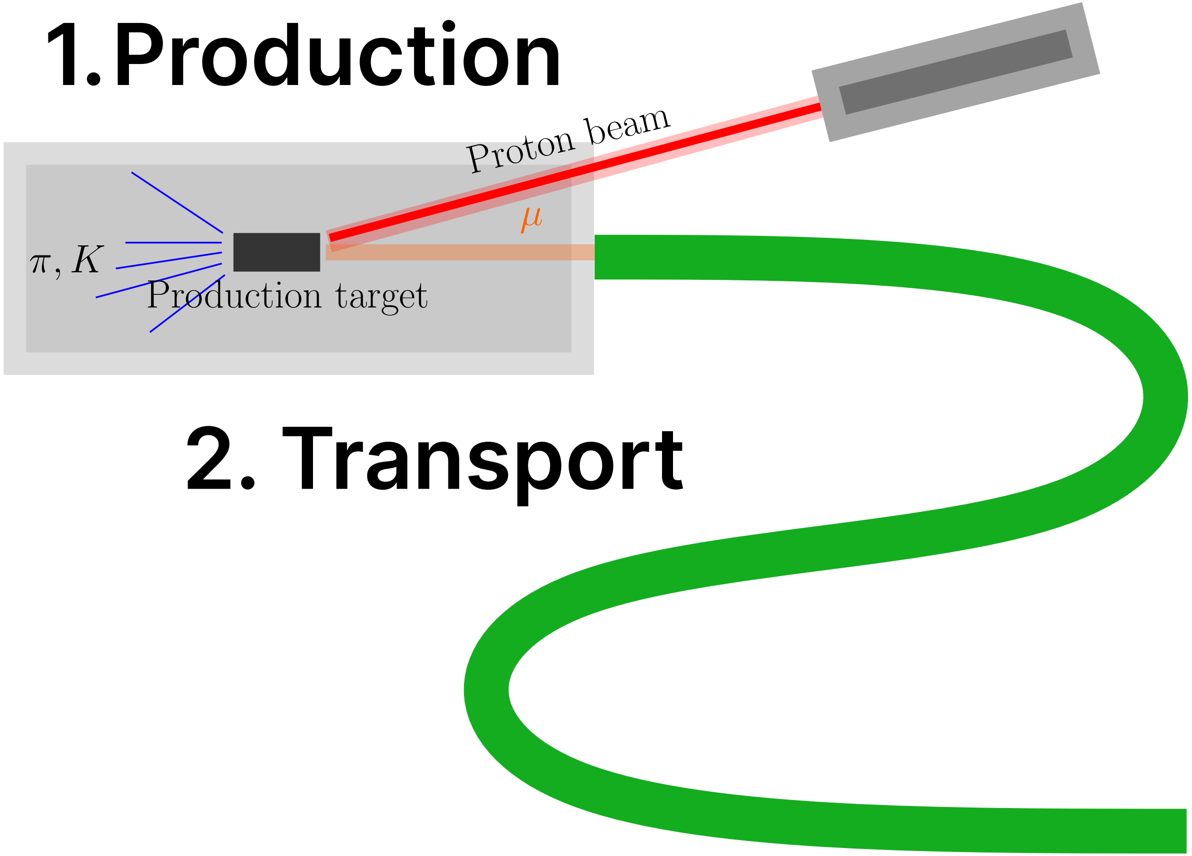
Mu2e

1. Production



Mu2e

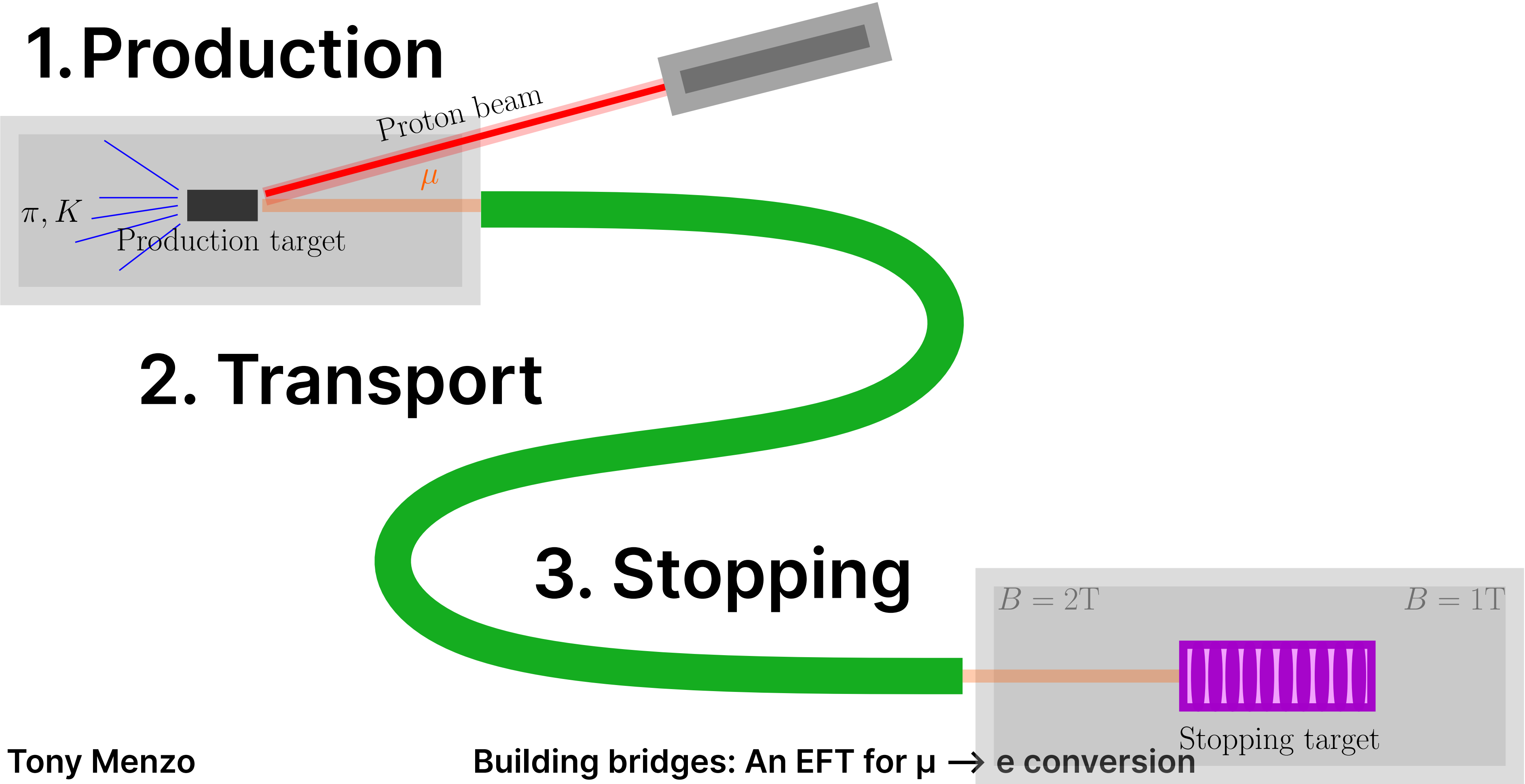
1. Production



2. Transport

Mu2e

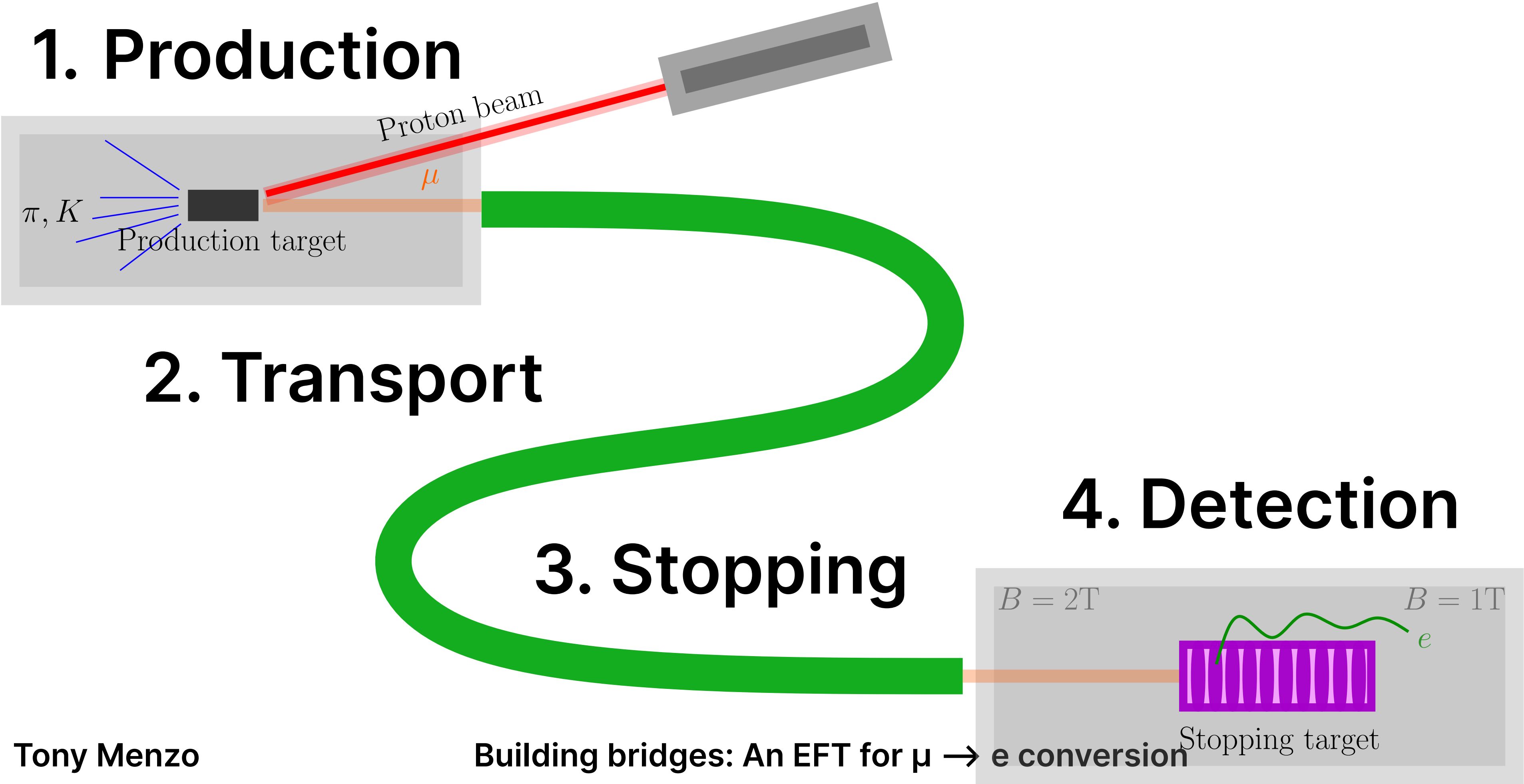
1. Production



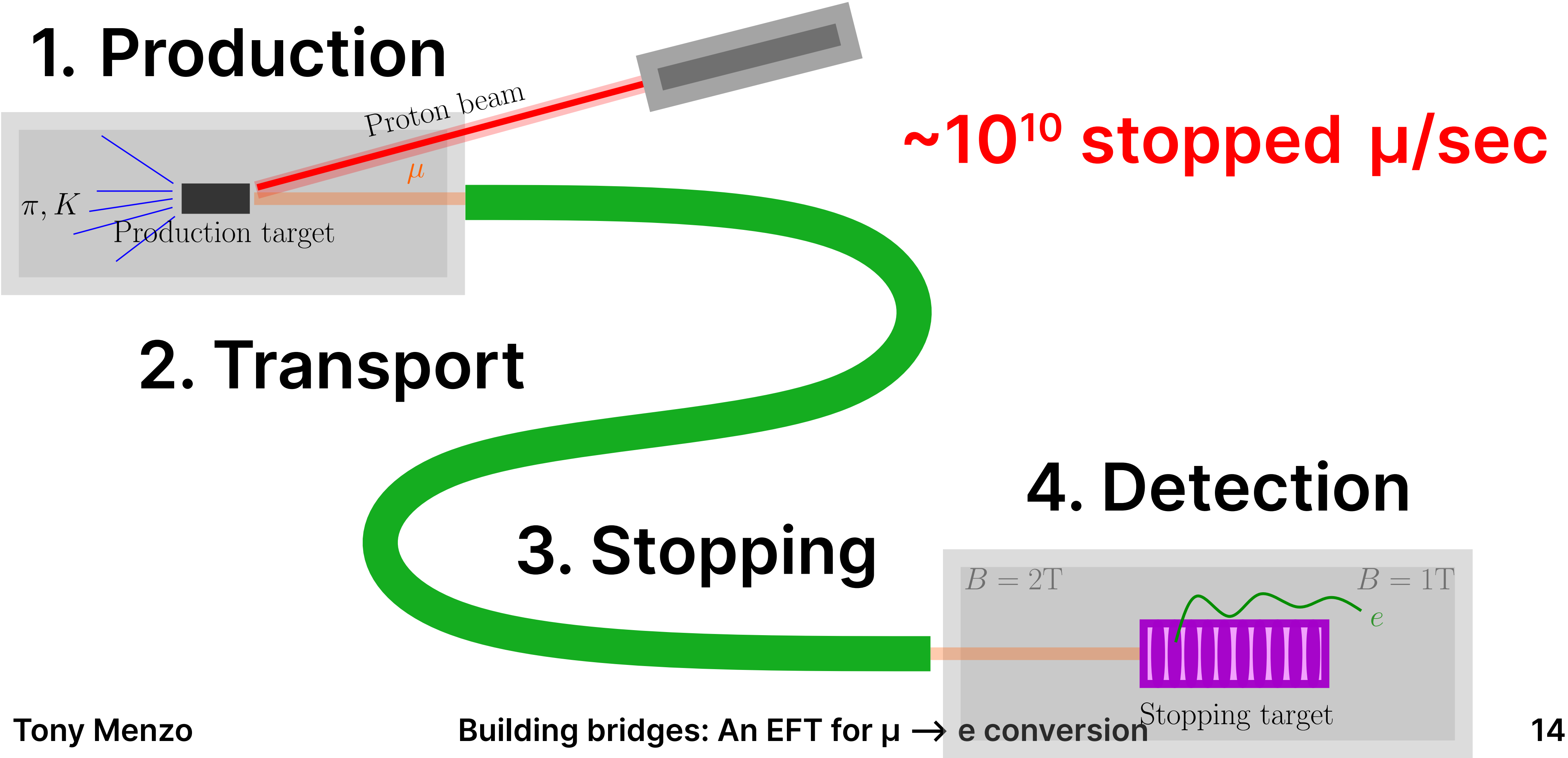
2. Transport

3. Stopping

Mu2e

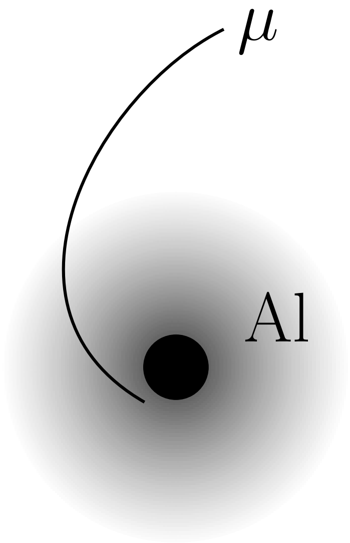

1. Production
Proton beam
μ
π, K
Production target
2. Transport
3. Stopping
4. Detection
$B = 2\mathrm{T}$
$B = 1\mathrm{T}$
e
Stopping target

Mu2e


1. Production
Proton beam
μ
π, K
Production target
~10^{10} stopped μ/sec
2. Transport
3. Stopping
4. Detection
$B = 2\mathrm{T}$
$B = 1\mathrm{T}$
e
Stopping target

Trapped muon

- When trapped the muon quickly cascades into 1s orbital



Three options:

1. Decay in orbit
2. Be captured by the nucleus
3. Convert to an electron
   a) Mono-energetic electron signal ($E_e = m_\mu$)

The μ → e rate

- **Dependent on UV physics + nuclear physics**
- **Can the rate be factorized?**

$$\Gamma \overset{?}{\propto} \text{leptonic} \times \text{nuclear}$$

- **Finally, can this be done in consistent theoretical truncation?**

The μ → e rate

- Dependent on UV physics + nuclear physics
- Can the rate be factorized?

$$\Gamma \overset{?}{\propto} \text{leptonic} \times \text{nuclear}$$

- Finally, can this be done in consistent theoretical truncation?

Effective field theory to the rescue!

Effective field theories

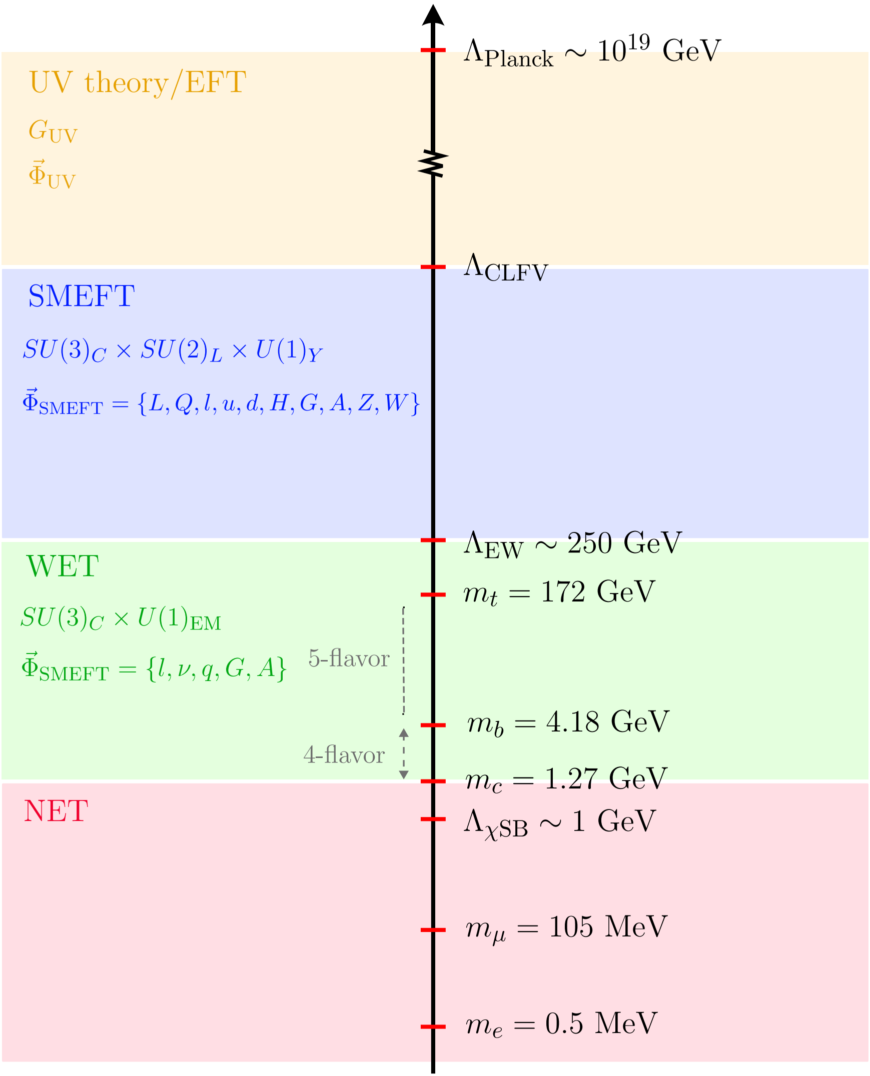
- Ingredients:
    1. Scale (Λ)
    2. Fields ($\vec{\Phi}$)
    3. Interactions ($\{\mathcal{O}\}$)
        (a) Parameters (c)
- Expansion parameter(s) - $(1/\Lambda)^d$, or other relevant dimensionless parameters

EFT towers

- Consider a series of EFTs
- EFT tower ingredients
  1. Scales (Λ_i)
  2. Fields ($\vec{\Phi}_i$)
  3. Interactions ($\{\mathcal{O}\}_i$)
  4. Matching conditions $c_I^{\rm IR}(\{\vec{c}^{\rm UV}\})$

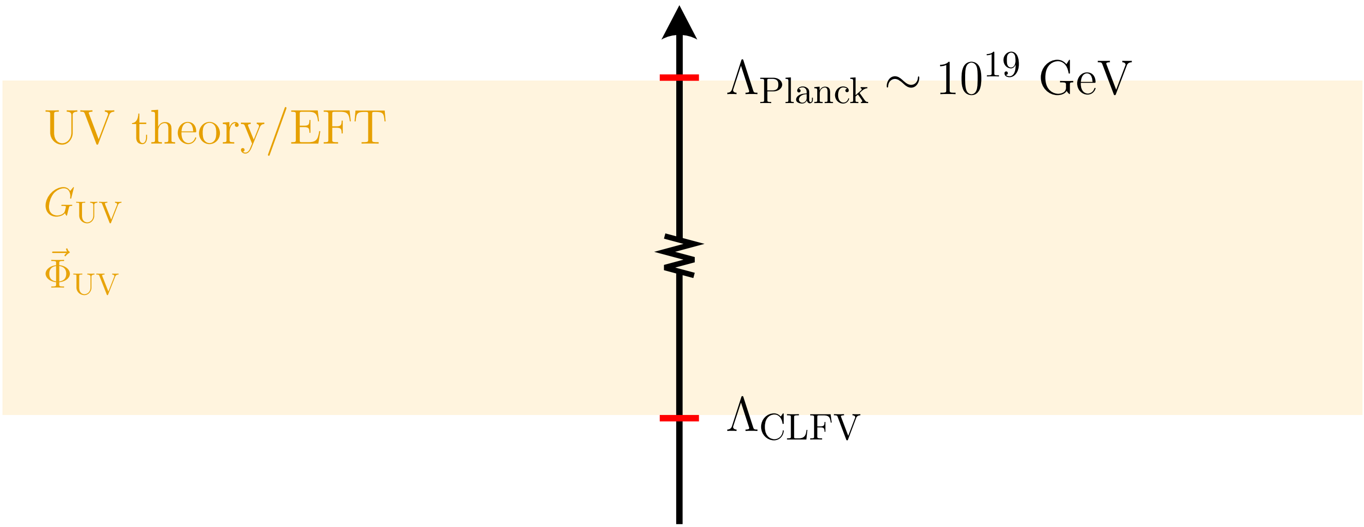
EFT tower

- The full tower: three/ four connected EFTs come together to output the conversion rate



EFT tower

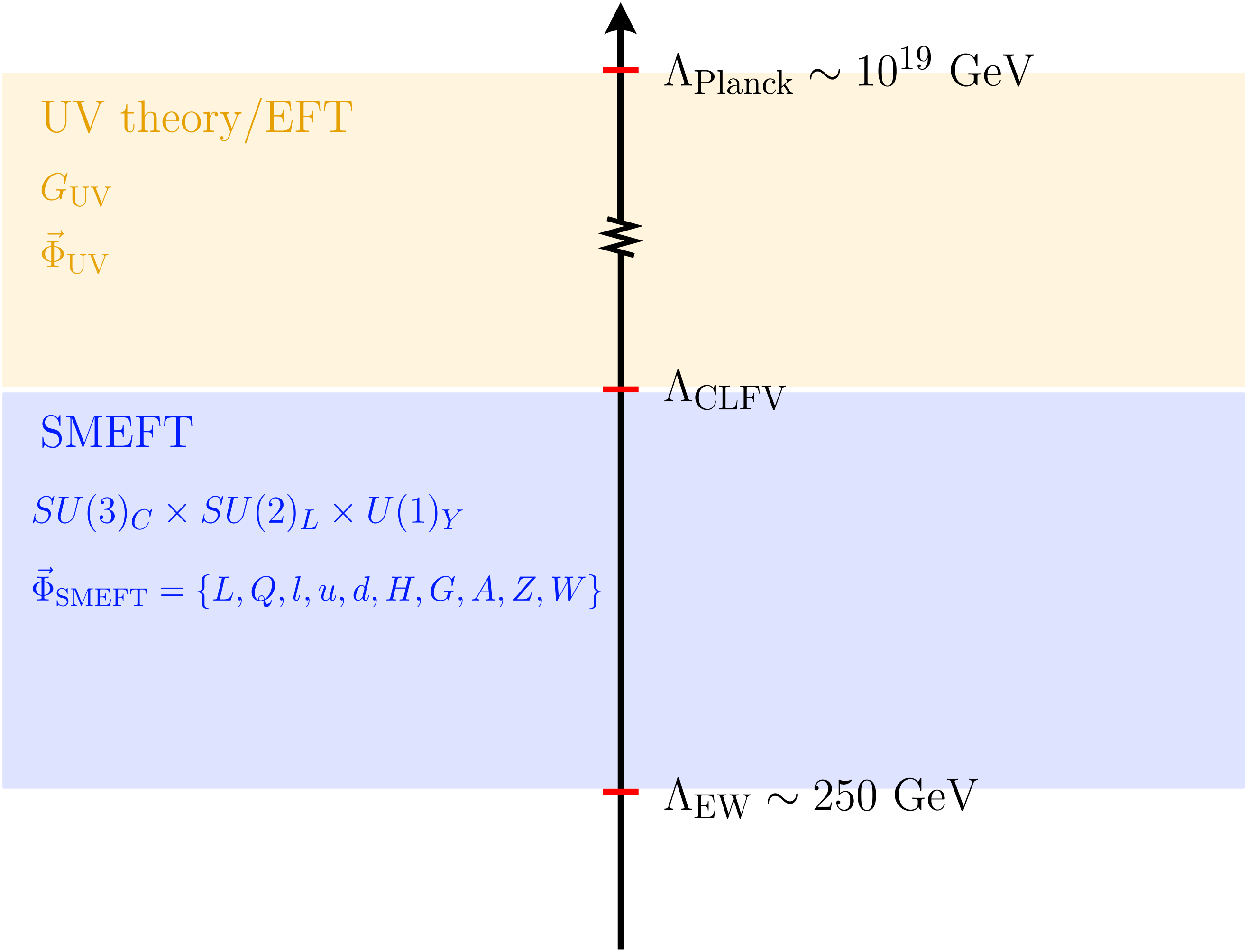
- UV



$\Lambda_{\mathrm{Planck}} \sim 10^{19}$ GeV
UV theory/EFT
G_{UV}
$\vec{\Phi}_{\mathrm{UV}}$
Λ_{CLFV}

EFT tower

- SMEFT



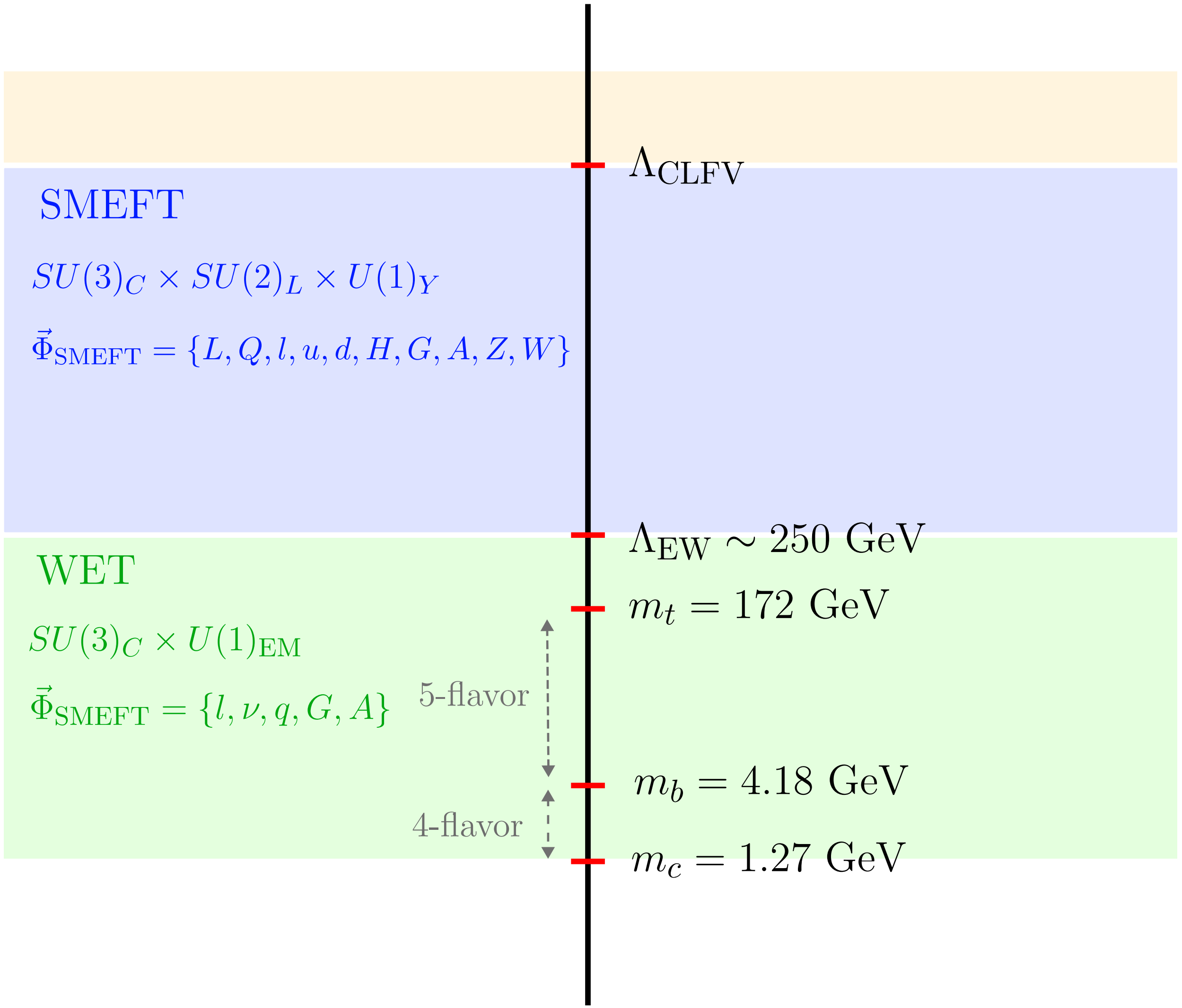
EFT tower

- SMEFT operators

$$
\begin{aligned}
Q_{e\varphi}: &\quad (\varphi^\dagger\varphi)(\bar{L}_2 e_R \varphi), \quad (\varphi^\dagger\varphi)(\bar{\ell}_1 \mu_R \varphi), \\
Q_{eW}: &\quad (\bar{\ell}_2 \sigma^{\mu\nu} e_R)\tau^I \varphi W^I_{\mu\nu}, \quad (\bar{\ell}_1 \sigma^{\mu\nu} \mu_R)\tau^I \varphi W^I_{\mu\nu}, \\
Q_{eB}: &\quad (\bar{\ell}_2 \sigma^{\mu\nu} e_R)\varphi B_{\mu\nu}, \quad (\bar{\ell}_1 \sigma^{\mu\nu} \mu_R)\varphi B_{\mu\nu}, \\
Q^{(1)}_{\varphi\ell}: &\quad (\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar{\ell}_2 \gamma^\mu \ell_1), \\
Q^{(3)}_{\varphi\ell}: &\quad (\varphi^\dagger i \overleftrightarrow{D}^I_\mu \varphi)(\bar{\ell}_2 \gamma^\mu \tau^I \ell_1), \\
Q_{\varphi e}: &\quad (\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar{\mu}_R \gamma^\mu \tau^I e_R), \\
Q^{(1)}_{\ell q}: &\quad (\bar{\ell}_2 \gamma^\mu \ell_1)(\bar{q}\gamma^\mu q), \\
Q^{(3)}_{\ell q}: &\quad (\bar{\ell}_2 \gamma^\mu \tau^I \ell_1)(\bar{q}\gamma^\mu \tau^I q), \\
Q_{eu}: &\quad (\bar{\mu}_R \gamma^\mu e_R)(\bar{u}_R \gamma^\mu u_R), \\
Q_{ed}: &\quad (\bar{\mu}_R \gamma^\mu e_R)(\bar{d}_R \gamma^\mu d_R), \\
Q_{\ell u}: &\quad (\bar{\ell}_2 \gamma^\mu \ell_1)(\bar{u}_R \gamma^\mu u_R), \\
Q_{\ell d}: &\quad (\bar{\ell}_2 \gamma^\mu \ell_1)(\bar{d}_R \gamma^\mu d_R), \\
Q_{qe}: &\quad (\bar{q}\gamma^\mu q)(\bar{\mu}_R \gamma^\mu e_R), \\
Q_{\ell edq}: &\quad (\bar{\ell}_2 e_R)(\bar{d}_R q), \quad (\bar{\ell}_1 \mu_R)(\bar{d}_R q), \\
Q^{(1)}_{\ell equ}: &\quad (\bar{\ell}^j_2 e_R)\varepsilon_{jk}(\bar{q}^k u), \quad (\bar{\ell}^j_1 \mu_R)\varepsilon_{jk}(\bar{q}^k u), \\
Q^{(3)}_{\ell equ}: &\quad (\bar{\ell}^j_2 \sigma_{\mu\nu} e_R)\varepsilon_{jk}(\bar{q}^k \sigma^{\mu\nu} u), \quad (\bar{\ell}^j_1 \sigma_{\mu\nu} \mu_R)\varepsilon_{jk}(\bar{q}^k \sigma^{\mu\nu} u),
\end{aligned}
$$

EFT tower

- WET



EFT tower

- **WET operators**

$$\mathcal{L}_{\text{eff}}^{\text{WET}} = \sum_{a,d} \hat{\mathcal{C}}_a^{(d)} \mathcal{Q}_a^{(d)},$$

$$\mathcal{Q}_{1,q}^{(6)} = (\bar{e}\gamma_\alpha\mu)(\bar{q}\gamma^\alpha q)\,, \qquad \mathcal{Q}_{2,q}^{(6)} = (\bar{e}\gamma_\alpha\gamma_5\mu)(\bar{q}\gamma^\alpha q)\,,$$

$$\mathcal{Q}_{3,q}^{(6)} = (\bar{e}\gamma_\alpha\mu)(\bar{q}\gamma^\alpha\gamma_5 q)\,, \qquad \mathcal{Q}_{4,q}^{(6)} = (\bar{e}\gamma_\alpha\gamma_5\mu)(\bar{q}\gamma^\alpha\gamma_5 q)\,.$$

$$\mathcal{Q}_{5,q}^{(6)} = (\bar{e}\mu)(\bar{q}q)\,, \qquad \mathcal{Q}_{6,q}^{(6)} = (\bar{e}i\gamma_5\mu)(\bar{q}q)\,,$$

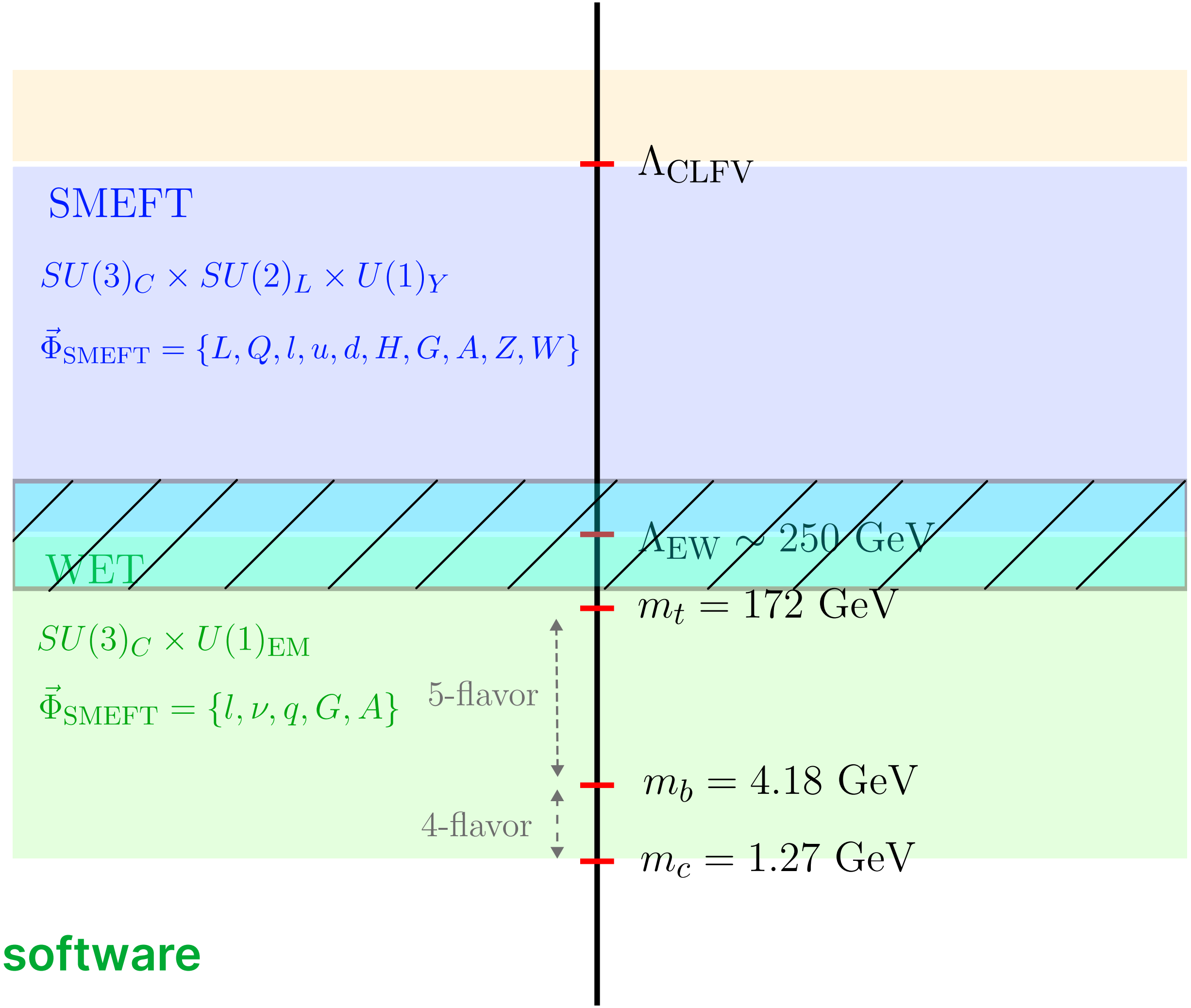
$$\mathcal{Q}_{7,q}^{(6)} = (\bar{e}\mu)(\bar{q}i\gamma_5 q)\,, \qquad \mathcal{Q}_{8,q}^{(6)} = (\bar{e}i\gamma_5\mu)(\bar{q}i\gamma_5 q)\,,$$

$$\mathcal{Q}_{9,q}^{(6)} = (\bar{e}\sigma^{\alpha\beta}\mu)(\bar{q}\sigma_{\alpha\beta}q)\,, \qquad \mathcal{Q}_{10,q}^{(6)} = (\bar{e}i\sigma^{\alpha\beta}\gamma_5\mu)(\bar{q}\sigma_{\alpha\beta}q)\,.$$

EFT tower

- **SMEFT-WET matching**

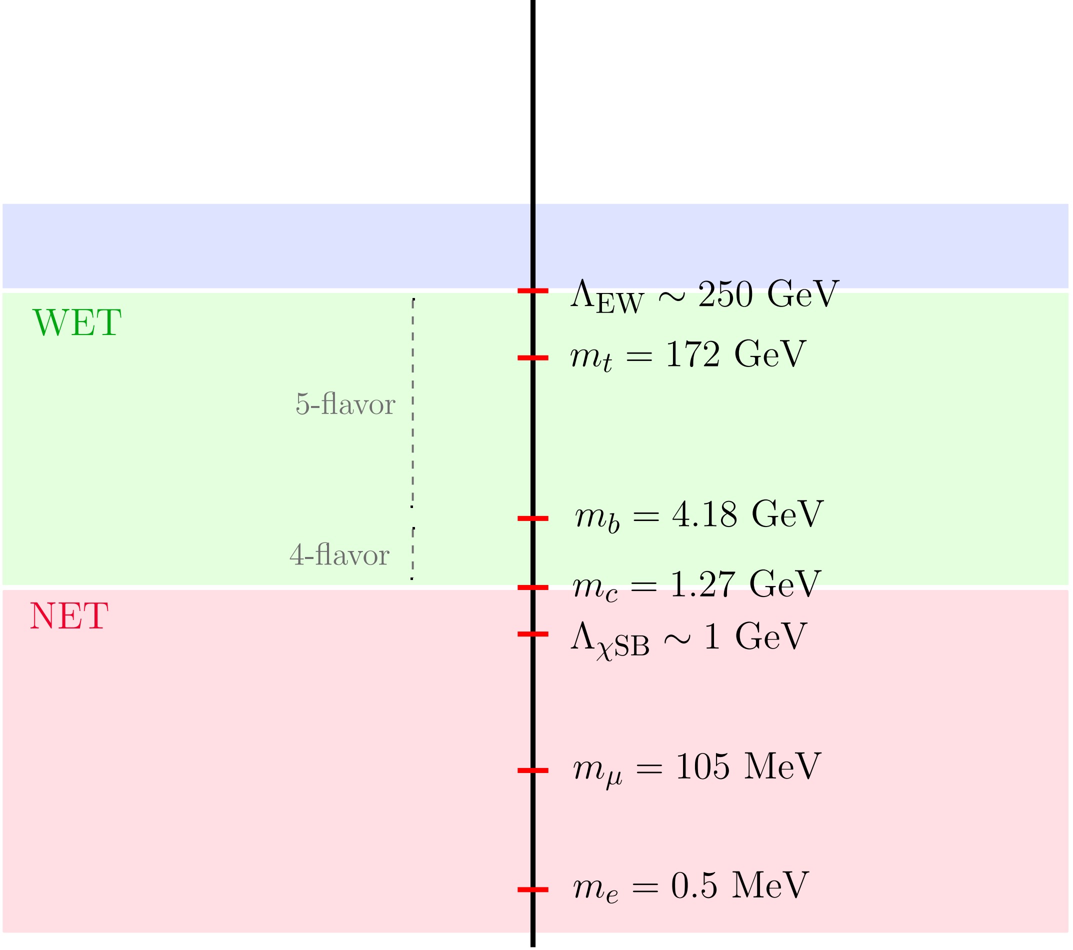
*Very well-developed literature related to running and matching for both EFTs, e.g. 1308.2627, 1310.4838, 1312.2014, 1709.04486, 1711.05270



Implemented in efficient public software

EFT tower

- NRET



EFT tower

- NRET


2208.07945

arXiv:2208.07945v3 [nucl-th] 1 Apr 2023

Nuclear-level Effective Theory of $\mu \to e$ Conversion: Formalism and Applications

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(Dated: April 4, 2023)

Over the next decade new $\mu \to e$ conversion searches at Fermilab (Mu2e) and J-PARC (COMET, DeeMe) are expected to advance limits on charged lepton flavor violation (CLFV) by more than four orders of magnitude. By considering the consequence of P and CP on elastic $\mu \to e$ conversion and the structure of possible charge and current densities, we show that rates are governed by six nuclear responses and a single scale, q/m_N, where $q \approx m_\mu$ is the momentum transferred from the leptons to the nucleus. To relate this result to microscopic formulations of CLFV, we construct in nonrelativistic effective theory (NRET) the CLFV nucleon-level interaction, pointing out the relevance of the dimensionless scales $y = (\frac{qb}{2})^2 > |\vec{v}_N| > |\vec{v}_\mu| > |\vec{v}_T|$, where b is the nuclear size, $\vec{v}_N$ and $\vec{v}_\mu$ are the nucleon and muon intrinsic velocities, and $\vec{v}_T$ is the target recoil velocity. We discuss previous work, noting the lack of a systematic treatment of the various small parameters. Because the parameter y is not small, a proper calculation of $\mu \to e$ conversion requires a full multipole expansion of the nuclear response functions, an apparently daunting task with Coulomb-distorted electron partial waves. We demonstrate that the multipole expansion can be carried out to high precision by introducing a simplifying local momentum q_{eff} for the electron. Previous work has been limited to simple charge or spin interactions, thereby treating the nucleus effectively as a point particle. We show that such formulations are not compatible with the general form of the $\mu \to e$ conversion rate, failing to generate three of the six allowed nuclear response functions. The inclusion of the nucleon velocity $\vec{v}_N$ yields an NRET with 16 operators and a rate of the general form. Consequently, in the current discovery era for CLFV, it provides the most sensible starting point for experimental analysis, defining what can and cannot be determined about CLFV from the highly exclusive process of $\mu \to e$ conversion. Finally, we expand the NRET operator basis to account for the effects of $\vec{v}_\mu$, associated with the muon's lower component, generating corrections to the CLFV coefficients of the point-nucleus response functions. Using advanced shell-model methods, we compute $\mu \to e$ conversion rates for a series of experimental targets, deriving bounds on the coefficients of the CLFV operators. These calculations are the first to include a general basis of CLFV operators, full evaluation of the associated nuclear response functions, and an accurate treatment of electron and muon Coulomb effects. We discuss target selection as an experimental "knob" that can be turned to probe the microscopic origins of CLFV. We describe two types of coherence that enhance certain CLFV operators and selection rules that blind elastic $\mu \to e$ conversion to others. We discuss the matching of the NRET onto higher level effective field theories, such as those constructed at the light quark level, noting opportunities to build on existing work in direct detection of dark matter. We discuss the relation of $\mu \to e$ conversion to $\mu \to e + \gamma$ and $\mu \to 3e$, showing how MEG II and Mu3e results will complement those of Mu2e and COMET. Finally we describe a accompanying script – in Mathematica and Python versions – that can be used to compute $\mu \to e$ conversion rates in various nuclear targets for the full set of NRET operators.

I. INTRODUCTION

Muon-to-electron conversion, in which a muon bound to a nucleus converts to a mono-energetic outgoing electron, occurs at an observable level only if there are new sources of flavor violation, beyond those responsible for neutrino mixing [1–4]. It has yond the standard model [5–7]. This has motivated a series of experimental advances that, in sum, have improved limits on $\mu \to e$ conversion rates by ≈12 orders of magnitude over the past 75 years [8].

The experimental attributes of $\mu \to e$ conversion are quite attractive. Intense muon beams exist, with rates on target of $\approx 10^{11}$/s expected in the

EFT tower

- NRET
- Expansion in dimensionless scales specific to $\mu \to e$

$$y \equiv \left(\frac{q}{2b}\right)^2 > |\vec{v}_N| > |\vec{v}_\mu| > |\vec{v}_T|$$

EFT tower

- NRET

$^{27}\mathrm{Al}$

- Expansion in dimensionless scales specific to μ → e

$$y \equiv \left(\frac{q}{2b}\right)^2 > |\vec{v}_N| > |\vec{v}_\mu| > |\vec{v}_T|$$

≈ 0.2

≈ 0.05

Harmonic oscillator parameter

$b = 1.85\ \mathrm{fm}$

Leptonic momentum transfer

$q \approx m_\mu = 1/1.86\ \mathrm{fm}$

EFT tower

- **NRET single-nucleon operators**

$$\mathcal{O}_1 = 1_L \; 1_N, \qquad \mathcal{O}_2' = 1_L \; i\hat{q} \cdot \vec{v}_N,$$

$$\mathcal{O}_3 = 1_L \; i\hat{q} \cdot [\vec{v}_N \times \vec{\sigma}_N], \qquad \mathcal{O}_4 = \vec{\sigma}_L \cdot \vec{\sigma}_N,$$

$$\mathcal{O}_5 = \vec{\sigma}_L \cdot (i\hat{q} \times \vec{v}_N), \qquad \mathcal{O}_6 = i\hat{q} \cdot \vec{\sigma}_L \; i\hat{q} \cdot \vec{\sigma}_N,$$

$$\mathcal{O}_7 = 1_L \; \vec{v}_N \cdot \vec{\sigma}_N, \qquad \mathcal{O}_8 = \vec{\sigma}_L \cdot \vec{v}_N,$$

$$\mathcal{O}_9 = \vec{\sigma}_L \cdot (i\hat{q} \times \vec{\sigma}_N), \qquad \mathcal{O}_{10} = 1_L \; i\hat{q} \cdot \vec{\sigma}_N,$$

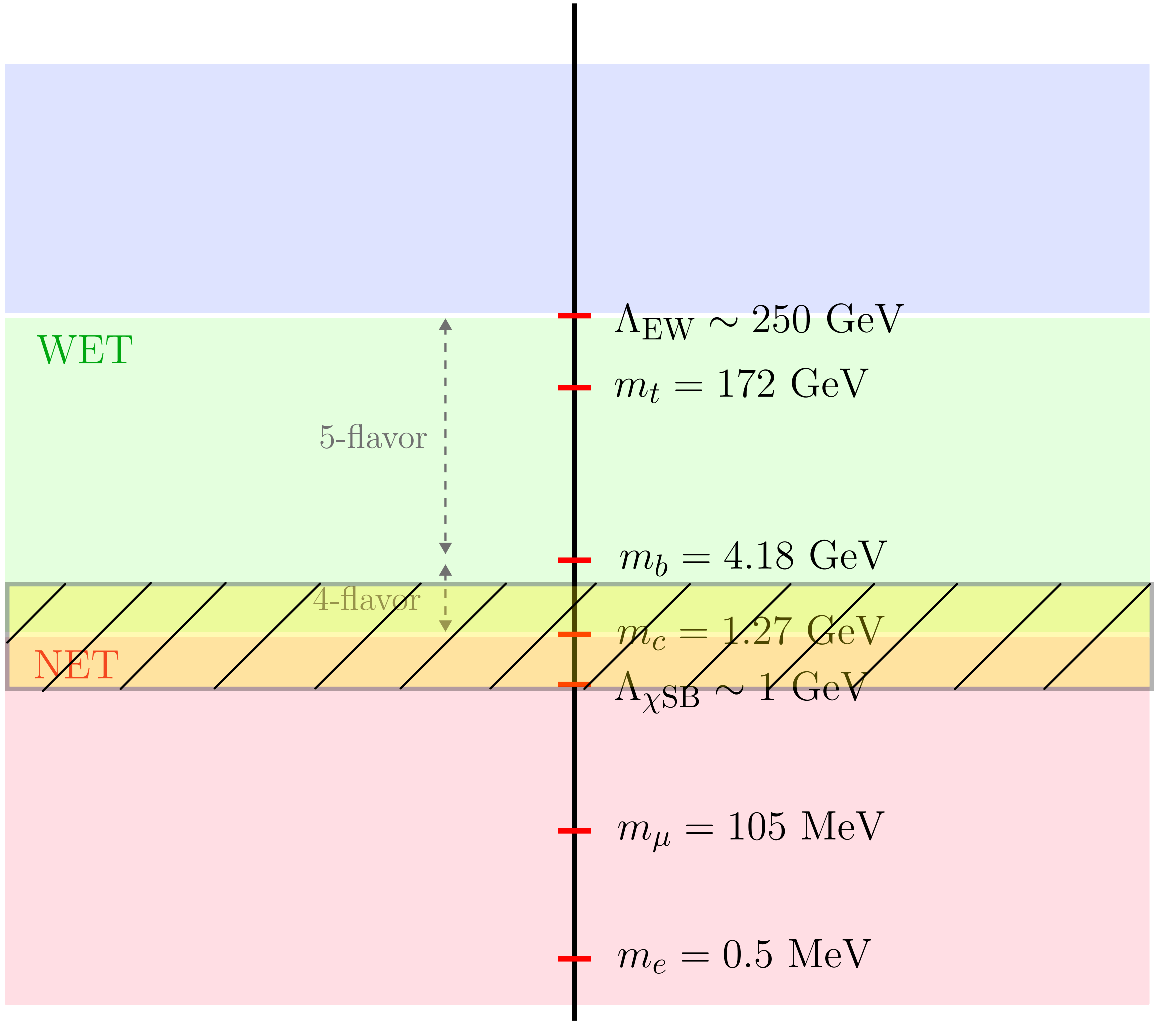
$$\mathcal{O}_{11} = i\hat{q} \cdot \vec{\sigma}_L \; 1_N, \qquad \mathcal{O}_{12} = \vec{\sigma}_L \cdot [\vec{v}_N \times \vec{\sigma}_N],$$

$$\mathcal{O}_{13}' = \vec{\sigma}_L \cdot (i\hat{q} \times [\vec{v}_N \times \vec{\sigma}_N]), \qquad \mathcal{O}_{14} = i\hat{q} \cdot \vec{\sigma}_L \; \vec{v}_N \cdot \vec{\sigma}_N,$$

$$\mathcal{O}_{15} = i\hat{q} \cdot \vec{\sigma}_L \; i\hat{q} \cdot [\vec{v}_N \times \vec{\sigma}_N], \qquad \mathcal{O}_{16}' = i\hat{q} \cdot \vec{\sigma}_L \; i\hat{q} \cdot \vec{v}_N.$$

EFT tower

- Matching between WET and NRET (hadronization)



EFT tower

- **Matching between WET and NRET (hadronization)**
- **Parameterize with nuclear form factors**

$$\langle N'|\bar{q}\gamma^\mu q|N\rangle = \bar{u}'_N\Big[F_1^{q/N}(q^2)\gamma^\mu - \frac{i}{2m_N}F_2^{q/N}(q^2)\sigma^{\mu\nu}q_\nu\Big]u_N\,,$$

$$\langle N'|\bar{q}\gamma^\mu\gamma_5 q|N\rangle = \bar{u}'_N\Big[F_A^{q/N}(q^2)\gamma^\mu\gamma_5 - \frac{1}{2m_N}F_{P'}^{q/N}(q^2)\gamma_5 q^\mu\Big]u_N\,,$$

$$\langle N'|m_q\bar{q}q|N\rangle = F_S^{q/N}(q^2)\,\bar{u}'_N u_N\,,$$

$$\langle N'|m_q\bar{q}i\gamma_5 q|N\rangle = F_P^{q/N}(q^2)\,\bar{u}'_N i\gamma_5 u_N\,,$$

$$\langle N'|\frac{\alpha_s}{12\pi}G^{a\mu\nu}G^a_{\mu\nu}|N\rangle = F_G^N(q^2)\,\bar{u}'_N u_N\,,$$

$$\langle N'|\frac{\alpha_s}{8\pi}G^{a\mu\nu}\tilde{G}^a_{\mu\nu}|N\rangle = F_{\tilde{G}}^N(q^2)\,\bar{u}'_N i\gamma_5 u_N\,,$$

$$\langle N'|\bar{q}\sigma^{\mu\nu}q|N\rangle = \bar{u}'_N\Big[\hat{F}_{T,0}^{q/N}(q^2)\,\sigma^{\mu\nu} - \frac{i}{2m_N}\gamma^{[\mu}q^{\nu]}\hat{F}_{T,1}^{q/N}(q^2) - \frac{i}{m_N^2}q^{[\mu}k_{12}^{\nu]}\hat{F}_{T,2}^{q/N}(q^2)\Big]u_N\,,$$

$$\langle N'|\frac{\alpha}{12\pi}F^{\mu\nu}F_{\mu\nu}|N\rangle = F_\gamma^N(q^2)\,\bar{u}'_N u_N\,,$$

$$\langle N'|\frac{\alpha}{8\pi}F^{\mu\nu}\tilde{F}_{\mu\nu}|N\rangle = F_{\tilde{\gamma}}^N(q^2)\,\bar{u}'_N i\gamma_5 u_N\,.$$

EFT tower

- **Matching between WET and NRET (hadronization)**
- **Matching expressions →**

$$c_1^N = -\frac{\alpha}{\pi q}\hat{\mathcal{C}}_1^{(5)}\sum_q Q_q F_1^{q/N} + \sum_q \hat{\mathcal{C}}_{1,q}^{(6)} F_1^{q/N} + \sum_q \frac{1}{m_q}\hat{\mathcal{C}}_{5,q}^{(6)} F_S^{q/N}$$
$$- \frac{q}{m_N}\sum_q \hat{\mathcal{C}}_{9,q}^{(6)}\left(\hat{F}_{T,0}^{q/N} - \hat{F}_{T,1}^{q/N} + 4\hat{F}_{T,2}^{q/N}\right)$$
$$+ \hat{\mathcal{C}}_1^{(7)} F_G^N + \hat{\mathcal{C}}_5^{(7)} F_\gamma^N + (q + m_+)\sum_q \hat{\mathcal{C}}_{9,q}^{(7)} F_1^{q/N}$$
$$- \frac{q^2}{2m_N}\sum_q \hat{\mathcal{C}}_{13,q}^{(7)}\left[\hat{F}_{T,0}^{q/N} - \hat{F}_{T,1}^{q/N} + \left(4 + \frac{q^2}{m_N^2}\right)\hat{F}_{T,2}^{q/N}\right],$$

$$c_2^N = i\Big[\sum_q \hat{\mathcal{C}}_{1,q}^{(6)} F_1^{q/N} + m_+\sum_q \hat{\mathcal{C}}_{9,q}^{(7)} F_1^{q/N} + \frac{q^2}{2m_N}\sum_q \hat{\mathcal{C}}_{13,q}^{(7)}\left(\hat{F}_{T,1}^{q/N} - 4\hat{F}_{T,2}^{q/N}\right)\Big],$$

$$\vdots$$

EFT tower

$$c_1^N = -\frac{\alpha}{\cdots}\hat{\mathcal{C}}_1^{(5)} \sum Q_q F_1^{q/N} + \sum \hat{\mathcal{C}}^{(6)}_{1,q} F_1^{q/N} + \sum \frac{1}{\cdots}\hat{\mathcal{C}}^{(6)}_{5,q} F_S^{q/N}$$

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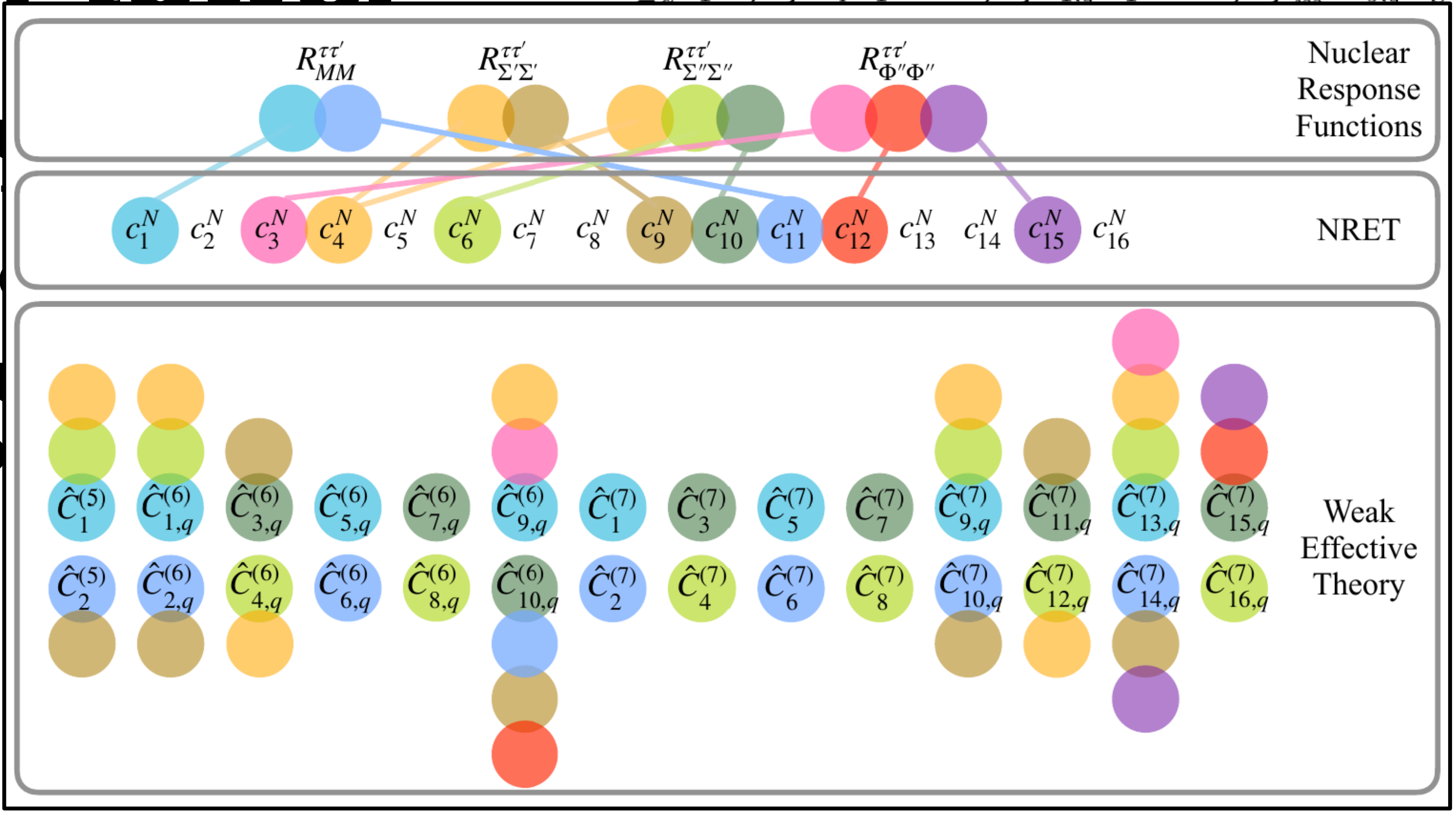
$$\cdots \hat{F}^{q/N}_{T,1} - 4\hat{F}^{q/N}_{T,2}\Big)\Big],$$



The finale

- **The conversion rate:** $\mathcal{O}(y)$

$$\Gamma(\mu \to e) = \frac{1}{2\pi} \frac{q_{\rm eff}^2}{1 + q/M_T} \left|\phi_{1s}^{Z_{\rm eff}}(\vec{0})\right|^2 \sum_{\tau,\tau'} \Big[R_{MM}^{\tau\tau'} W_{MM}^{\tau\tau'}(q_{\rm eff}) + R_{\Sigma'\Sigma'}^{\tau\tau'} W_{\Sigma'\Sigma'}^{\tau\tau'}(q_{\rm eff}) + R_{\Sigma''\Sigma''}^{\tau\tau'} W_{\Sigma''\Sigma''}^{\tau\tau'}(q_{\rm eff}) \Big],$$

$$R_{MM}^{\tau\tau'} = c_1^\tau c_1^{\tau'*} + c_{11}^\tau c_{11}^{\tau'*},$$
$$R_{\Sigma'\Sigma'}^{\tau\tau'} = c_4^\tau c_4^{\tau'*} + c_9^\tau c_9^{\tau'*},$$
$$R_{\Sigma''\Sigma''}^{\tau\tau'} = \left(c_4^\tau - c_6^\tau\right)\left(c_4^{\tau'} - c_6^{\tau'}\right)^* + c_{10}^\tau c_{10}^{\tau'*},$$

The finale

- The conversion rate: $\mathcal{O}(y)$

Dependent on UV

$$\Gamma(\mu \to e) = \frac{1}{2\pi}\frac{q_{\rm eff}^2}{1+q/M_T}\left|\phi_{1s}^{Z_{\rm eff}}(\vec{0})\right|^2 \sum_{\tau,\tau'}\Big[R_{MM}^{\tau\tau'} W_{MM}^{\tau\tau'}(q_{\rm eff}) + R_{\Sigma'\Sigma'}^{\tau\tau'} W_{\Sigma'\Sigma'}^{\tau\tau'}(q_{\rm eff}) + R_{\Sigma''\Sigma''}^{\tau\tau'} W_{\Sigma''\Sigma''}^{\tau\tau'}(q_{\rm eff})\Big],$$

$$R_{MM}^{\tau\tau'} = c_1^\tau c_1^{\tau'*} + c_{11}^\tau c_{11}^{\tau'*},$$
$$R_{\Sigma'\Sigma'}^{\tau\tau'} = c_4^\tau c_4^{\tau'*} + c_9^\tau c_9^{\tau'*},$$
$$R_{\Sigma''\Sigma''}^{\tau\tau'} = \left(c_4^\tau - c_6^\tau\right)\left(c_4^{\tau'} - c_6^{\tau'}\right)^* + c_{10}^\tau c_{10}^{\tau'*},$$

The finale

- **The conversion rate:** $\mathcal{O}(\vec{v}_N)$

*All nuclear responses allowed by symmetries are generated

$$\omega = \frac{G_F^2}{\pi} \frac{q_{\rm eff}^2}{1+\frac{q}{M_T}} |\phi_{1s}^{Z_{\rm eff}}(\vec{0})|^2 \sum_{\tau=0,1} \sum_{\tau'=0,1} \Bigg\{ \left[\tilde{R}_{MM}^{\tau\tau'} \, W_{MM}^{\tau\tau'}(q_{\rm eff}) + \tilde{R}_{\Sigma''\Sigma''}^{\tau\tau'} \, W_{\Sigma''\Sigma''}^{\tau\tau'}(q_{\rm eff}) + \tilde{R}_{\Sigma'\Sigma'}^{\tau\tau'} \, W_{\Sigma'\Sigma'}^{\tau\tau'}(q_{\rm eff}) \right]$$
$$+ \frac{q_{\rm eff}^2}{m_N^2} \left[\tilde{R}_{\Phi''\Phi''}^{\tau\tau'} \, W_{\Phi''\Phi''}^{\tau\tau'}(q_{\rm eff}) + \tilde{R}_{\tilde{\Phi}'\tilde{\Phi}'}^{\tau\tau'} \, W_{\tilde{\Phi}'\tilde{\Phi}'}^{\tau\tau'}(q_{\rm eff}) + \tilde{R}_{\Delta\Delta}^{\tau\tau'} \, W_{\Delta\Delta}^{\tau\tau'}(q_{\rm eff}) \right]$$
$$- \frac{2q_{\rm eff}}{m_N} \left[\tilde{R}_{\Phi''M}^{\tau\tau'} \, W_{\Phi''M}^{\tau\tau'}(q_{\rm eff}) + \tilde{R}_{\Delta\Sigma'}^{\tau\tau'} \, W_{\Delta\Sigma'}^{\tau\tau'}(q_{\rm eff}) \right] \Bigg\} \quad (59)$$

$$R_{MM}^{\tau\tau'} = c_1^\tau c_1^{\tau'*} + c_{11}^\tau c_{11}^{\tau'*},$$
$$R_{\Sigma'\Sigma'}^{\tau\tau'} = c_4^\tau c_4^{\tau'*} + c_9^\tau c_9^{\tau'*},$$
$$R_{\Sigma''\Sigma''}^{\tau\tau'} = \left(c_4^\tau - c_6^\tau\right)\left(c_4^{\tau'} - c_6^{\tau'}\right)^* + c_{10}^\tau c_{10}^{\tau'*},$$

$$\tilde{R}_{\Phi''\Phi''}^{\tau\tau'} = \tilde{c}_3^\tau \tilde{c}_3^{\tau'*} + (\tilde{c}_{12}^\tau - \tilde{c}_{15}^\tau)(\tilde{c}_{12}^{\tau'*} - \tilde{c}_{15}^{\tau'*})$$
$$\tilde{R}_{\Phi''M}^{\tau\tau'} = \mathrm{Re}[\tilde{c}_3^\tau \tilde{c}_1^{\tau'*} - (\tilde{c}_{12}^\tau - \tilde{c}_{15}^\tau)\, \tilde{c}_{11}^{\tau'*}]$$
$$\tilde{R}_{\Delta\Delta}^{\tau\tau'} = \tilde{c}_5^\tau \tilde{c}_5^{\tau'*} + \tilde{c}_8^\tau \tilde{c}_8^{\tau'*}$$
$$\tilde{R}_{\Delta\Sigma'}^{\tau\tau'} = \mathrm{Re}[\tilde{c}_5^\tau \tilde{c}_4^{\tau'*} + \tilde{c}_8^\tau \tilde{c}_9^{\tau'*}]$$
$$\tilde{R}_{\tilde{\Phi}'\tilde{\Phi}'}^{\tau\tau'} = \tilde{c}_{12}^\tau \tilde{c}_{12}^{\tau'*} + \tilde{c}_{13}^\tau \tilde{c}_{13}^{\tau'*}$$

Nuclear response hierarchy

$$W^{00}_{MM} \sim \mathcal{O}(A^2) \gg \left\{ W^{00}_{\Sigma'\Sigma'}, W^{00}_{\Sigma''\Sigma''}, \frac{q^2_{\text{eff}}}{m^2_N} W^{00}_{\Phi''\Phi''} \right\} \gg \left\{ \frac{q^2_{\text{eff}}}{m^2_N} W^{00}_{\Delta\Delta}, \frac{q^2_{\text{eff}}}{m^2_N} W^{00}_{\tilde{\Phi}'\tilde{\Phi}'} \right\}.$$

Nuclear response hierarchy

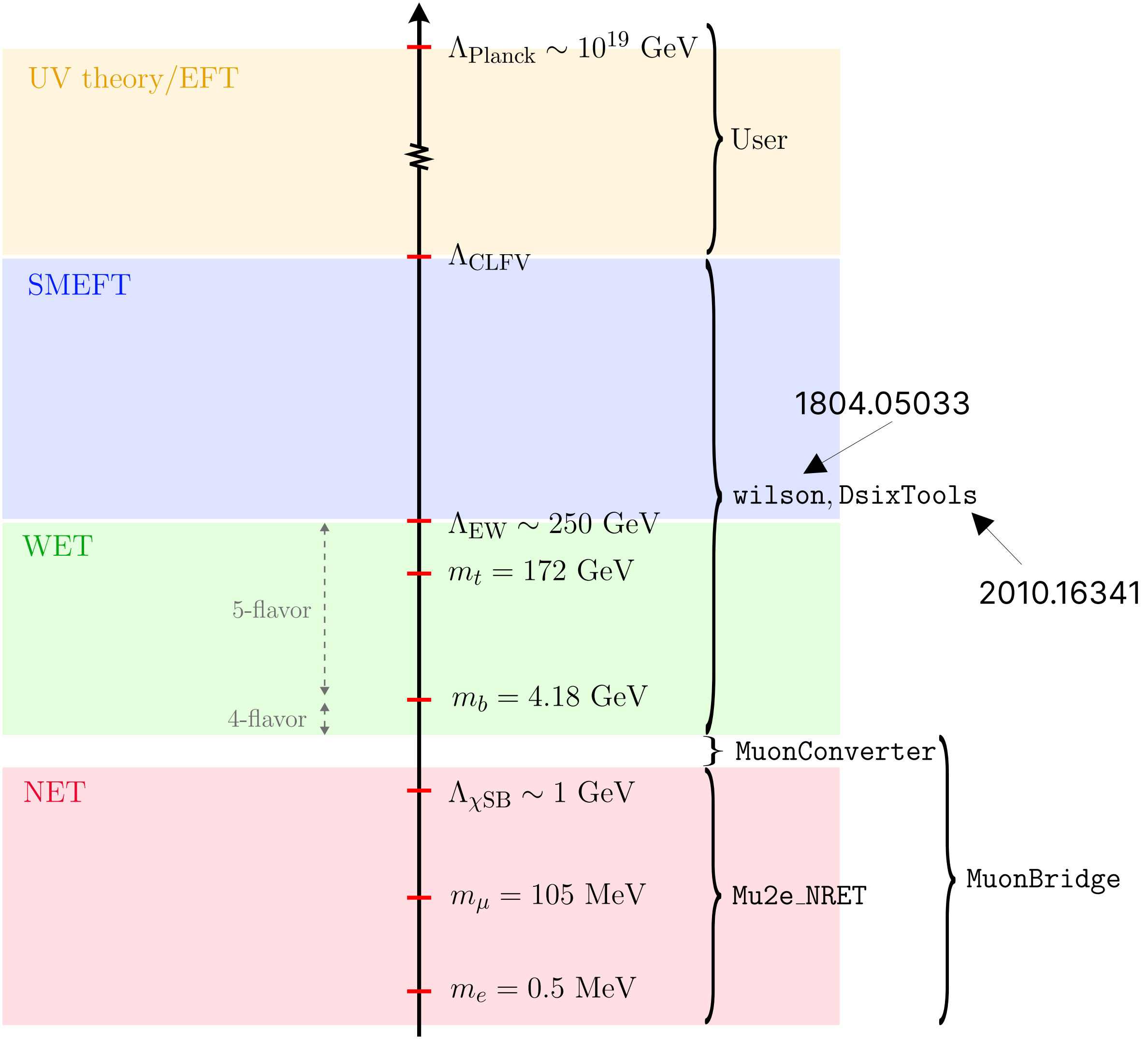
$$W^{00}_{MM} \sim \mathcal{O}(A^2) \gg \left\{ W^{00}_{\Sigma'\Sigma'}, W^{00}_{\Sigma''\Sigma''}, \frac{q_{\rm eff}^2}{m_N^2} W^{00}_{\Phi''\Phi''} \right\} \gg \left\{ \frac{q_{\rm eff}^2}{m_N^2} W^{00}_{\Delta\Delta}, \frac{q_{\rm eff}^2}{m_N^2} W^{00}_{\tilde{\Phi}'\tilde{\Phi}'} \right\}.$$

*Can become semi-coherent in some nuclei (such as Al), the interference becomes relevant. Hierarchy is modified:

$$W^{00}_{M} \gg q/m_N \; W^{00}_{\Phi'' M} \gg q^2/m_N^2 \; W^{00}_{\Phi''}$$

Code

Amalgamation of new and existing EFT software



Usage

In practice the code can be used in ~2 ways

1. Bottom-up analyses where μ → e is investigated in the context of an EFT

$$\{\mathcal{O}_1, \mathcal{O}_2, \cdots\} \rightarrow \Gamma(\mu N \rightarrow eN)$$

2. Top-down analyses where an explicit UV model is matched onto the SMEFT and run down to the nuclear scale

$$M_{\mathrm{UV}}(\Lambda_{\mathrm{UV}}, \vec{\Phi}, \mathcal{L}_{\mathrm{UV}}) \rightarrow \Gamma(\mu N \rightarrow eN)$$

Bottom-up

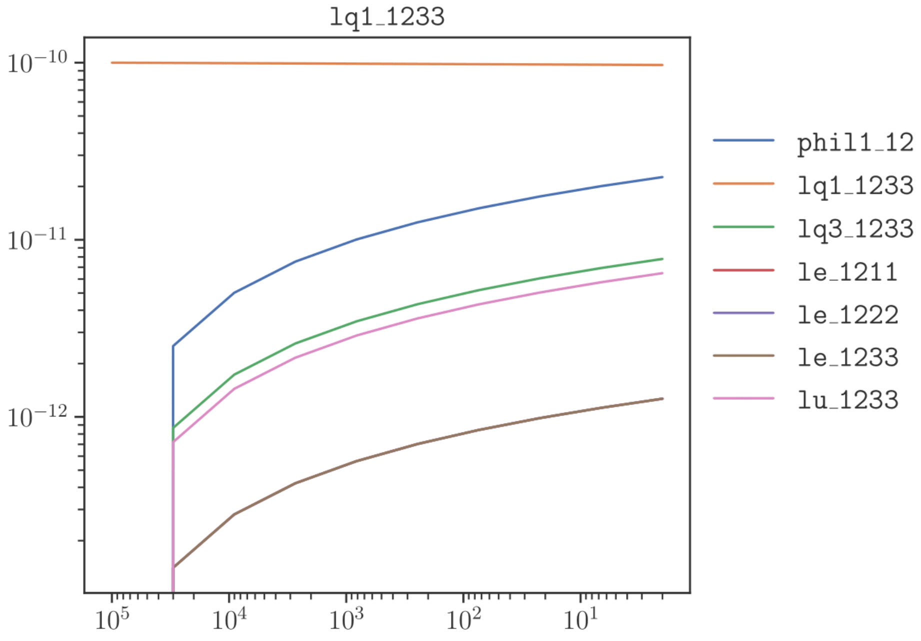
1. Turn on one SMEFT operator suppressed by a scale that we will vary

$$\texttt{lq1\_1233} = (\bar{\ell}_L \gamma^\mu \ell_L)_{12} (\bar{q}_L \gamma_\mu q_L)_{33}$$

Bottom-up

1. Turn on one SMEFT operator suppressed by a scale that we will vary

2. Run to 2 GeV

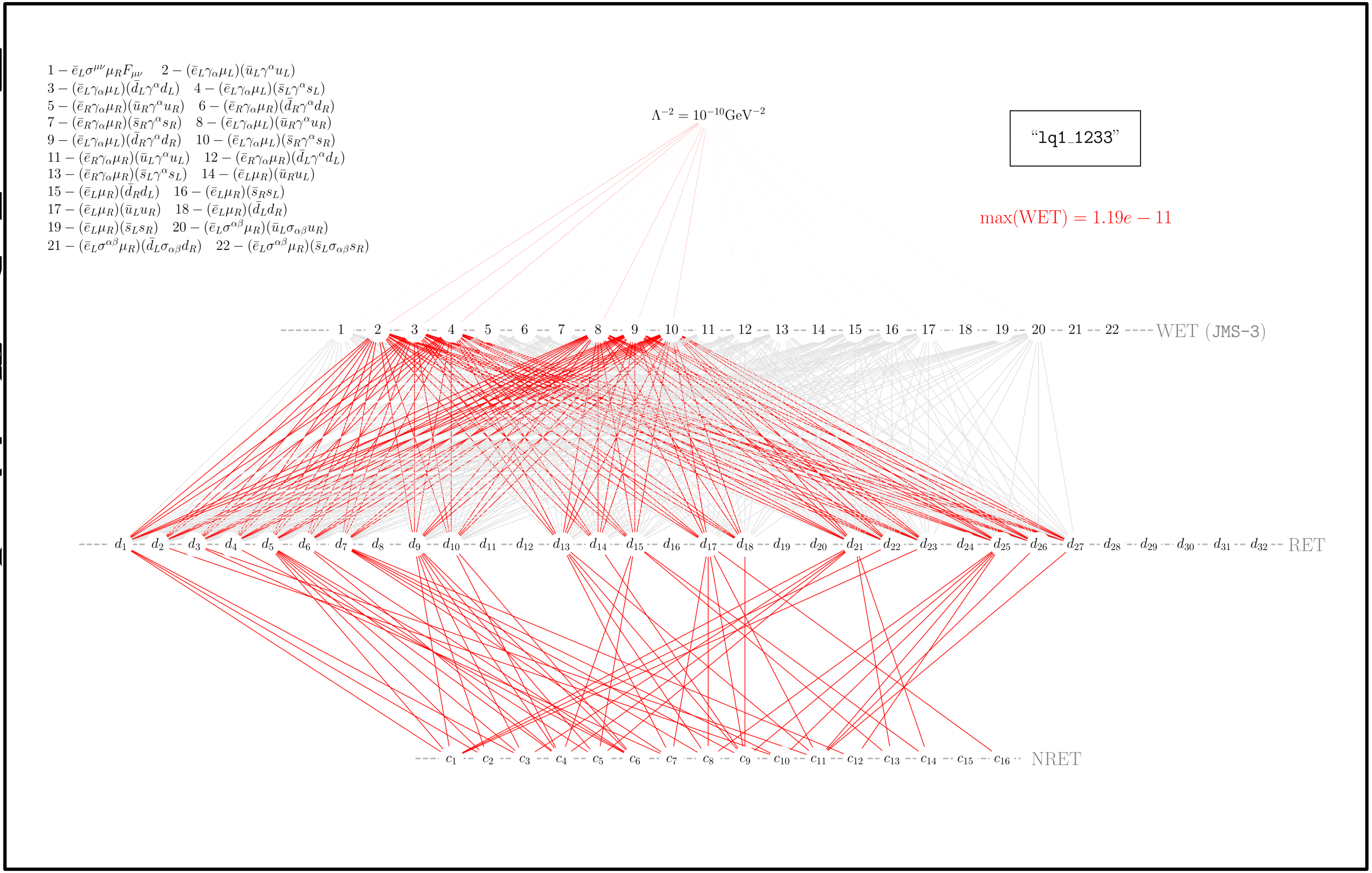


Bottom-up

1. Turn on one SMEFT operator suppressed by a scale that we will vary
2. Run to 2 GeV
3. Compute the coefficients using the tower

Bo

1. Tu t w
2. R
3. C
4. R

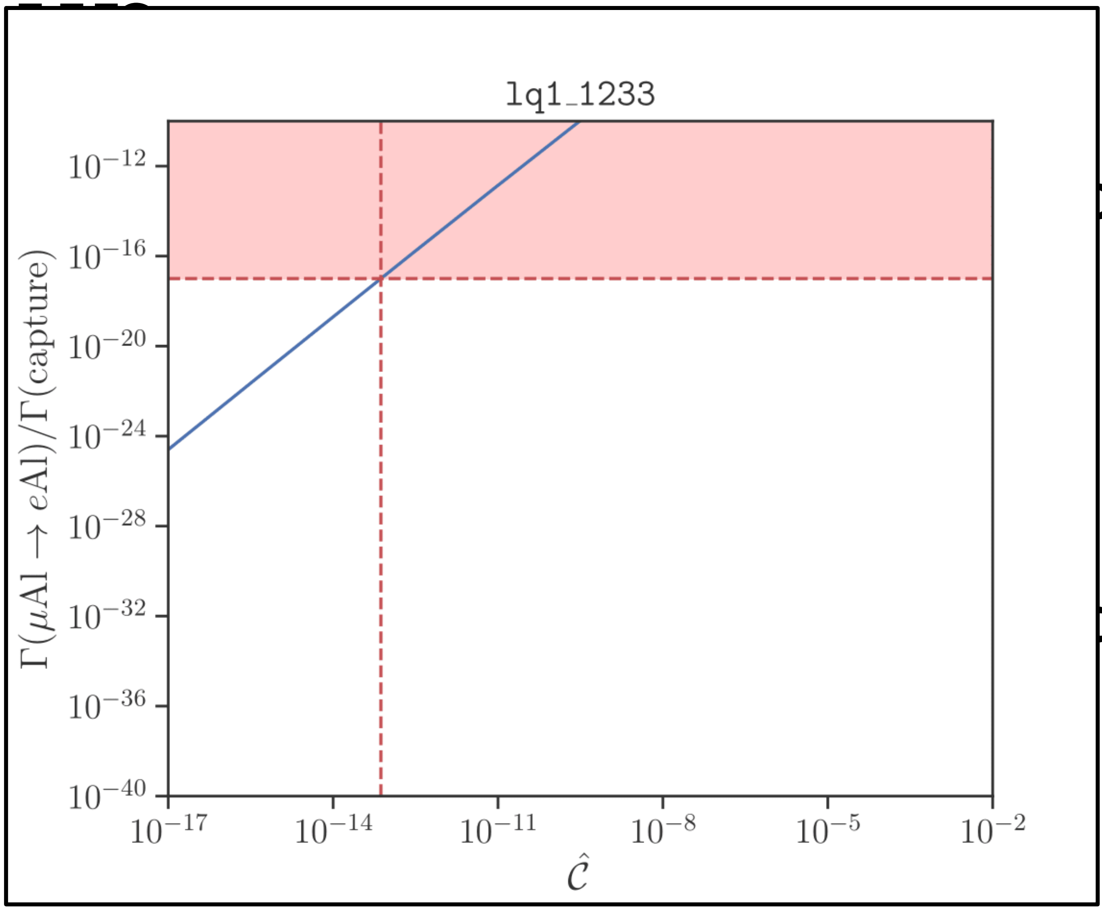


Bottom-up

1. Turn on one SMEFT operator suppressed by a scale that we will vary
2. Run to 2 GeV
3. Compute the coefficients using the tower
4. Compute CR and repeat until bound is saturated

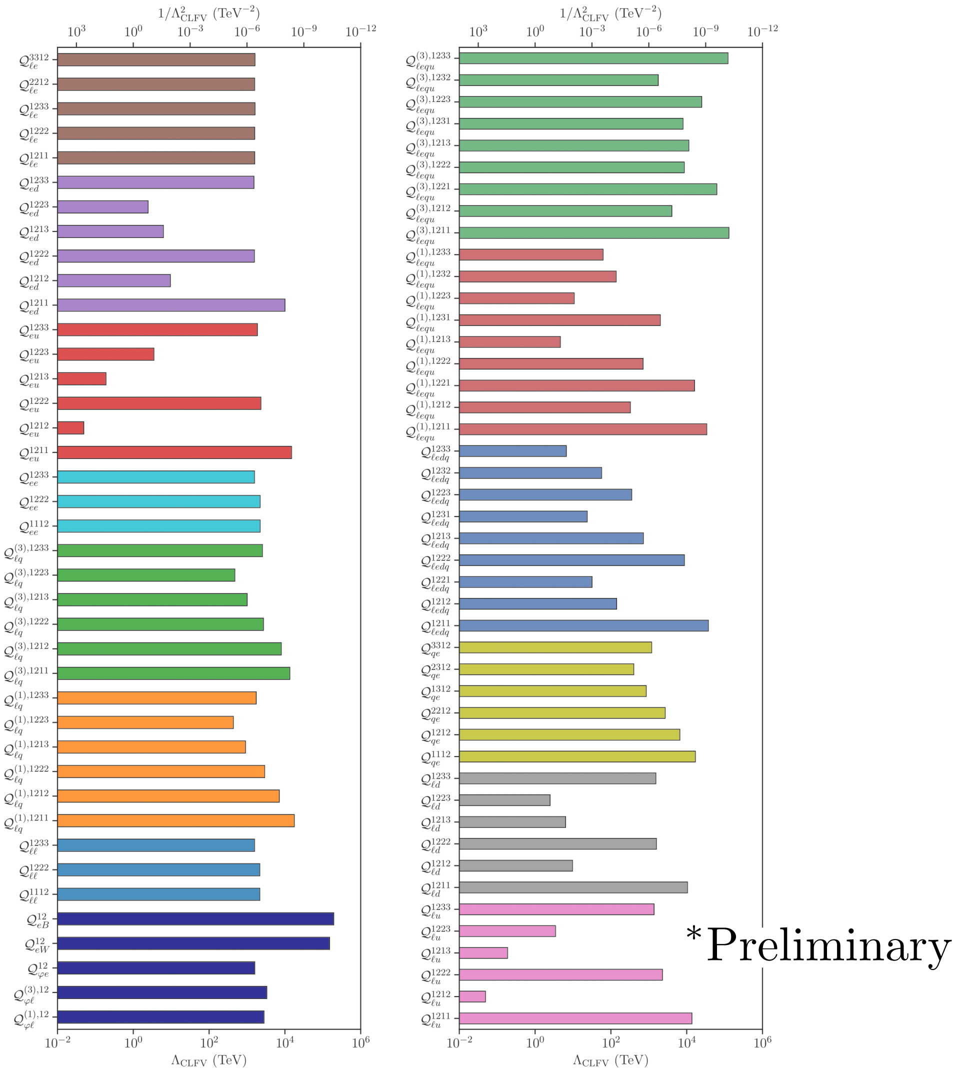
Bottom

1. Turn on on … cale that
   we will var
2. Run to 2 G
3. Compute t
4. Compute (… d



Bottom-up

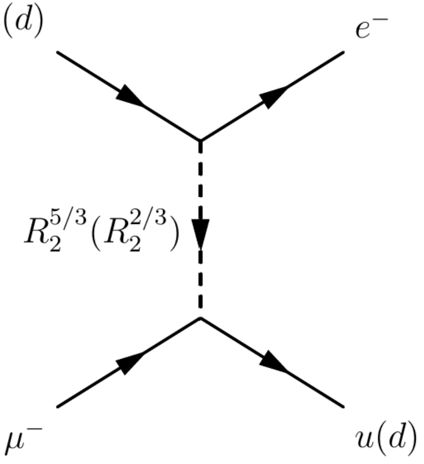
- Single operator bounds



Top-down

- Tensor contributions are important!
- Consider the following leptoquark model

$$\mathcal{L} \supset -y_{2\,ij}^{RL}\,\bar{u}_R^i R_2^a \epsilon^{ab} L_L^{j,b} + y_{2\,ij}^{LR}\,\bar{e}_R^i R_2^{a\,*} Q_L^{j,a} + \text{h.c.}$$



Top-down

- Integrate out the leptoquark and match to SMEFT

*37 new parameters

$$C_{\substack{\ell u\\12ii}} = -\frac{1}{2m_{LQ}^2} y_{2\,i2}^{RL} y_{2\,i1}^{RL*},$$

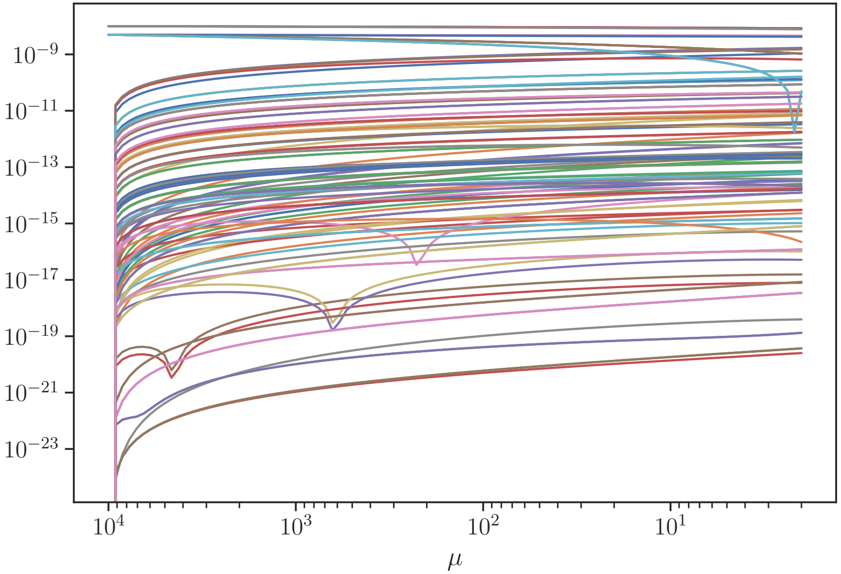
$$C_{\substack{qe\\ii12}} = -\frac{1}{2m_{LQ}^2} y_{2\,2i}^{LR*} y_{2\,1i}^{LR},$$

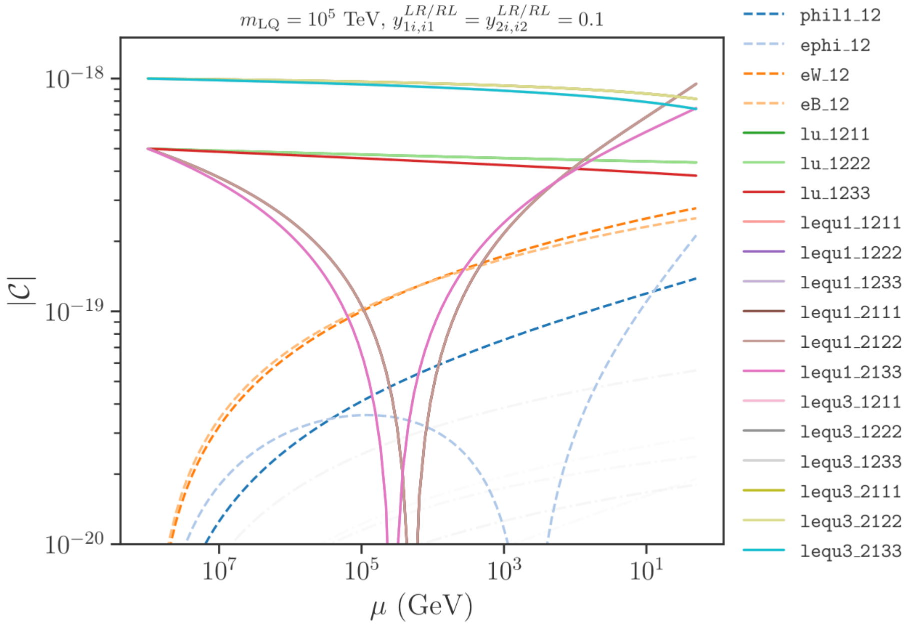
$$C^{(1)}_{\substack{\ell equ\\12ii}} = 2C^{(3)}_{\substack{\ell equ\\12ii}} = -\frac{1}{2m_{LQ}^2} y_{2\,2i}^{LR*} y_{2\,i1}^{RL*},$$

$$C^{(1)*}_{\substack{\ell equ\\21ii}} = 2C^{(3)*}_{\substack{\ell equ\\21ii}} = -\frac{1}{2m_{LQ}^2} y_{2\,i2}^{LR} y_{2\,1i}^{RL},$$

Top-down

- Run and compute rate



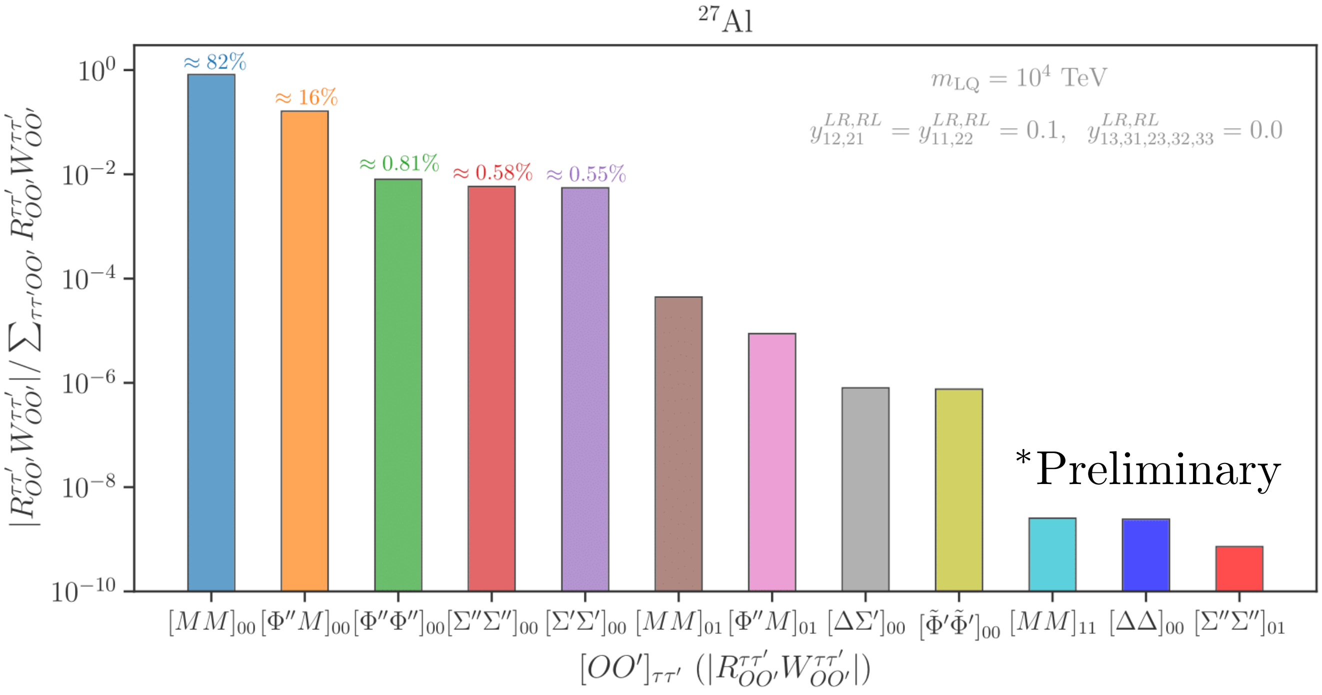


Top-down

- Response decomposition

*Tensor current induced repsonse is the subleading component

$$\Gamma \propto \sum_i R_i W_i$$



Conclusions

- Very flexible framework for computing $\mu \to e$ conversion rates from theories defined at arbitrarily high scales.
- Tensor currents contribute non-negligibly to the total conversion rate.
- MuonBridge software suite facilitates the computation (available soon in Python and Mathematica).

Conclusions

- Very flexible framework for computing $\mu \to e$ conversion rates from theories defined at arbitrarily high scales.
- Tensor currents contribute non-negligibly to the total conversion rate.
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Thank you :)