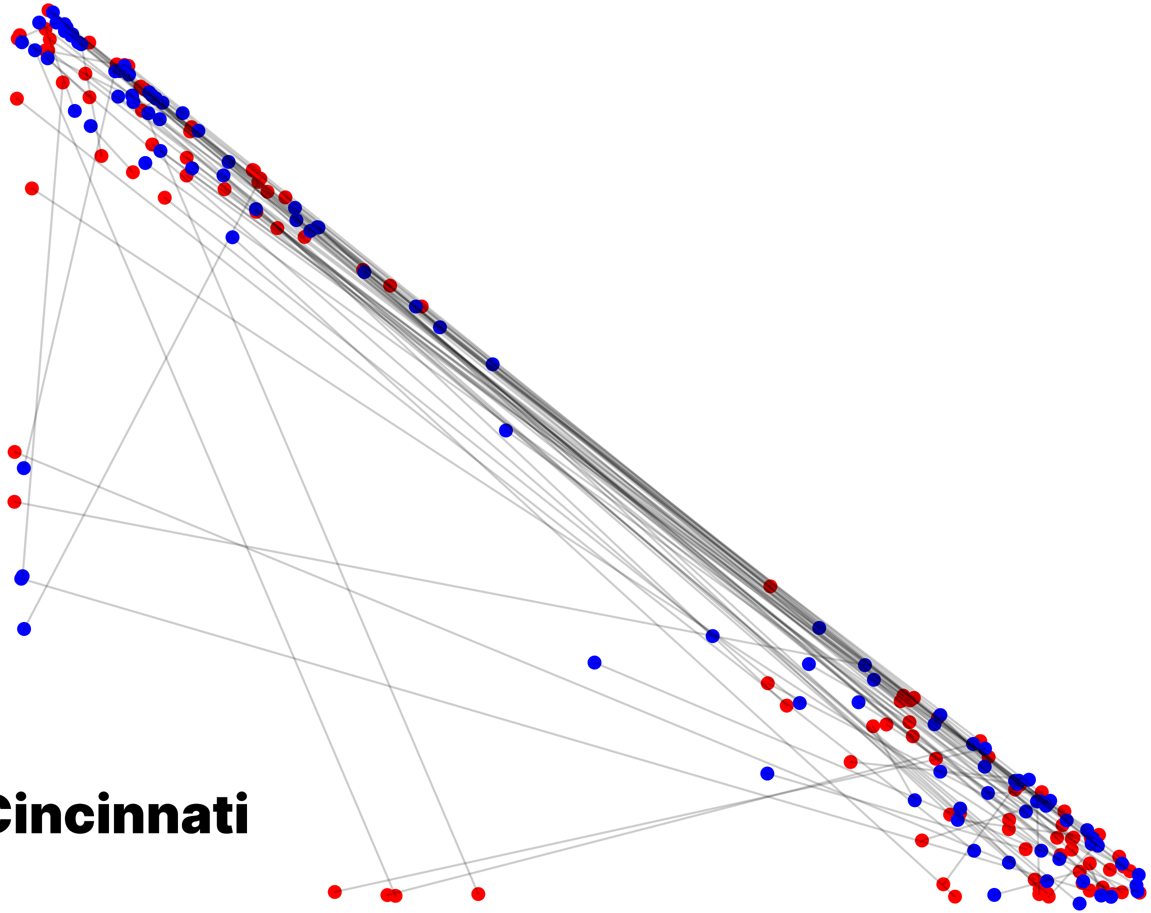
Earth Mover's Distance and CP-violation



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In collaboration with Adam Davis, Ahmed Youssef, and Jure Zupan

Based on 2301.13211

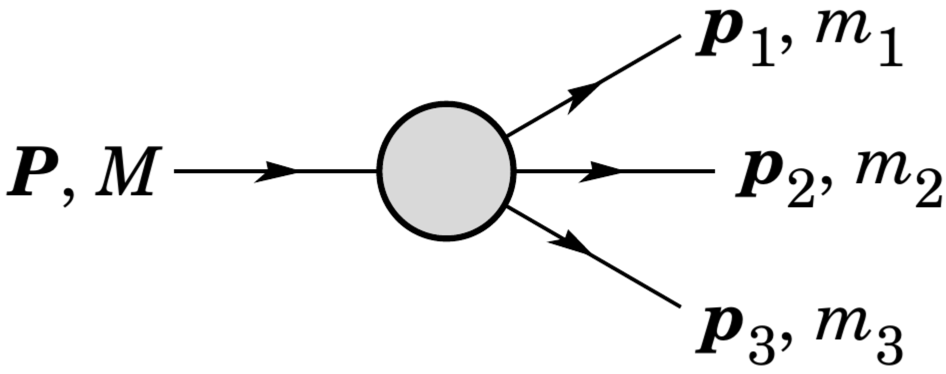
3-body CPV

Direct CPV: $\frac{\mathcal{M}(X \to \cdots)}{\mathcal{M}(\bar{X} \to \overline{\cdots})} \neq 1$

12 degrees of freedom
- 10 constraints

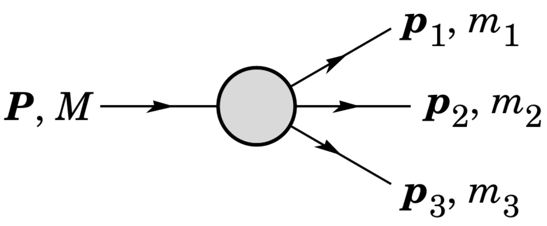
2 independent variables

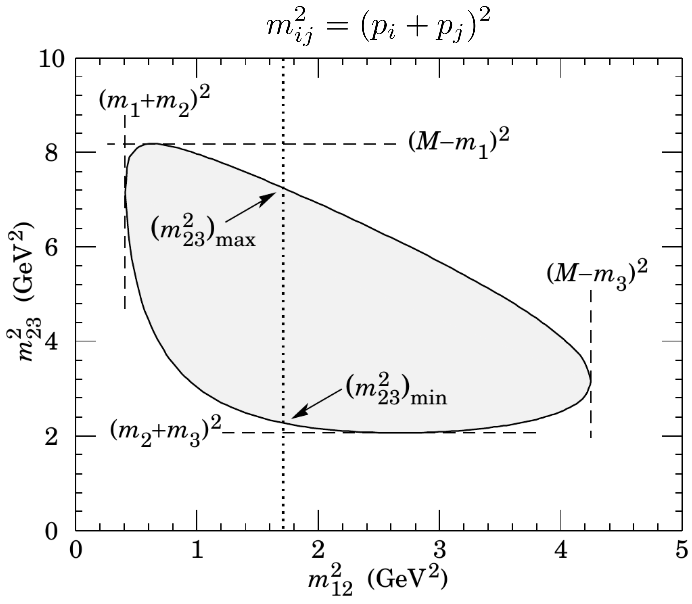
Phase space-dependent CPV



Dalitz analysis

Consider a 3-body decay:





What to remember:

$$d\Gamma \propto |\mathcal{M}|^2 dm_{12}^2 dm_{23}^2$$

Direct CPV manifests as local **density asymmetries** between conjugate Dalitz plots

Dalitz analysis

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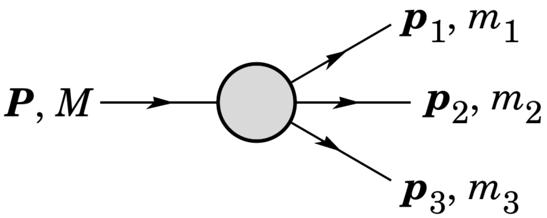

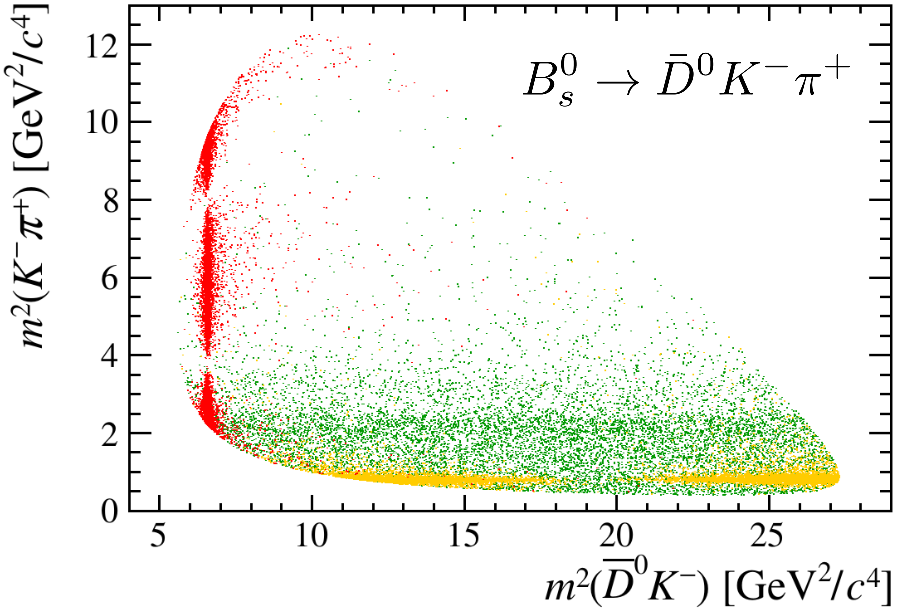

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$$d\Gamma \propto |\mathcal{M}|^2 dm_{12}^2 dm_{23}^2$$

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Tests of CPV

1. Binned test statistic → $A_{\mathrm{CP}} = \frac{N - \bar{N}}{N + \bar{N}}$
   - Binning washes out asymmetry
2. Model the amplitude
   - Extremely complicated
3. Unbinned test statistic
   - E.g.

$$T = \sum_{i,j>i}^{N} \frac{\psi_{ij}}{N(N-1)} + \sum_{i,j>i}^{\bar{N}} \frac{\psi_{ij}}{\bar{N}(\bar{N}-1)} - \sum_{i,j}^{N,\bar{N}} \frac{\psi_{ij}}{N\bar{N}}, \qquad \psi_{ij} \equiv \psi(d_{ij};\sigma) = e^{-d_{ij}^2/2\sigma^2}$$

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Statistical distance

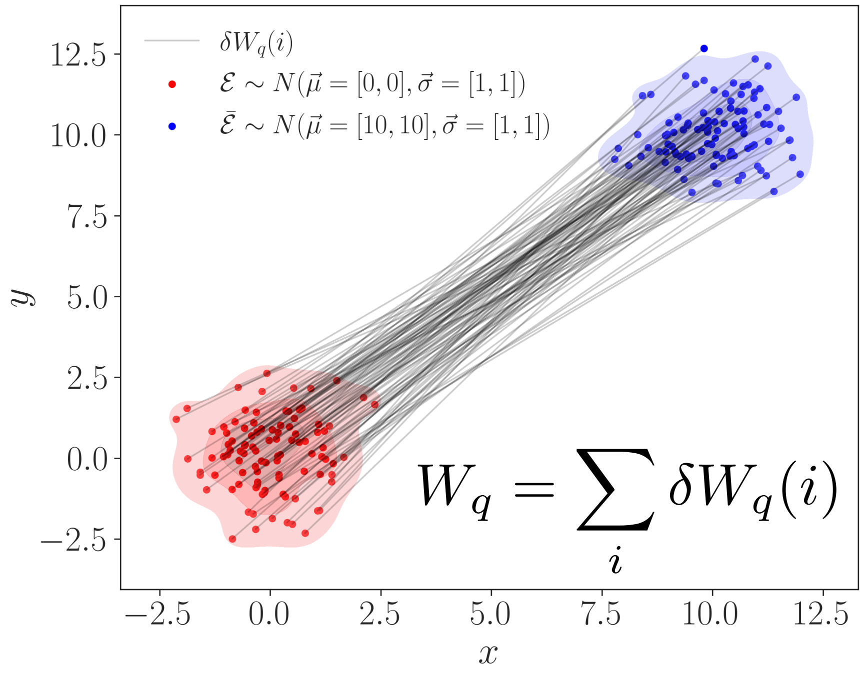
Input two statistical samples and output scalar value.

Larger values = dissimilar distributions
Smaller values = similar distributions

One example:

Wasserstein or Earth Mover's distance (EMD)

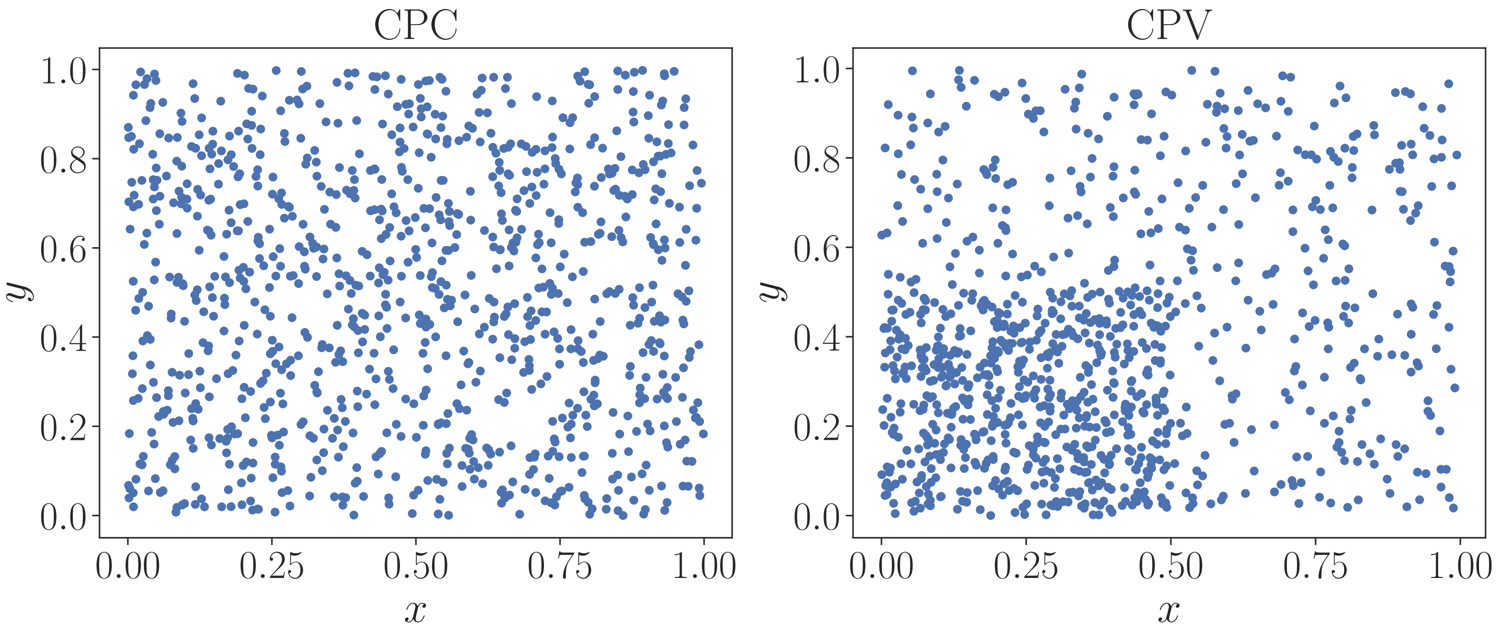
$$W_q(\mathcal{E},\bar{\mathcal{E}}) = \left[\min_{\{f_{ij}\geq 0\}} \sum_{i=1}^{N}\sum_{j=1}^{\bar{N}} f_{ij}\left(\hat{d}_{ij}\right)^q\right]^{1/q}$$



EMD and CPV

Direct CPV manifests as local **density asymmetries** between conjugate Dalitz plots.

An exagerated example:



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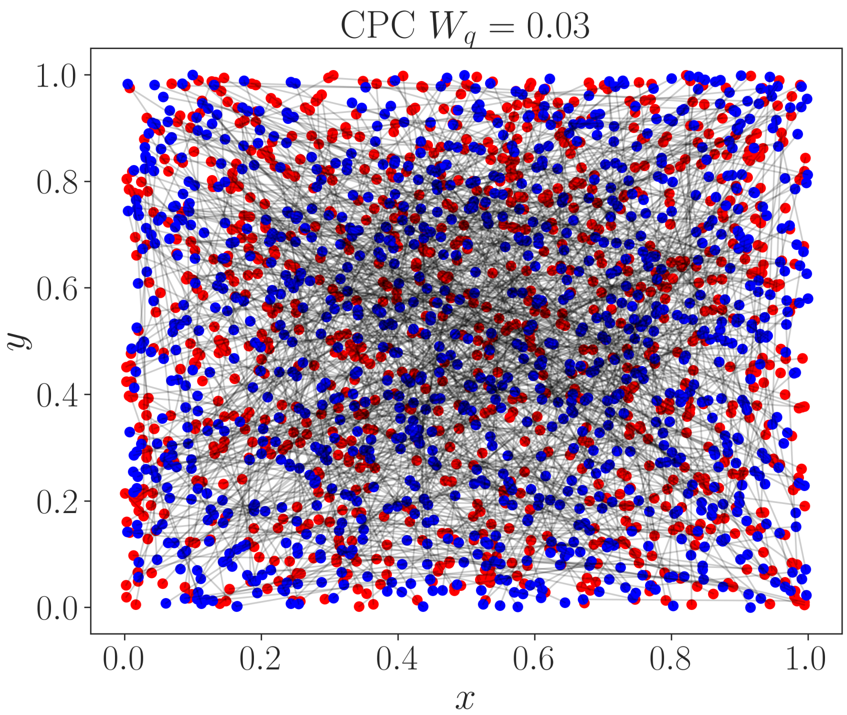
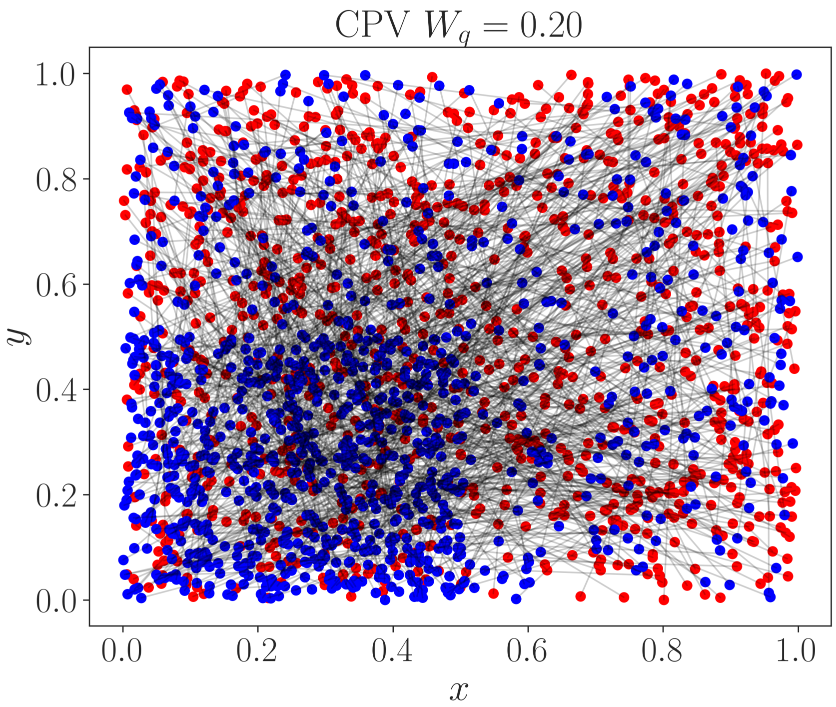
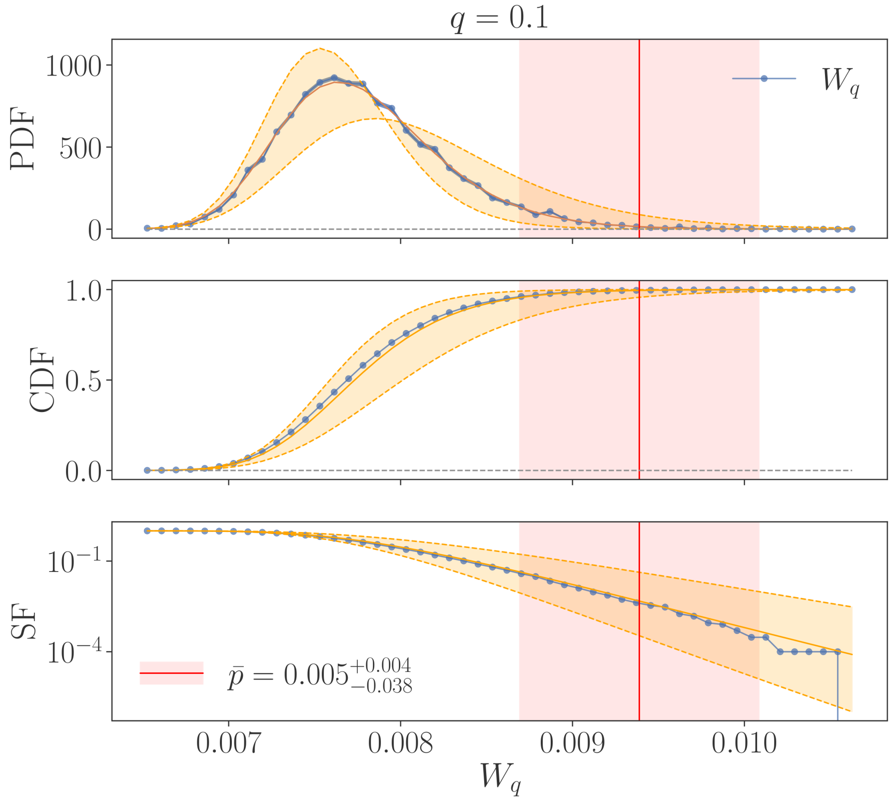
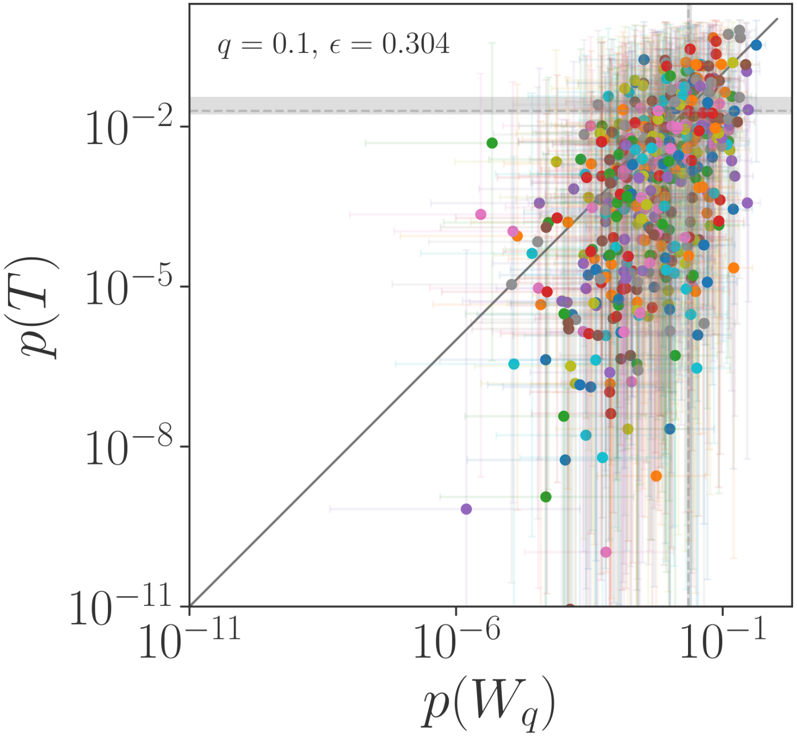




Performance as a statistic



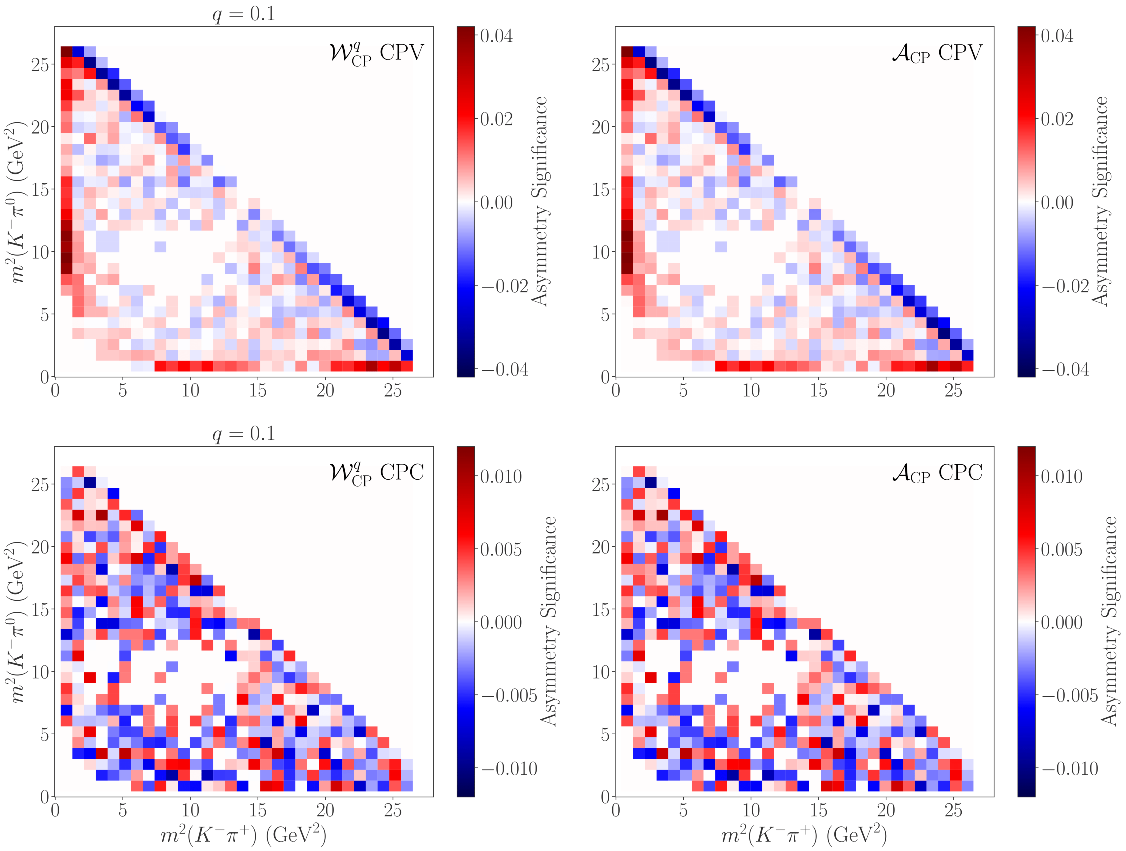


Visualization

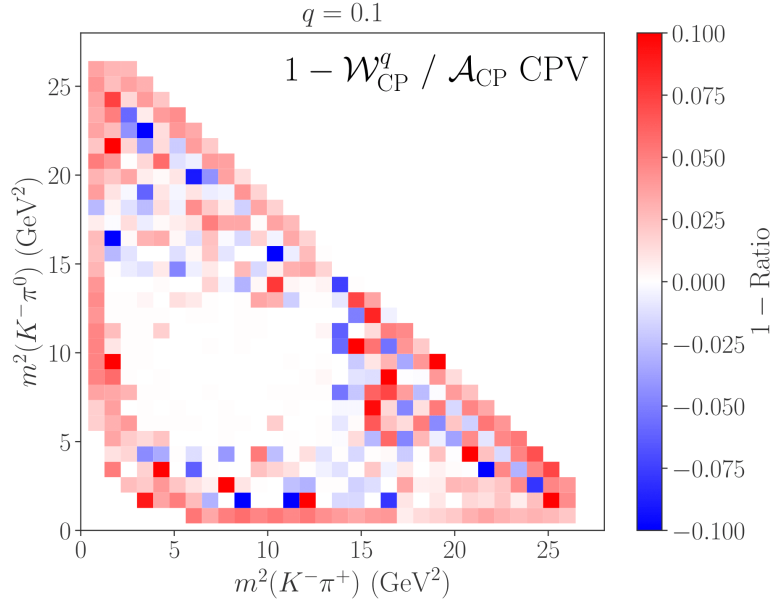
$$\mathcal{A}_{\rm CP}(s_{12}, s_{13}) = \frac{d\bar{\Gamma}(\bar{s}_{12}, \bar{s}_{13}) - d\Gamma(s_{12}, s_{13})}{d\bar{\Gamma}(\bar{s}_{12}, \bar{s}_{13}) + d\Gamma(s_{12}, s_{13})}$$

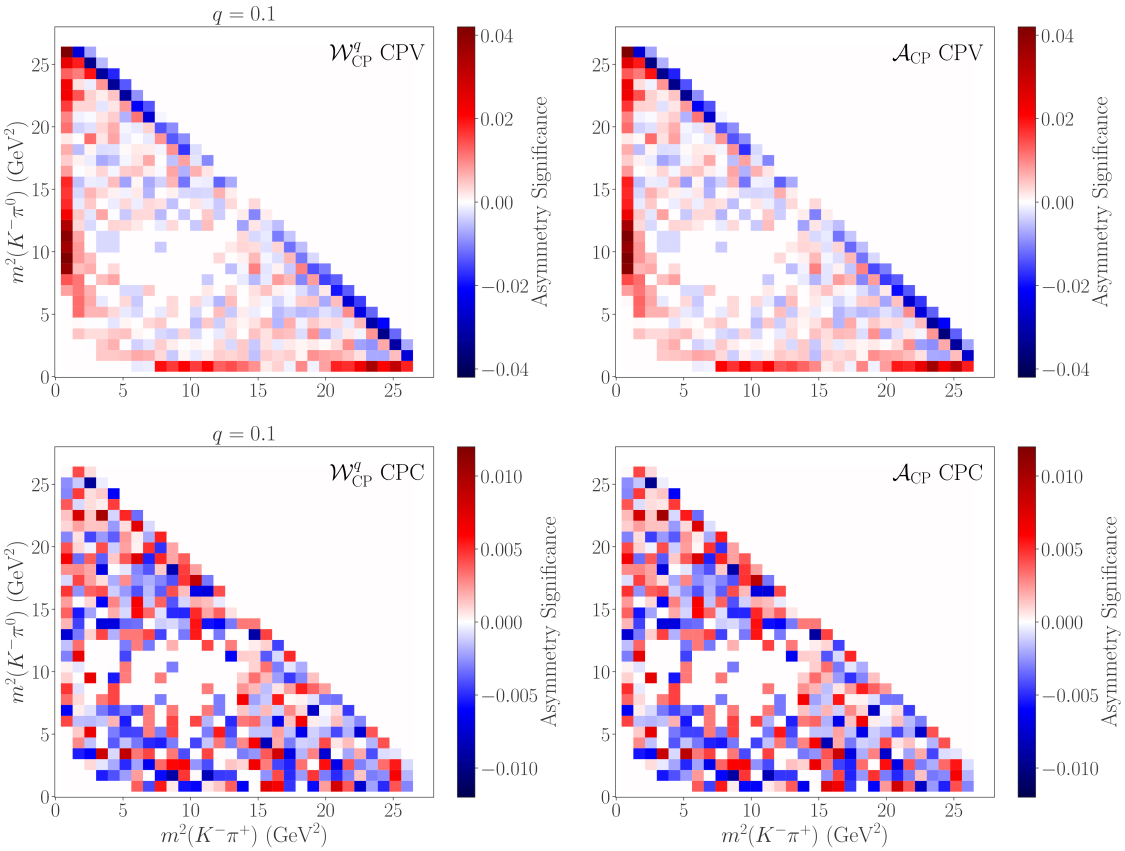
$$W_q = \sum_i \delta W_q(i)$$

$$\mathcal{W}^q_{\rm CP}(s_{12}, s_{13}) = \frac{\sum_i \delta \bar{W}_i - \sum_i \delta W_i}{\sum_i \delta \bar{W}_i + \sum_i \delta W_i}$$



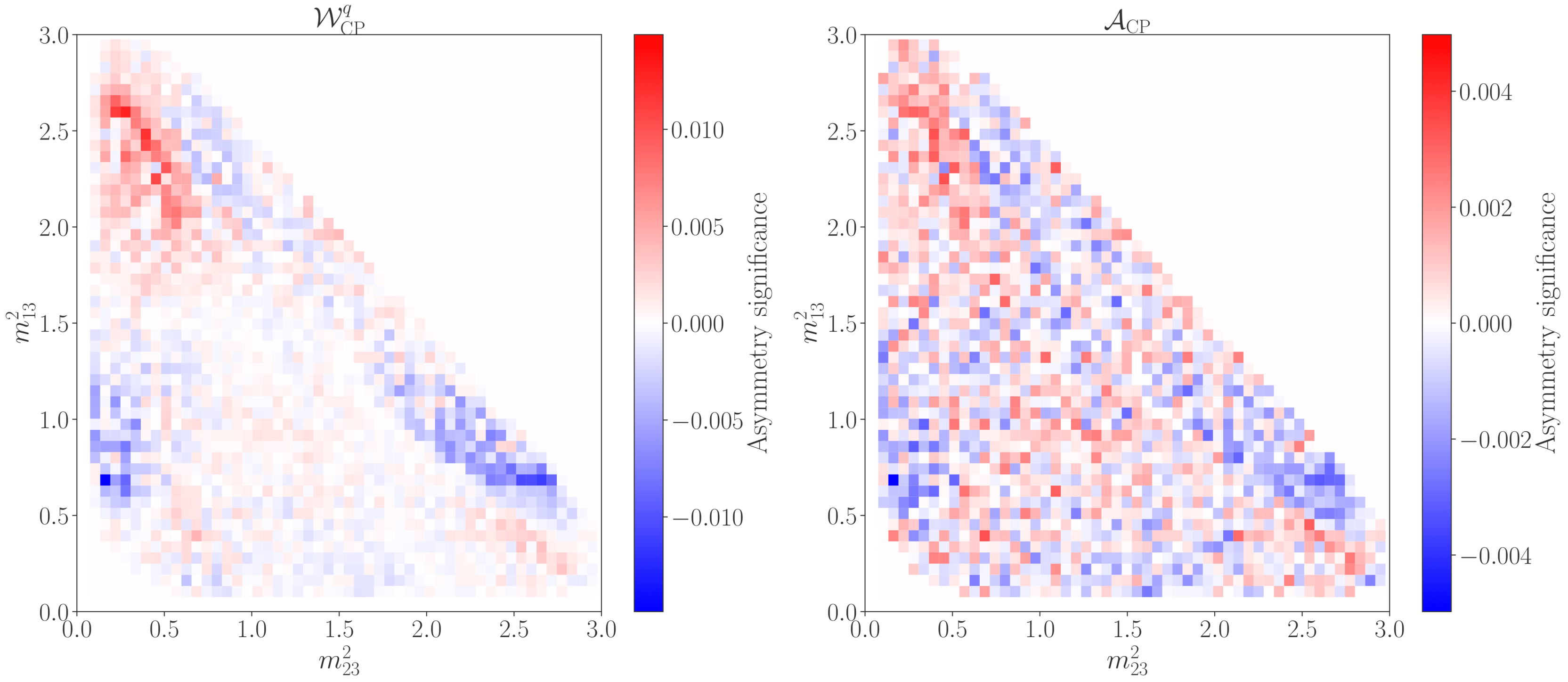
Visualization



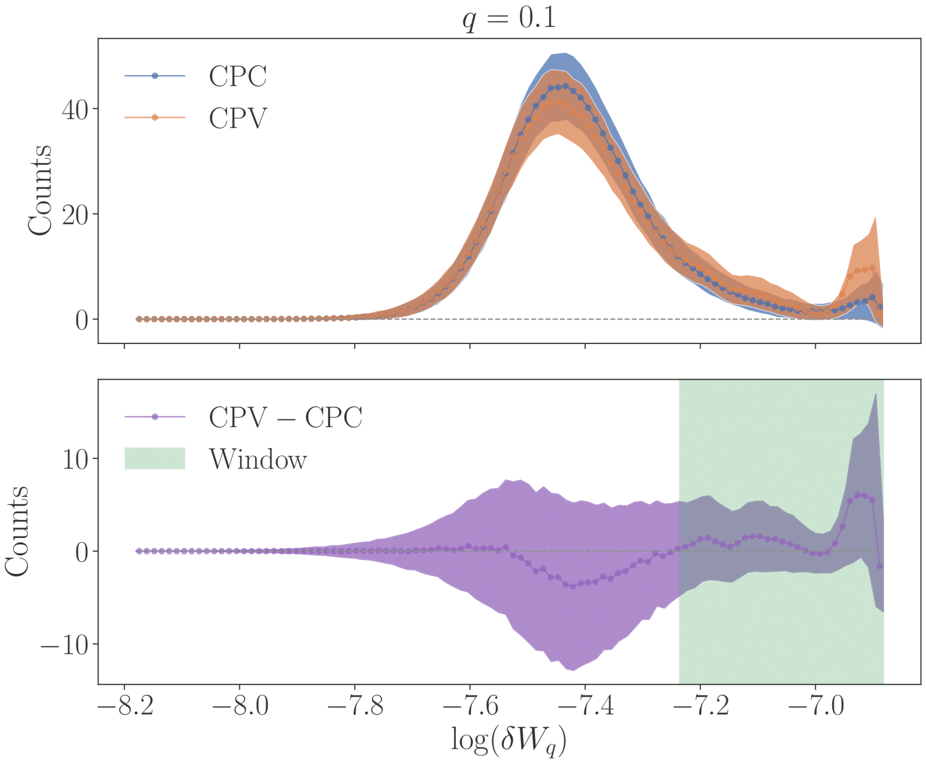


Visualization

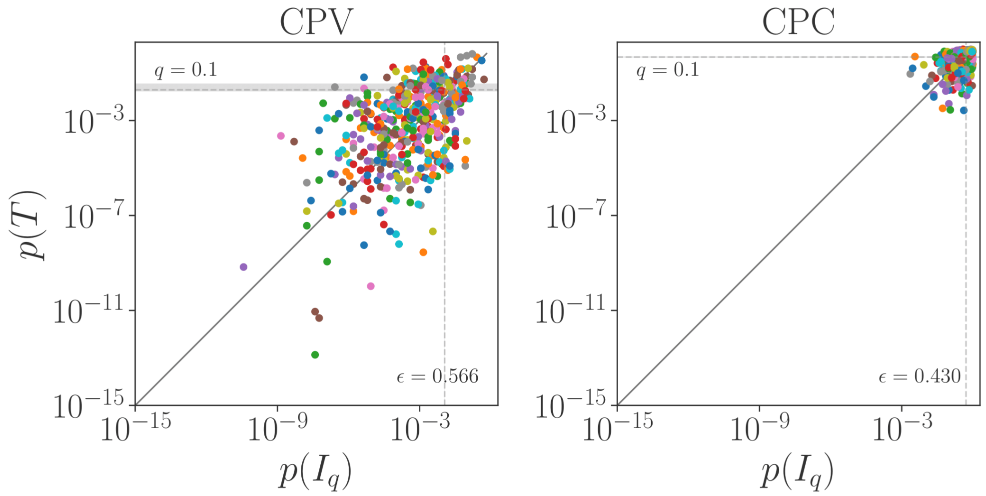
Binned variation for large dataset:



Windowed-Wasserstein statistic



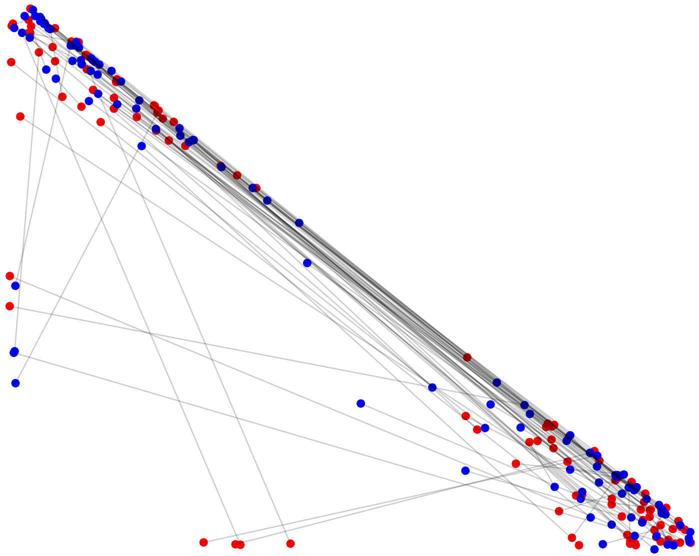
$$w(x) = \begin{cases} +1 & x \in [\delta W^{\text{window}}_{\text{min}}, \delta W^{\text{window}}_{\text{max}}] \\ -1 & x \in [\overline{\delta W^{\text{window}}_{\text{min}}}, \overline{\delta W^{\text{window}}_{\text{max}}}] \\ 0 & \text{otherwise} \end{cases}$$



Conclusion

The Wasserstein distance can be used as a robust, model independent, unbinned test of CP violation.

- **Future directions:**
  - Time dependent CPV
  - Flavor violation
  - Is there a better test?
  - Model independent way of classifying CPV resonances



BACK-UP

Direct CP violation in the Dalitz Plot

Neccesary condition for direct CPV: $\frac{\mathcal{M}(X \to \cdots)}{\mathcal{M}(\bar{X} \to \overline{\cdots})} \neq 1$

Requires at least two interferring amplitudes:

$$\mathcal{M}(X \to \cdots) = m_1 e^{i\phi_1} e^{i\delta_1} + m_2 e^{i\phi_2} e^{i\delta_2}$$

$$\mathcal{M}(\bar{X} \to \overline{\cdots}) = m_1 e^{-i\phi_1} e^{i\delta_1} + m_2 e^{-i\phi_2} e^{i\delta_2}$$

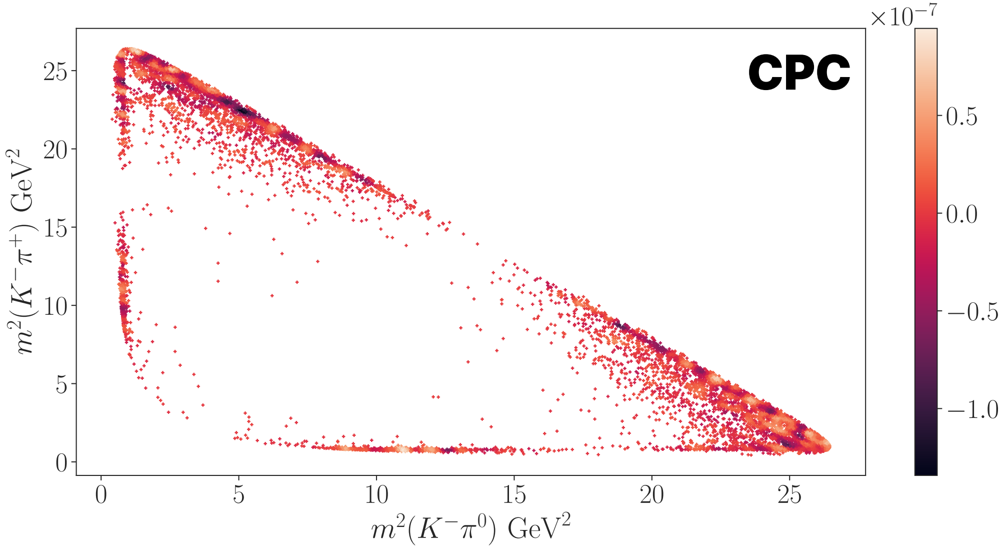
$$\downarrow$$

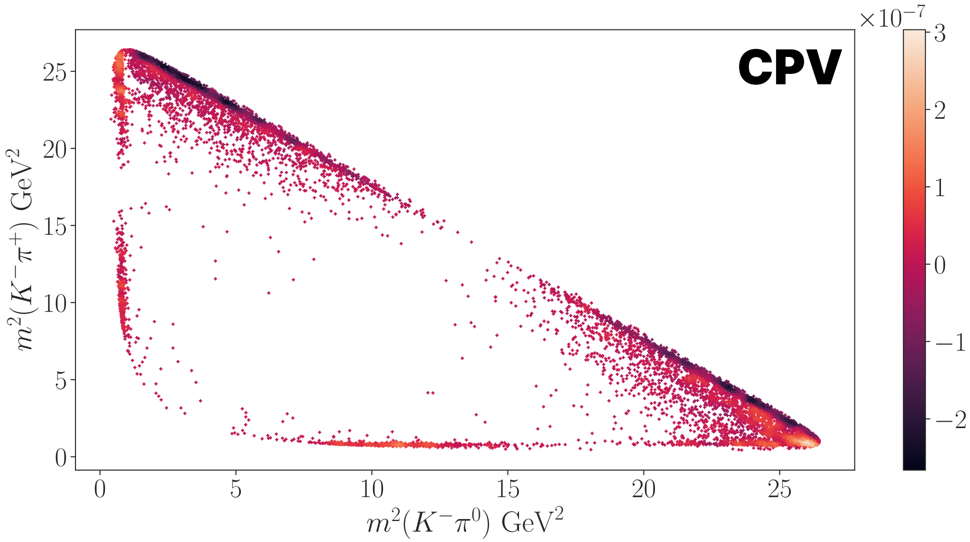
$$\frac{\Gamma(X) - \Gamma(\bar{X})}{\Gamma(X) + \Gamma(\bar{X})} = \frac{-m_1 m_2 \sin(\phi_1 - \phi_2) \sin(\delta_1 - \delta_2)}{|m_1|^2 + |m_2|^2 + 2 m_1 m_2 \cos(\phi_1 - \phi_2) \cos(\delta_1 - \delta_2)}$$

Unbinned tests for CPV

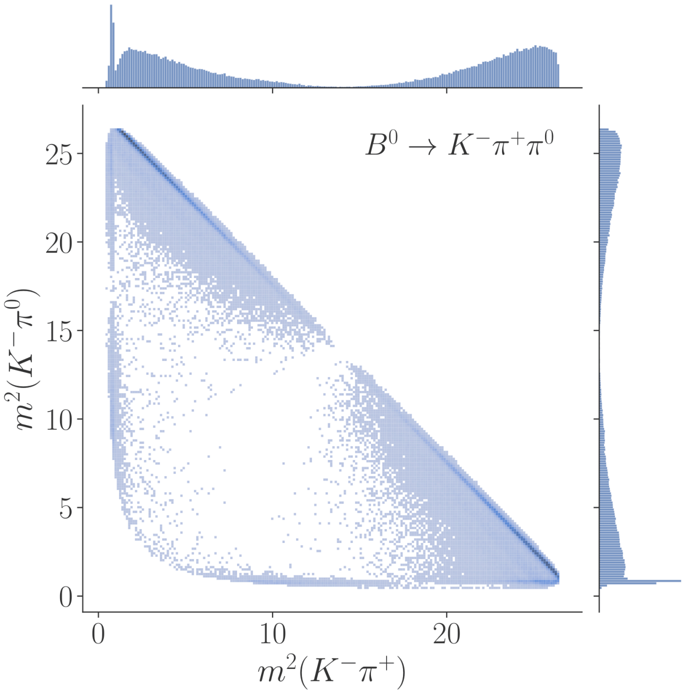
Energy Test: $O(n^2)$, tuned on σ

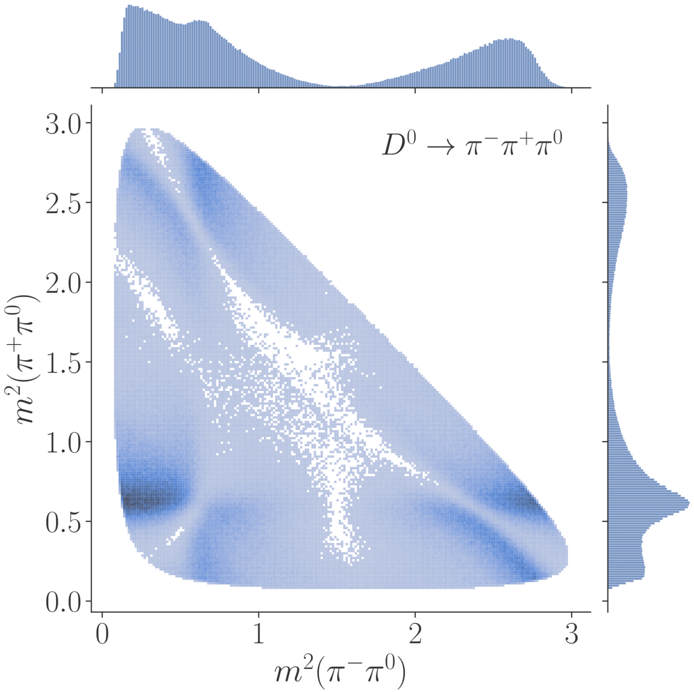
$$T = \sum_{i,j>i}^{N} \frac{\psi_{ij}}{N(N-1)} + \sum_{i,j>i}^{\bar{N}} \frac{\psi_{ij}}{\bar{N}(\bar{N}-1)} - \sum_{i,j}^{N,\bar{N}} \frac{\psi_{ij}}{N\bar{N}}, \qquad \psi_{ij} \equiv \psi(d_{ij};\sigma) = e^{-d_{ij}^2/2\sigma^2}$$





Data





Two analyses: Low statistics (1000 event datasets) and large statistics (10^5 event datasets)

EMD intuition