Rare Lepton Decays
and
Differentiable Hadronization Models
From New Physics Signatures to Data-Driven Event Generation

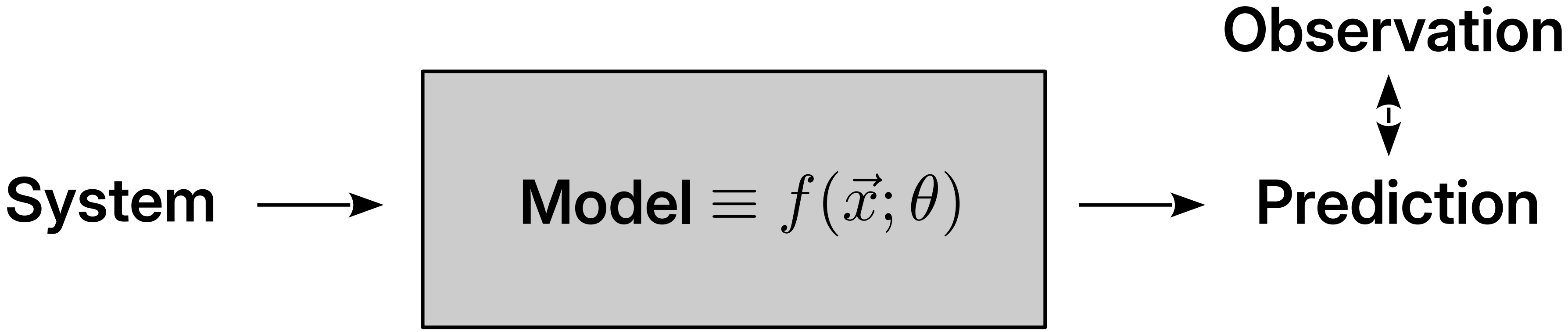
August 22nd, 2025

Tony Menzo

PhD candidate, University of Cincinnati

Physics

Description of systems using mathematical models verified directly or indirectly against physical observables.



Physics

Observation

↕

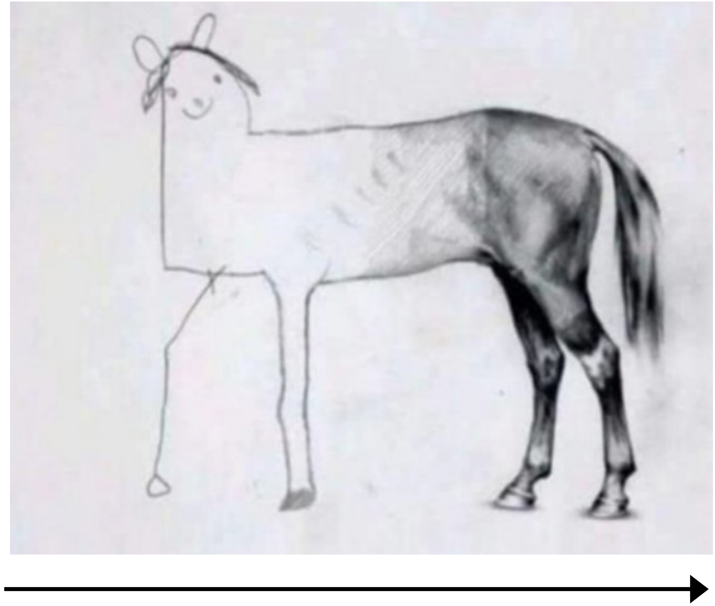
System → **Model** $\equiv f(\vec{x};\theta)$ → Prediction

Many classes of models:

Classical mechanics,
electricity and magnetism,
quantum mechanics, statistical
mechanics, ...

Physics

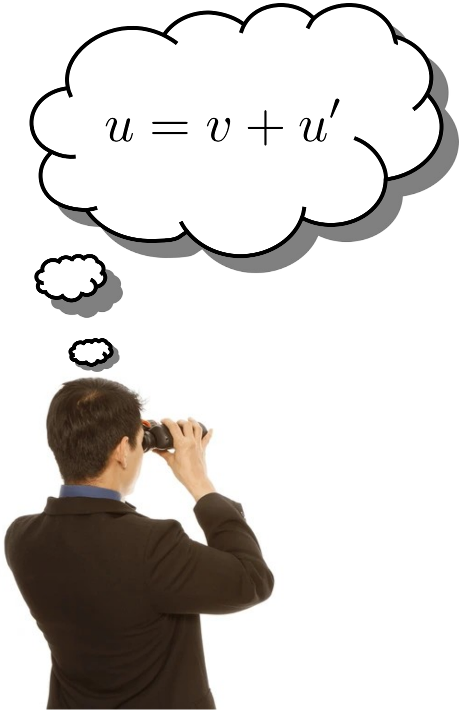
System → Model $\equiv f(x$

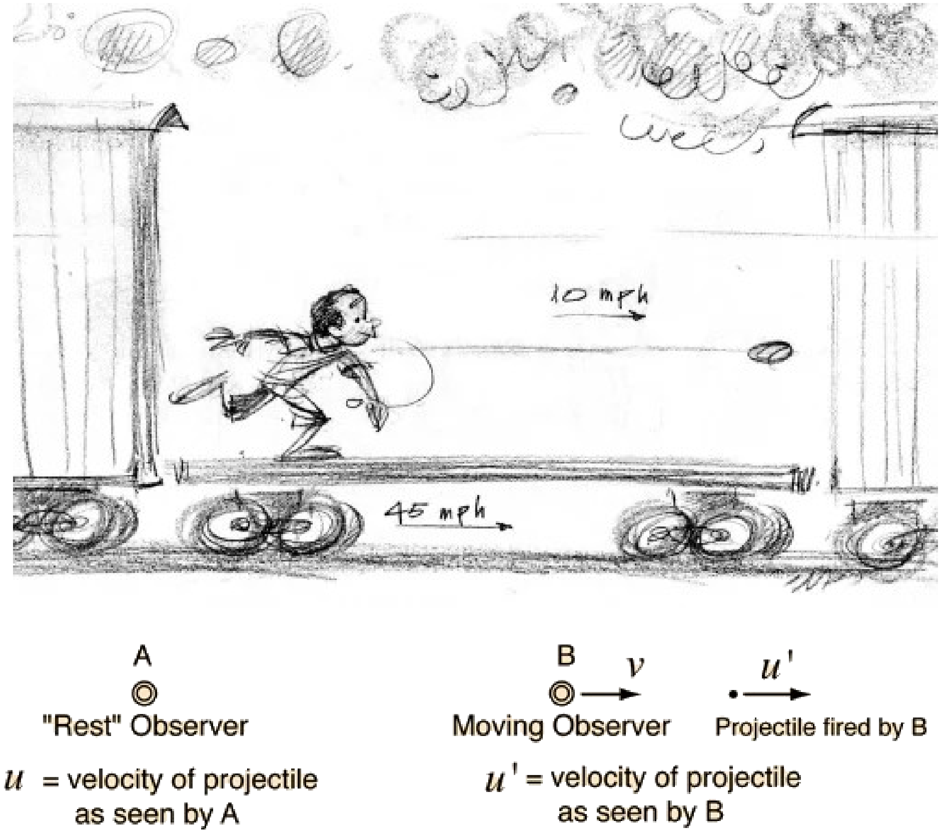


Many classes of models:

Classical mechanics, electricity and magnetism, quantum mechanics, statistical mechanics, ...

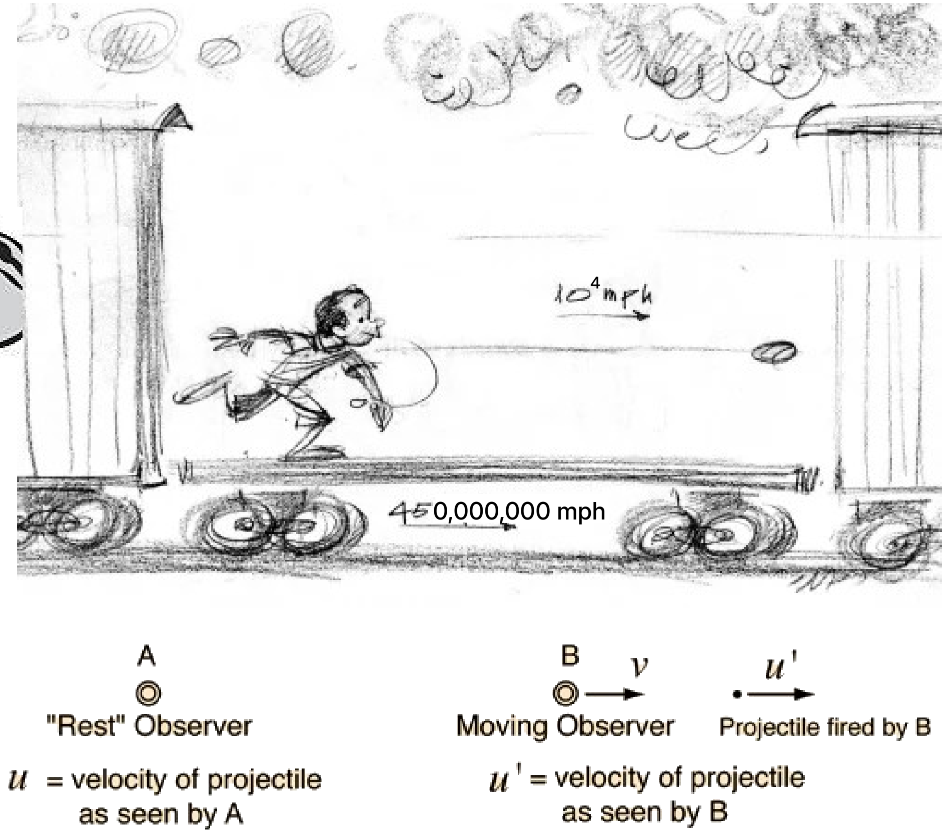
Fundamentals





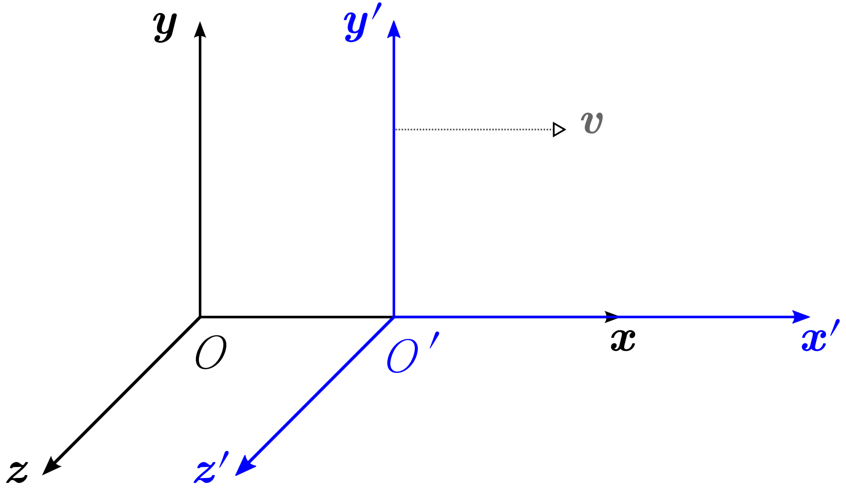
Fundamentals

$$u = \frac{v + u'}{1 + \frac{vu'}{c^2}}$$



Fundamentals

When/why do we need something more than classical mechanics?



$$x' = \gamma(x - vt), \quad t' = \gamma\left(t - \frac{v}{c^2}x\right), \quad \gamma \equiv \frac{1}{\sqrt{1 - v^2/c^2}}$$

Speed

Controlled by the speed of light: c

- The speed of light is constant in all frames!
- Physics should be independent of frame.

$v \ll c$	$v \sim c$
Galilean $\longrightarrow$	Lorentzian
(non-relativistic)	(relativistic)

Fundamentals

When/why do we need something more than classical mechanics?

$$E^2 = p^2c^2 + m^2c^4$$

Time dilation: $\Delta t = \gamma \Delta t'$

Length contraction: $L = \dfrac{L'}{\gamma}$

$$x' = \gamma(x - vt), \quad t' = \gamma\left(t - \frac{v}{c^2}x\right), \quad \gamma \equiv \frac{1}{\sqrt{1 - v^2/c^2}}$$

Speed

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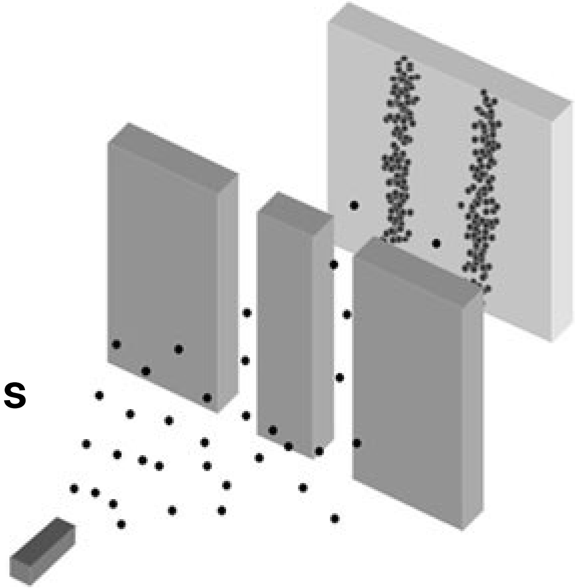
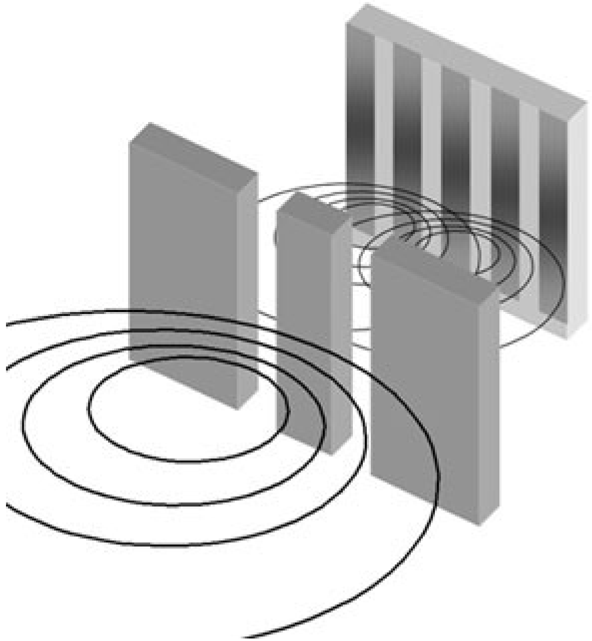
Fundamentals



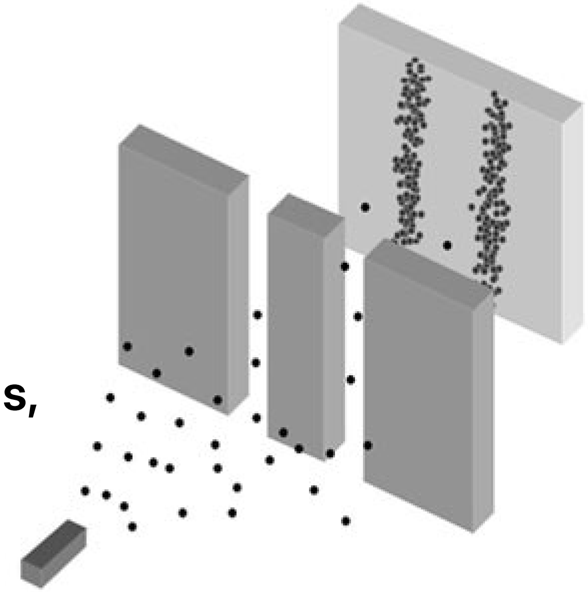
Baseballs

Fundamentals

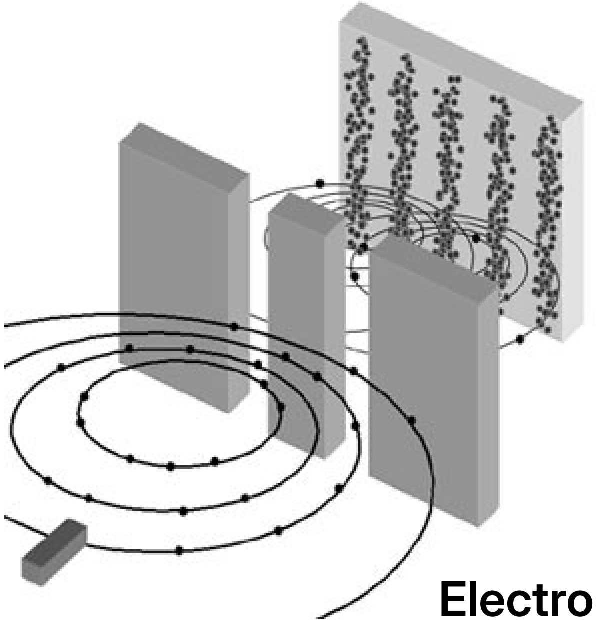


Fundamentals

Electrons,
Photons



$d \sim \lambda$



Electrons,
Photons

Fundamentals

When/why do we need something more than classical mechanics?

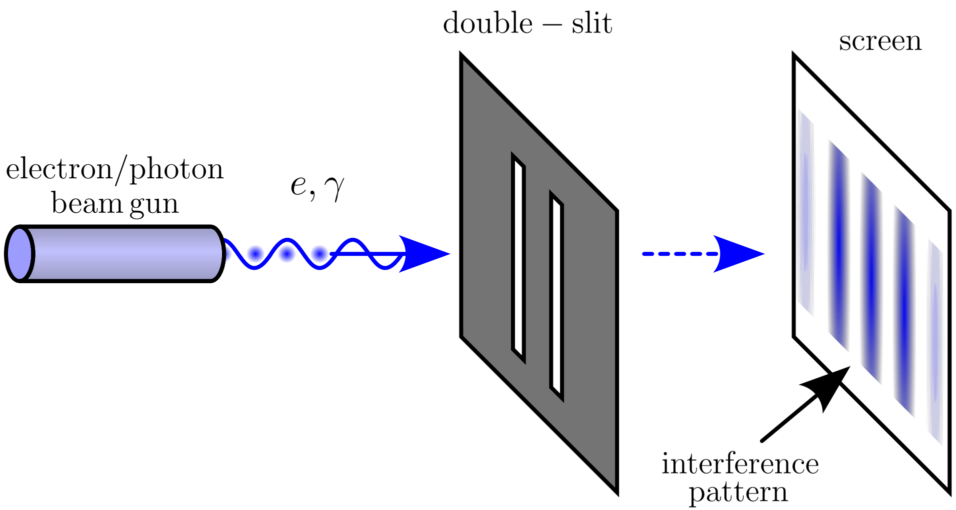
Size/Scale

Controlled by Planck's constant: $\hbar$

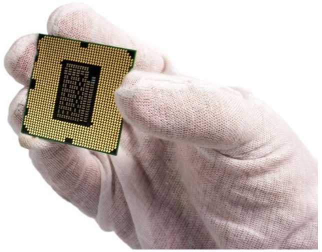
Matter is wavelike!

$$\lambda = \frac{2\pi\hbar}{mv},$$

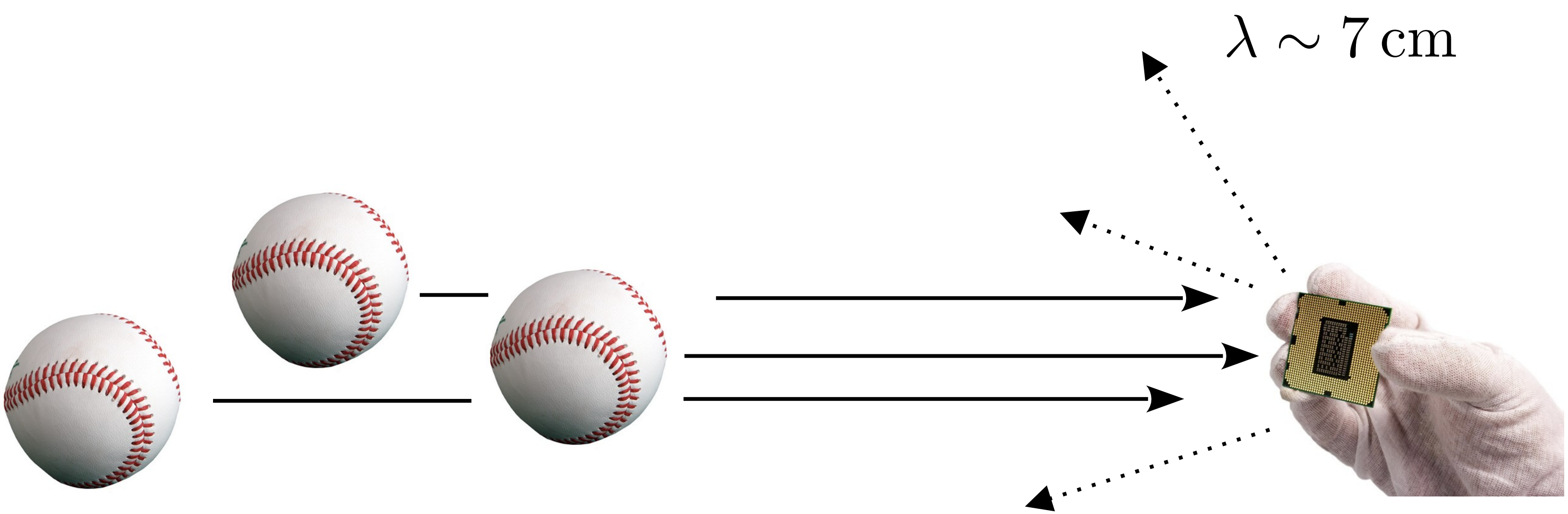
$d \gg \lambda$		$d \sim \lambda$
classical	$\rightarrow$	quantum
(deterministic)		(probabalistic)



Fundamentals

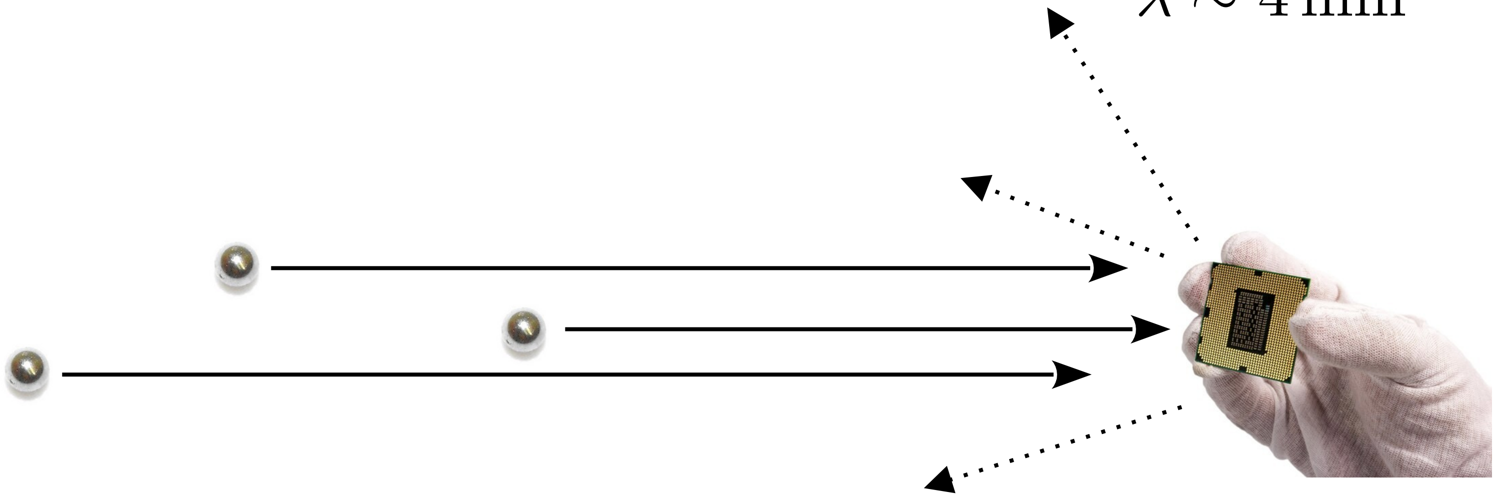


Fundamentals



Fundamentals

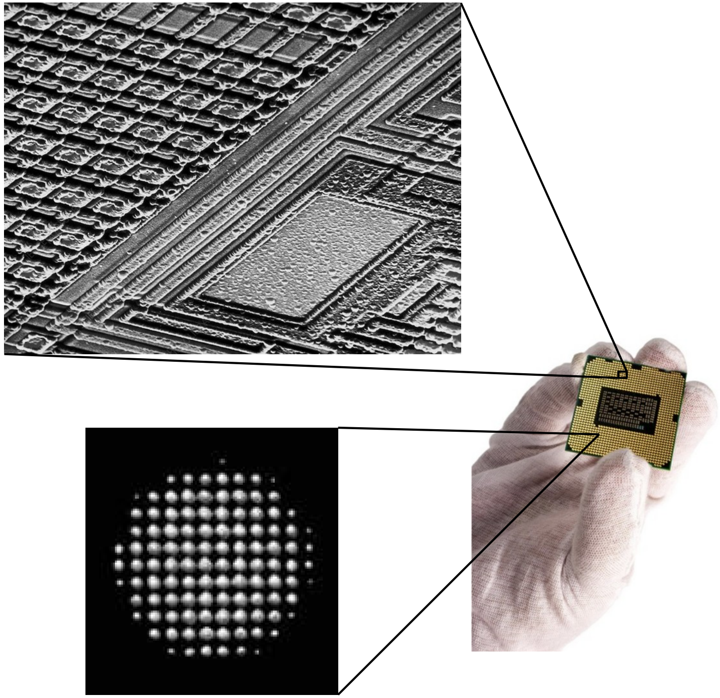
$\lambda \sim 4\,\mathrm{mm}$



Fundamentals

$$\lambda \sim 10^{-6}\,\mathrm{m}$$

Fundamentals

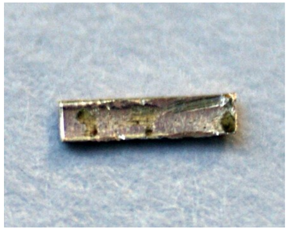


$$\lambda \sim 10^{-10}\,\mathrm{m}$$

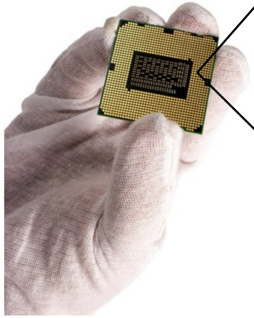
Fundamentals

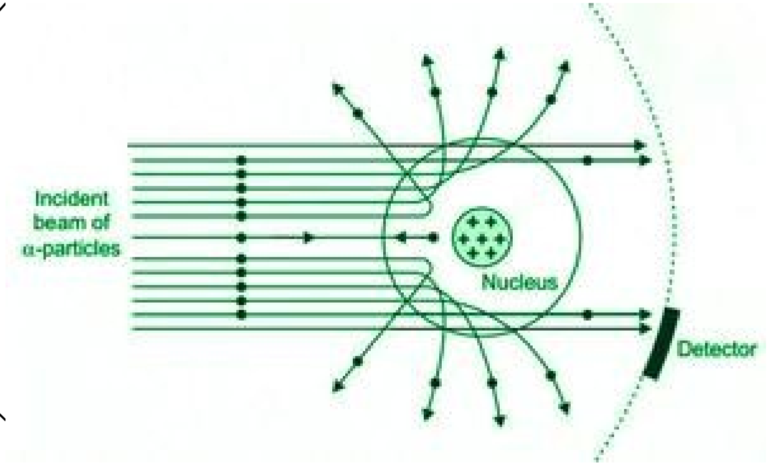
$$\lambda \sim 10^{-14}\,\mathrm{m} \sim \mathrm{MeV}$$

Radium



α

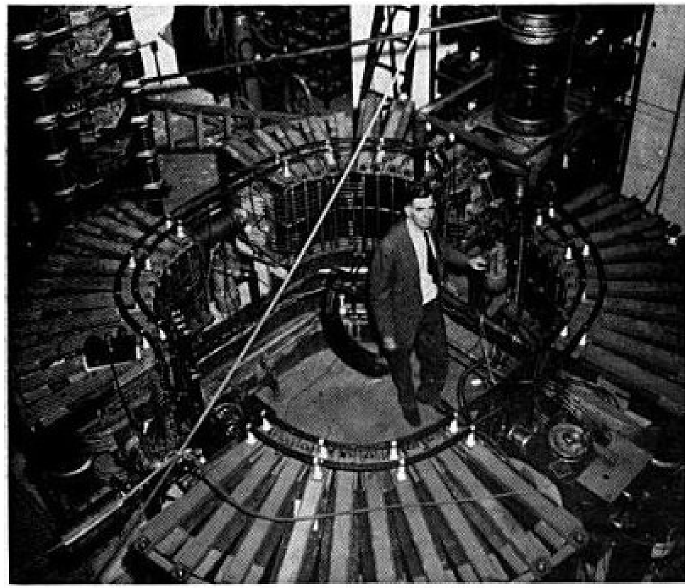




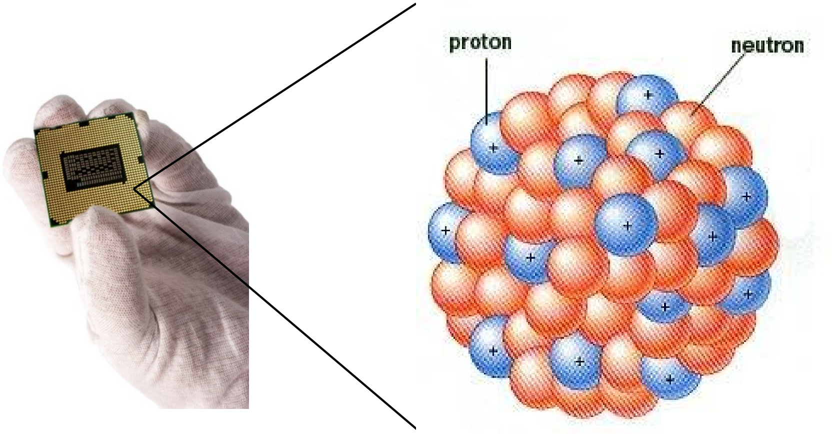
Fundamentals

$$\lambda \sim 10^{-15}\,\mathrm{m} \sim \mathrm{GeV}$$

Synchrotron



e

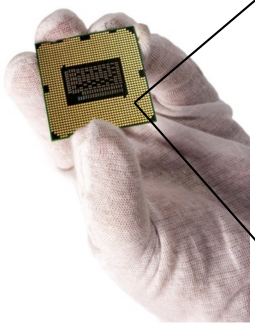


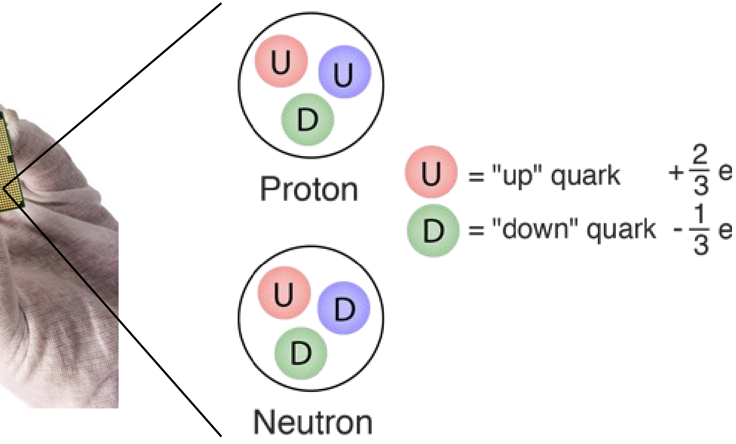
Fundamentals

$$\lambda \sim 10^{-18}\,\mathrm{m} \sim 20\,\mathrm{GeV}$$

SLAC-MIT experiment

e

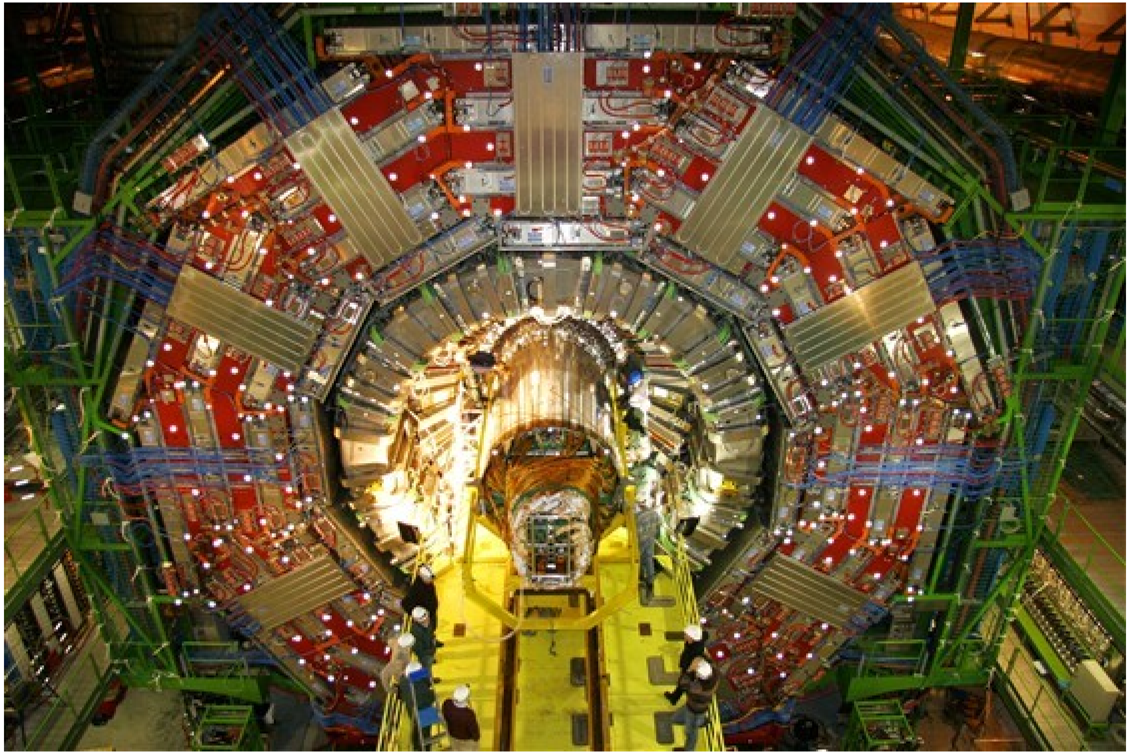


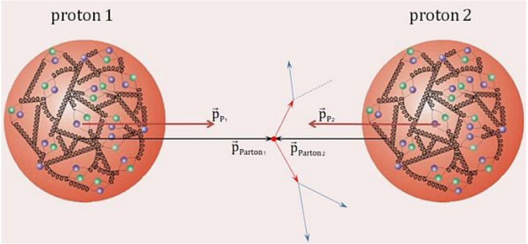


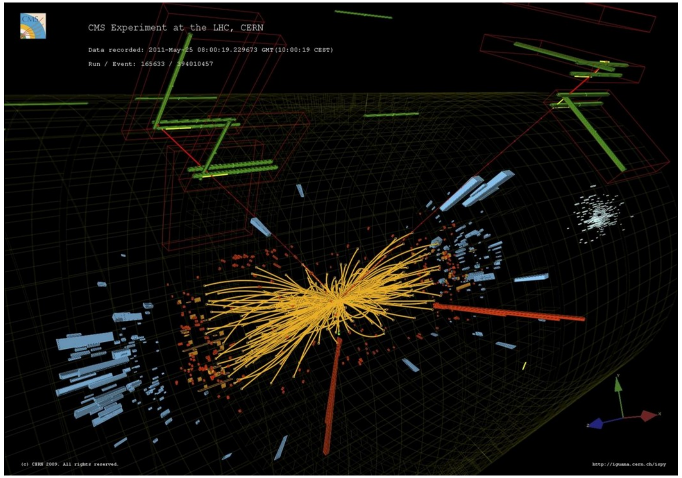
Fundamentals

Modern colliders - LHC

$$\lambda \sim 10^{-20}\,\mathrm{m} \sim 7\,\mathrm{TeV}$$







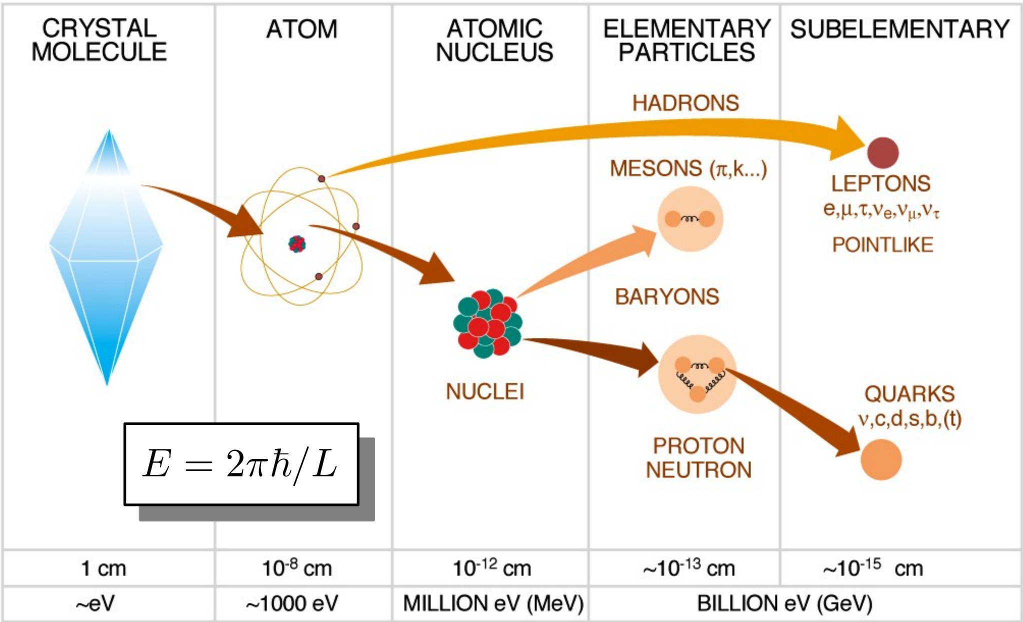
Fundamentals

When/why do we need something more than classical mechanics?

Size/Scale

The degree of freedoms and dynamics change as length scale decreases (energy increases).

Theory is inherently scale-dependent.



$$E = 2\pi\hbar/L$$

https://cds.cern.ch/record/841445

Fundamentals

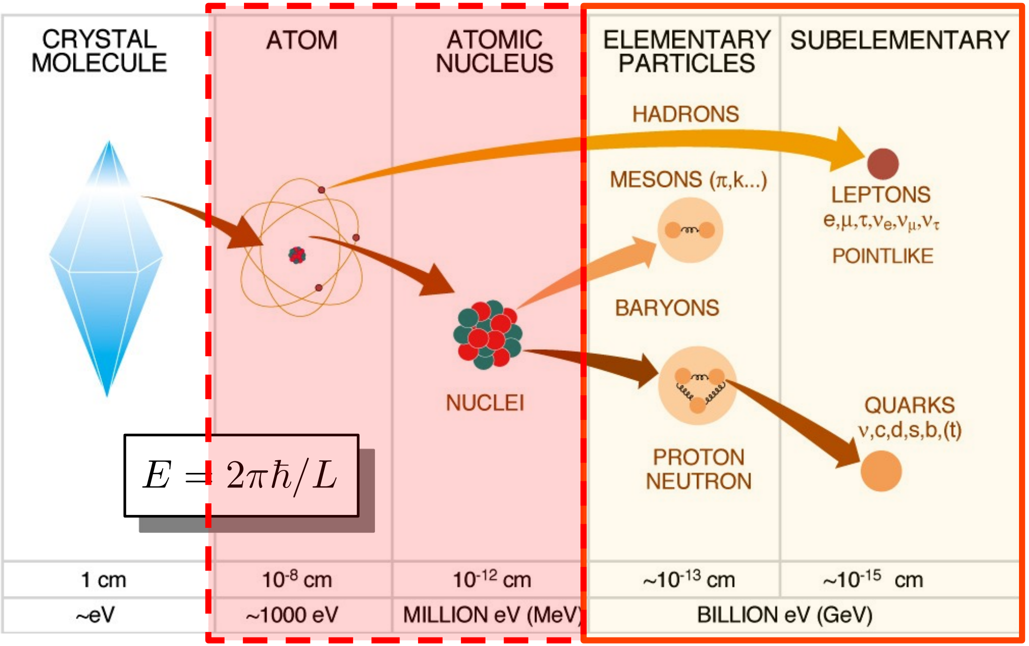
When/why do we need something more than classical mechanics?

High Energy Physics

Size/Scale

The degree of freedoms and dynamics change as length scale decreases (energy increases).

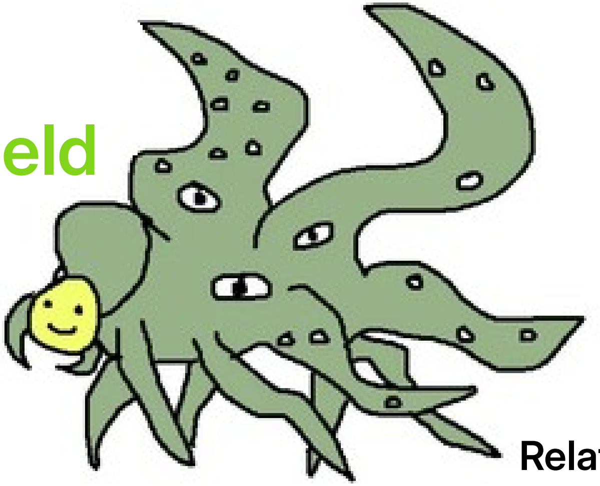
Theory is inherently scale-dependent.



https://cds.cern.ch/record/841445

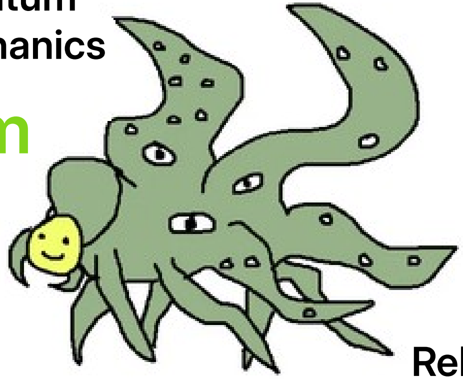
High Energy Physics (HEP)

What is the model?



High Energy Physics (HEP)

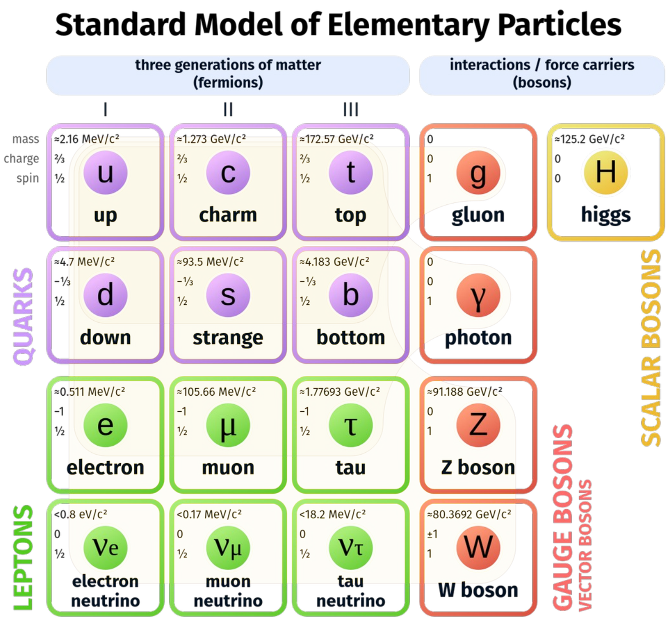
What is the model?

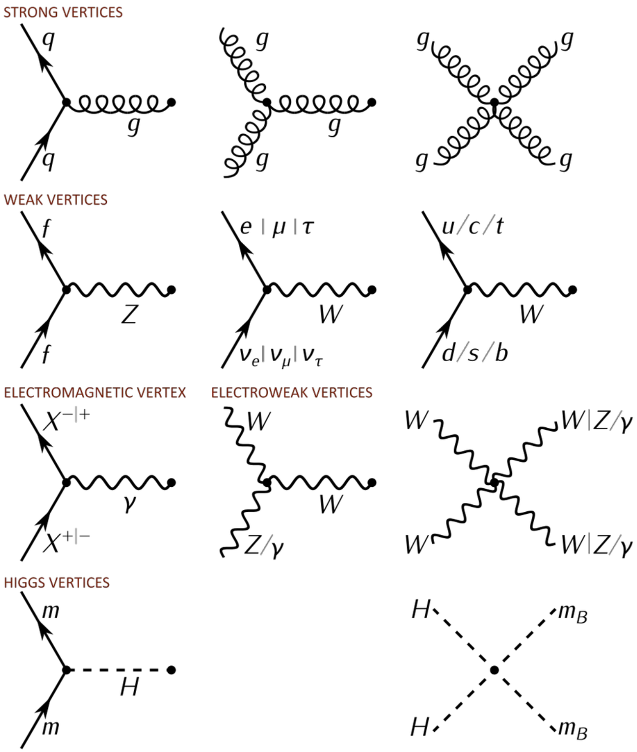


Ingredients:
- Symmetries
- Field content (particles)
- Lagrangian (interactions)

High Energy Physics (HEP)

The *Standard Model*





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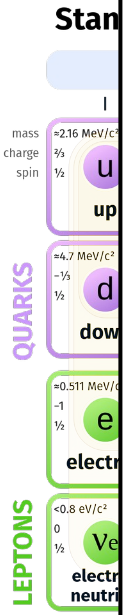
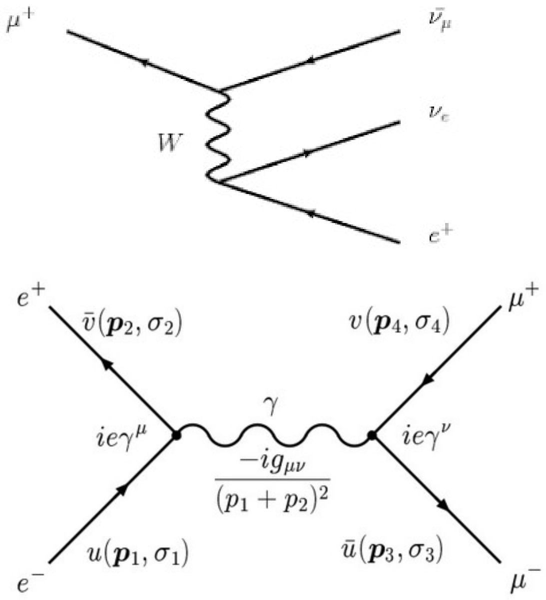
High Energy Physics (HEP)

The *Standard Model*





Decay rates

Γ

Cross sections

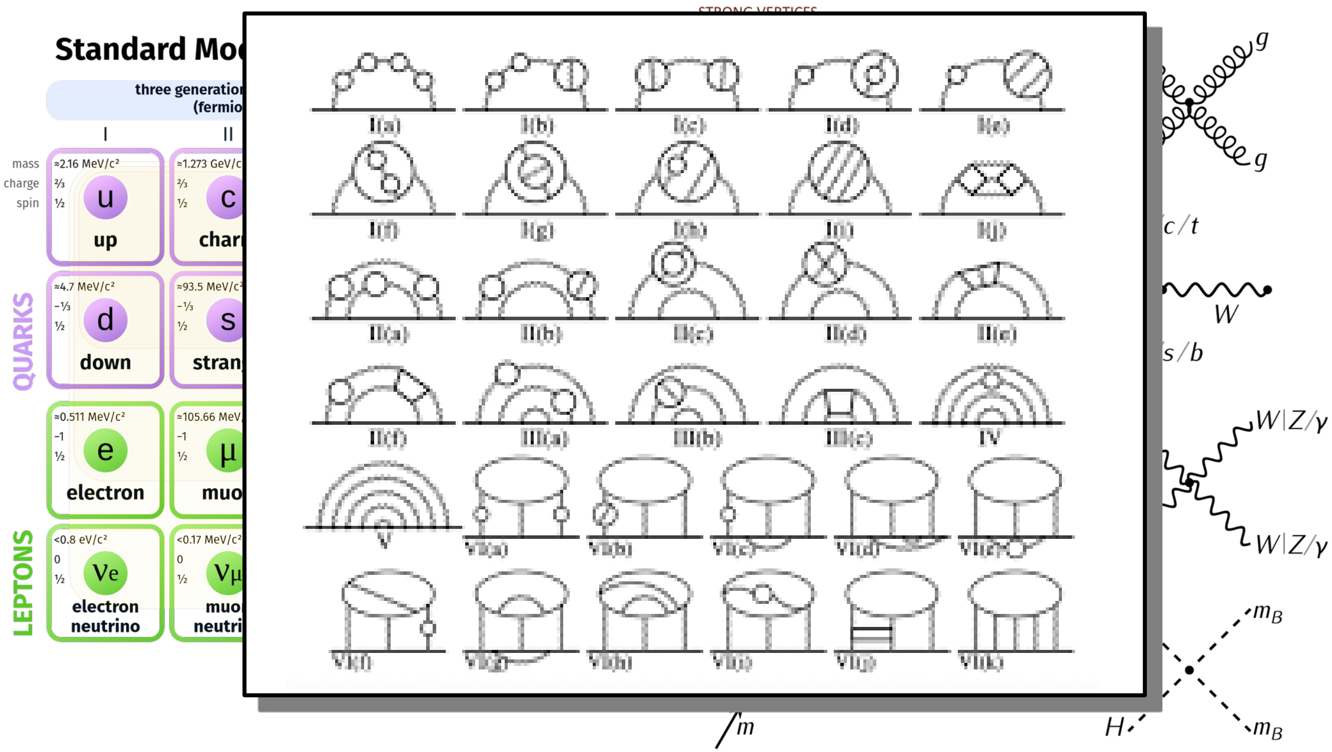
σ

Ingredients:

- Symmetries
- Field content (particles)
- Lagrangian (interactions)

High Energy Physics (HEP)

The *Standard Model*



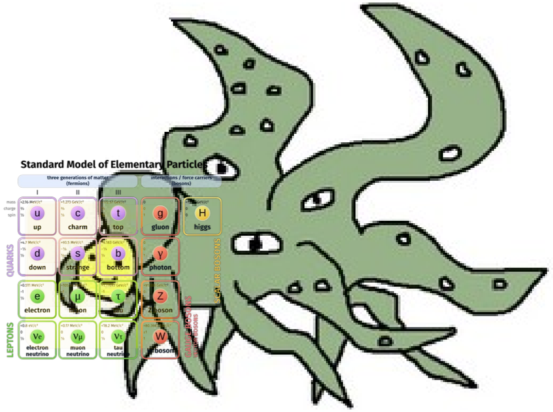
Ingredients:

- Symmetries
- Field content (particles)
- Lagrangian (interactions)

High Energy Physics (HEP)

The *Standard Model*

Particles, Kinematics →



→ Prediction

Observation

↕

Prediction

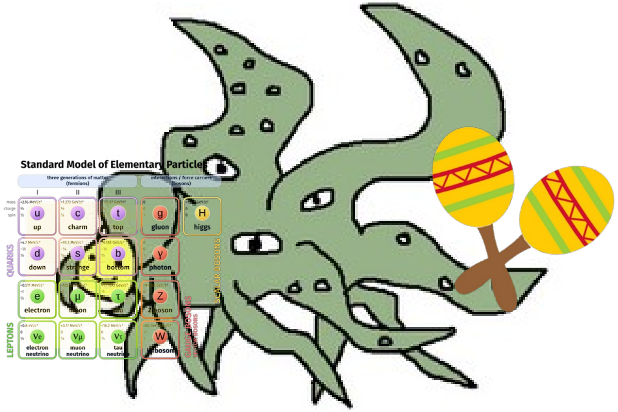
Γ, σ

HEP - Phenomenology

Physics beyond the
Standard Model (BSM)

Particles,
Kinematics

→



→

Observation

↕

Prediction

Γ', σ'

HEP - Experiment

Searching for BSM

Precise counting!

- Energy frontier: Collide at high energy collisions and search for new particle production
- Intensity frontier: Look for rare processes through large statistics

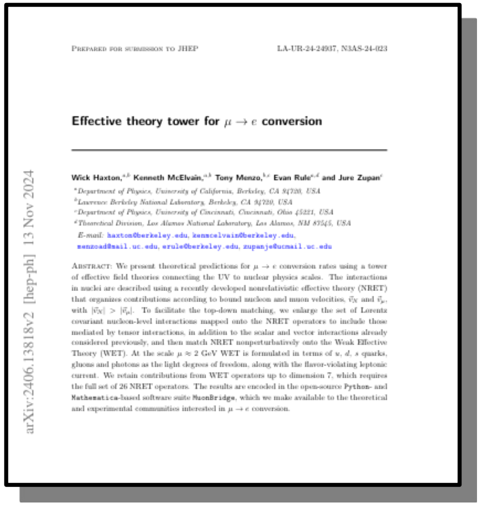
My dissertation

PART I - Rare lepton decays: Inserting new physics and looking for probeable signals at the intensity frontier


PREPARED FOR SUBMISSION TO JHEP

New physics in multi-electron muon decays

arXiv:2306.15631v2 [hep-ph] 13 Sep 2023


PREPARED FOR SUBMISSION TO JHEP

LA-UR-24-24937, N3AS-24-023

Effective theory tower for $\mu \to e$ conversion

arXiv:2406.13818v2 [hep-ph] 13 Nov 2024


Muon-induced baryon number violation

arXiv:2407.03450v1 [hep-ph] 3 Jul 2024


Direct Detection of Ultralight Dark Matter via Charged Lepton Flavor Violation

arXiv:2503.07722v2 [hep-ph] 22 May 2025

My dissertation

PART II – Methods to improve simulations of particle collisions at the energy frontier

My dissertation

Honorable mentions – Optimal transport based observables for CP violation, and new physics at fusion reactors


Prepared for submission to JHEP

arXiv:2301.13211v1 [hep-ph] 30 Jan 2023

Earth mover's distance as a measure of CP violation

Adam Davis,[a] Tony Menzo,[b] Ahmed Youssef,[b] Jure Zupan,[b]

[a]School of Physics and Astronomy, University of Manchester, M13 9PL, Manchester, UK
[b]Department of Physics, University of Cincinnati, Cincinnati, Ohio 45221, USA

E-mail: adam.davis@manchester.ac.uk, menzoad@mail.uc.edu, youssead@ucmail.uc.edu, zupanje@ucmail.uc.edu

Abstract: We introduce a new unbinned two sample test statistic sensitive to CP violation utilizing the optimal transport plan associated with the Wasserstein (earth mover's) distance. The efficacy of the test statistic is shown via two examples of CP asymmetric distributions with varying sample sizes: the Dalitz distributions of $B^0 \to K^+\pi^-\pi^0$ and of $D^0 \to \pi^+\pi^-\pi^0$ decays. The windowed version of the Wasserstein distance test statistic is shown to have comparable sensitivity to CP violation as the commonly used energy test statistic, but also retains information about the localized distributions of CP asymmetry over the Dalitz plot. For large statistic datasets we introduce two modified Wasserstein distance based test statistics – the binned and the sliced Wasserstein distance statistics, which show comparable sensitivity to CP violation, but improved computing time and memory scalings. Finally, general extensions and applications of the introduced statistics are discussed.


Prepared for submission to JHEP MIT-CTP/5816

arXiv:2502.12314v1 [hep-ph] 17 Feb 2025

Searching for exotic scalars at fusion reactors

Chaja Baruch,[a] Patrick J. Fitzpatrick,[b,c,e] Tony Menzo,[d] Yotam Soreq,[a] Sokratis Trifinopoulos,[e] Jure Zupan[d]

[a]Physics Department, Technion–Israel Institute of Technology, Haifa 3200003, Israel
[b]Dual CP Institute of High Energy Physics, C.P. 28045, Colima, México
[c]Instituto de Física, Universidad Nacional Autónoma de México, A.P. 20-364, Ciudad de México 01000, México
[d]Department of Physics, University of Cincinnati, Cincinnati, OH 45221, USA
[e]Center for Theoretical Physics, Massachusetts Institute of Technology, Cambridge, MA 02139, USA

E-mail: chajabaruch@campus.technion.ac.il, patrickf@campus.technion.ac.il, menzoad@mail.uc.edu, soreqy@physics.technion.ac.il, trifinos@mit.edu, zupanje@ucmail.uc.edu

Abstract: The energy created in deuterium-tritium fusion reactors originates from a high-intensity neutron flux interacting with the reactor's inner walls. The neutron flux can also be used to produce a self-sustaining reaction by lining the walls with lithium-rich 'breeding blankets', in which a fraction of neutrons interacts with lithium, creating the tritium fuel. The high-intensity neutron flux can also result in the production of dark sector particles, feebly interacting light scalars or pseudoscalars, via nuclear transitions within the breeding blanket. We estimate the potential size of such dark sector flux outside the reactor, taking into account all current constraints, and consider possible detection methods at current and future thermonuclear fusion reactors. As a by-product, we also recast the SNO axion bound for a CP even scalar. We find that year-long searches at current and future reactors can set leading constraints on dark scalar– and dark pseudoscalar–nucleon couplings.

Outline

1) Introduction to charged lepton flavor violation

2) Fun signatures at rare muon decay experiments

   - Multi-electron final states, exotic muon capture modes, time-dependent signals from ultralight dark matter

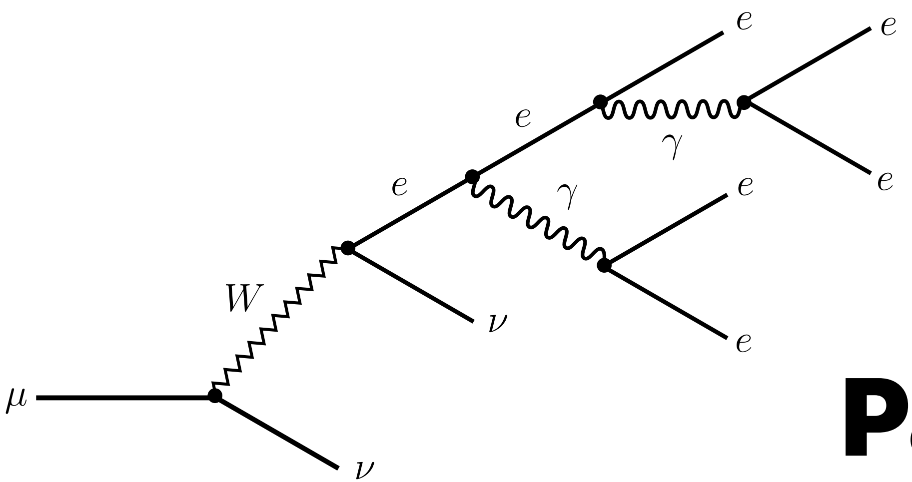
3) Introduction to event generators, hadronization

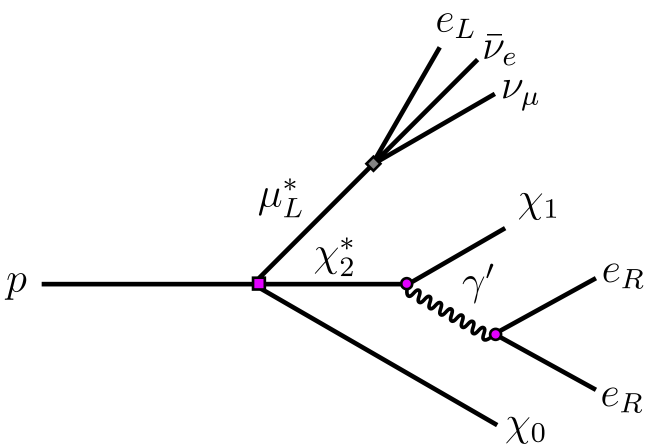
4) The inverse problem of hadronization

   - Pure ML solutions, hybrid solutions

5) Conclusions

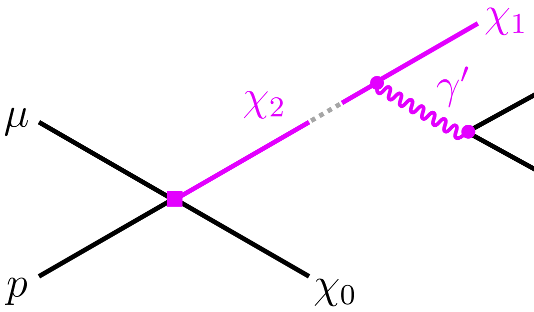
6) Acknowledgements

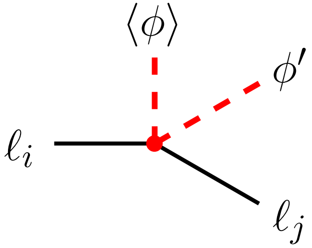


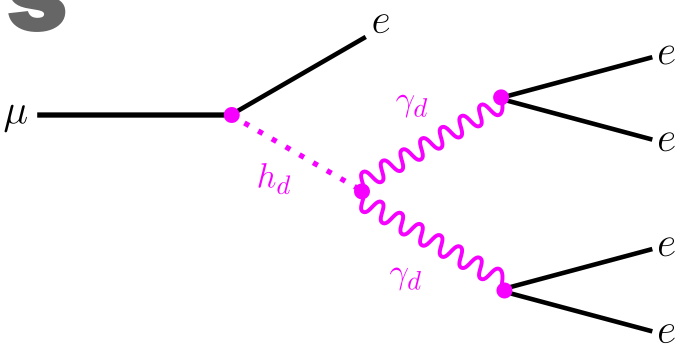


Part I:

Signatures of new physics in rare lepton decays





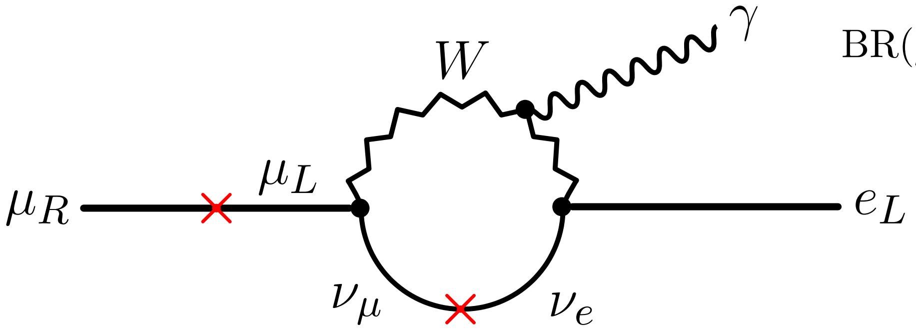


Standard lore - CLFV

- The Standard Model (SM) has an accidental global flavor symmetry

$$U(1)_e \times U(1)_\mu \times U(1)_\tau$$

- Because $m_\nu \neq 0$ charged-lepton-flavor violation (CLFV) can occur at one-loop



$$\mathrm{BR}(\mu \to e\gamma) \simeq \frac{3\alpha}{32\pi M_W^4}|U_{\mu 3}U_{e3}^*\Delta m_{31}^2 + U_{\mu 2}U_{e2}^*\Delta m_{21}^2|^2 \simeq 10^{-54}$$

$$\mathrm{BR}(\mu \to eee) \simeq \frac{\alpha}{3\pi}\left(\log\frac{m_\mu^2}{m_e^2} - 3\right) \times \mathrm{BR}(\mu \to e\gamma)$$

$$\mathrm{CR}(\mu \mathrm{N} \to e\mathrm{N}) \simeq \alpha \times \mathrm{BR}(\mu \to e\gamma)$$

Bottom line: Observing CLFV = new physics

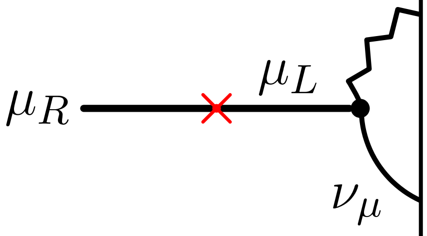
Standard lore - CLFV

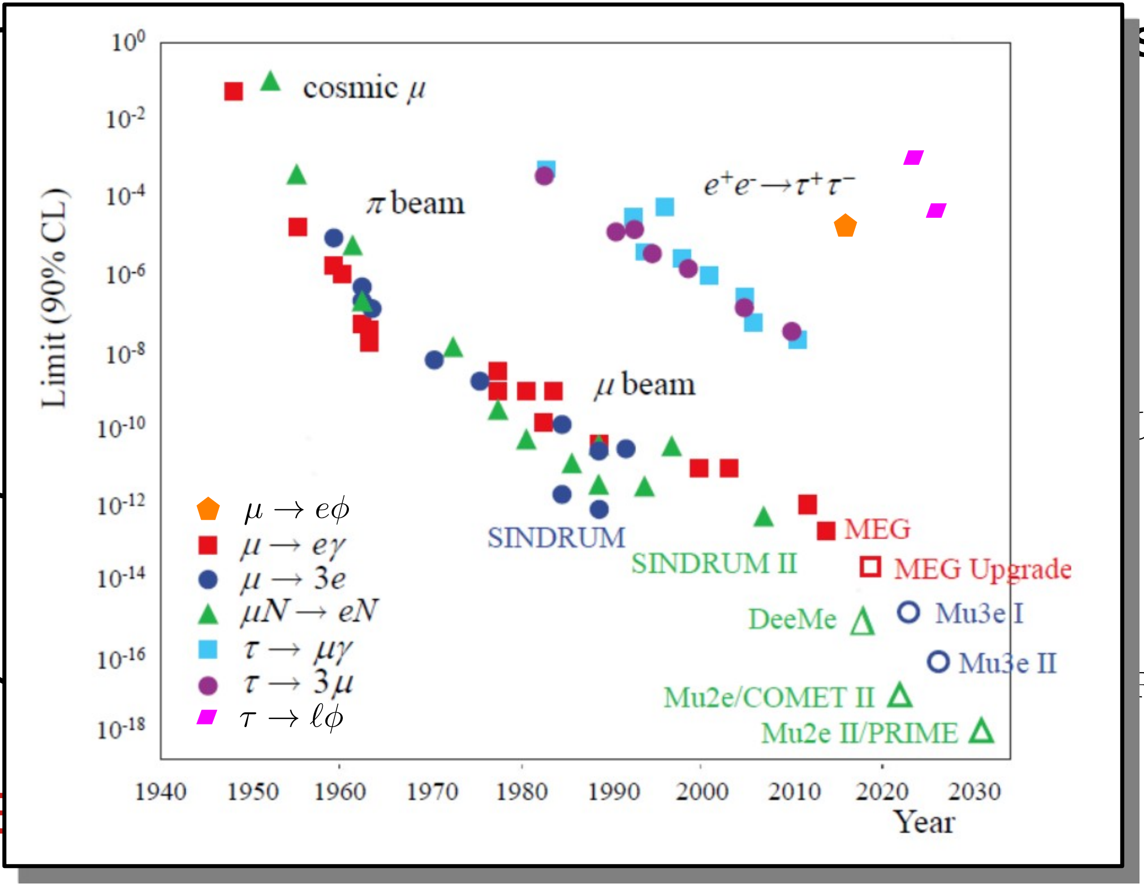
- The Standar… symmetry
- Because m_ν … n occur at one-loop

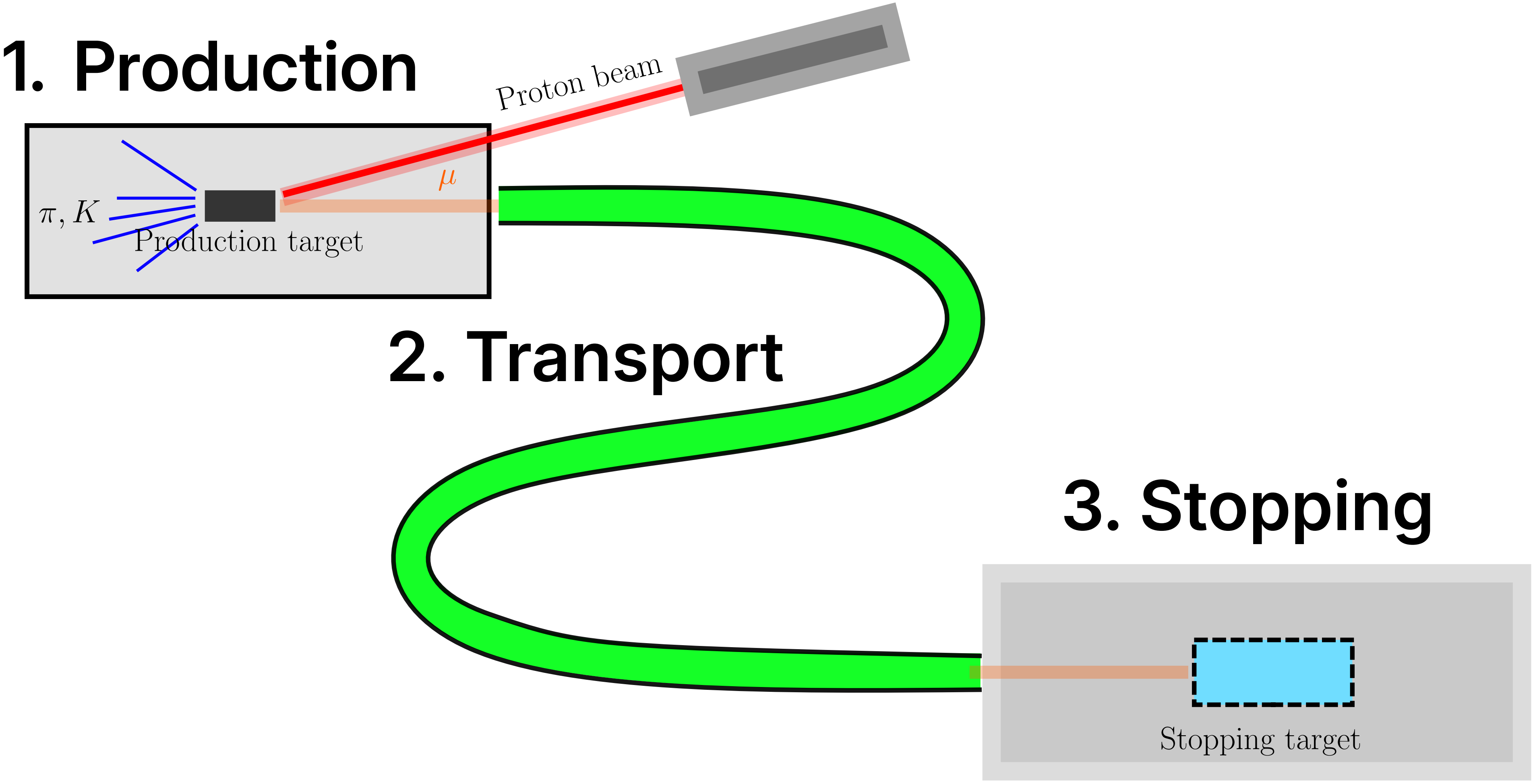
$\ldots U_{\mu 2} U_{e2}^{*} \Delta m_{21}^{2}|^{2} \simeq 10^{-54}$

$\ldots 3 \Big) \times \mathrm{BR}(\mu \to e\gamma)$

$\ldots \mathrm{R}(\mu \to e\gamma)$



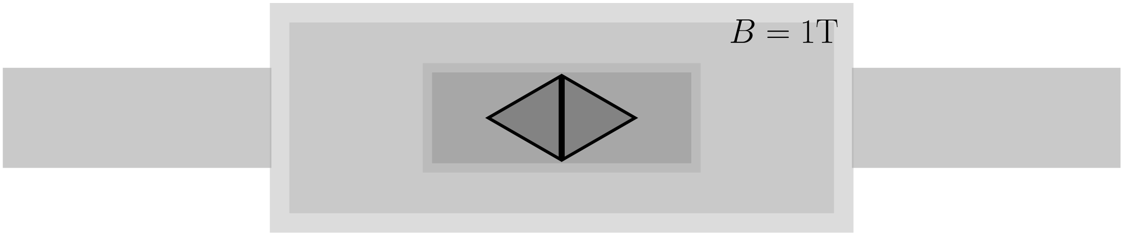



1. Production
Proton beam
μ
π, K
Production target
2. Transport
3. Stopping
Stopping target

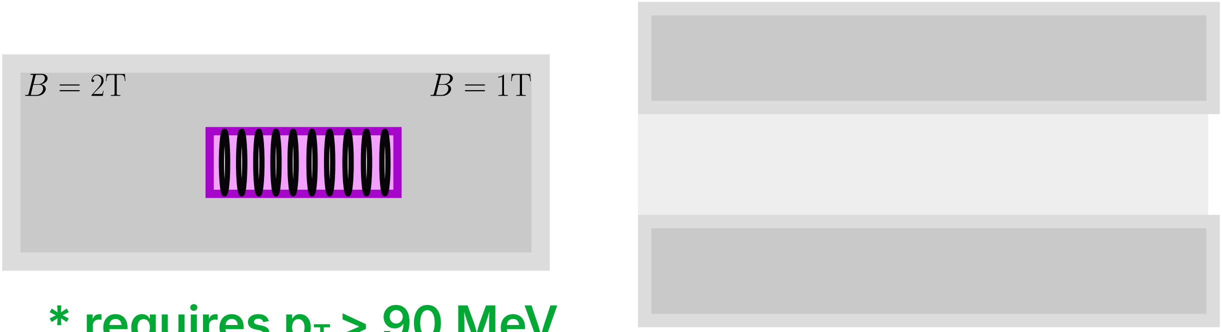
4. Detection (phenomenology is influenced by detector)

Mu3e (μ^+)

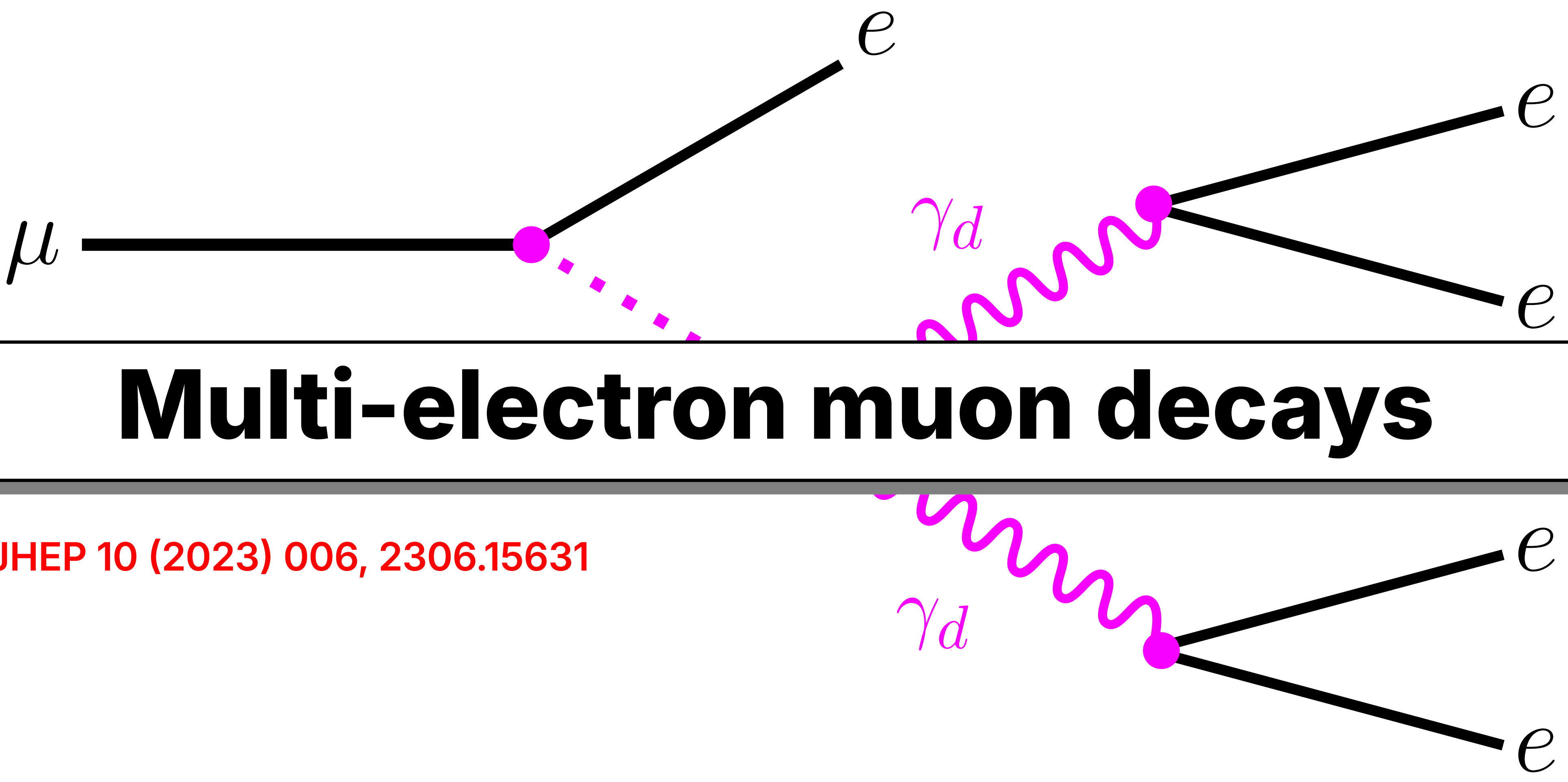
* requires $p_T > 10$ MeV



Mu2e (μ^-)



* requires $p_T > 90$ MeV



Multi-electron muon decays

JHEP 10 (2023) 006, 2306.15631

Mu3e → Mu5e

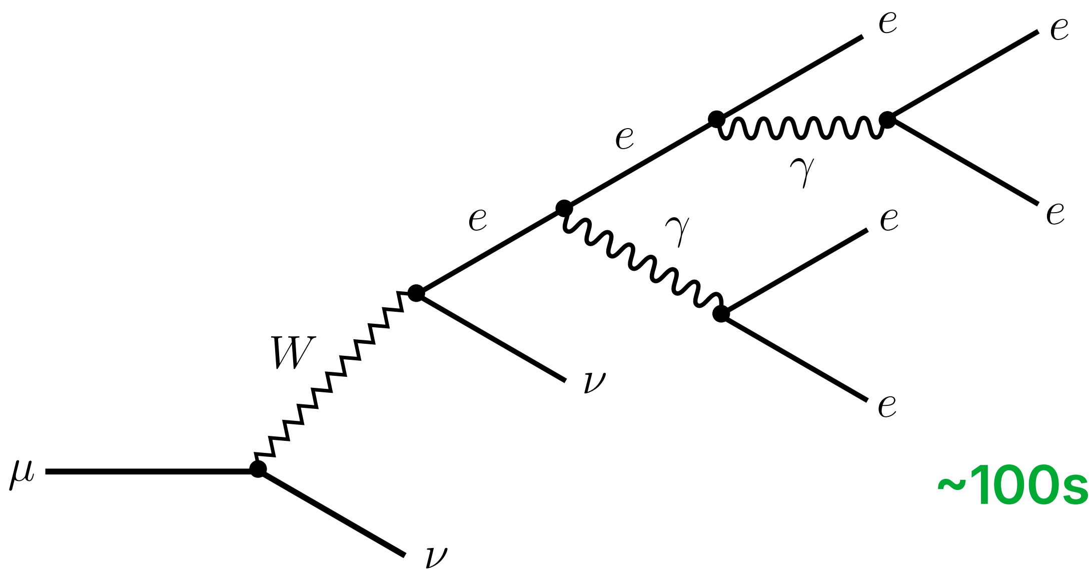
- Mu3e will see ~10^{15} total muon decays from the stopping target.
- With these statistics are there any other interesting channels?
  - What about $\mu \to 5e$?

μ → eeeeevv

- **SM background for Mu5e**

From MG5: $\mathcal{B}(\mu^+ \to e^+e^-e^+e^-e^+\nu_e\bar{\nu}_\mu) = (3.929 \pm 0.001) \times 10^{-10}$

$$\mathcal{B}\left(\mu^+ \to e^+e^-e^+e^-e^+\nu_e\bar{\nu}_\mu \,|\, \mathrm{all}\; p_{\mathrm{e}^\pm}^{\mathrm{T,true}} > 10\ \mathrm{MeV}\right) = (1.4 \pm 0.1) \times 10^{-14}$$



$\propto G_F^2 \alpha^4$

~100s of μ → 5evv events after cuts!

μ → eeeee

- Higgsed $U(1)_D$ + SM portal via kinetic mixing

$$\mathscr{L}_{\text{DS}} = (D_\mu \phi)^\dagger D^\mu \phi - \frac{1}{4} F_d^{\mu\nu} F_{d\,\mu\nu} - \frac{\varepsilon}{2} F_d^{\mu\nu} F_{\mu\nu} - \mu^2 (\phi^\dagger \phi) - \lambda (\phi^\dagger \phi)^2$$

$$\mathscr{L}_{\text{LFV}} = -\frac{C_{ij}}{\Lambda} \phi \left(\bar{L}_i H \right) \ell_j + \text{h.c.}$$

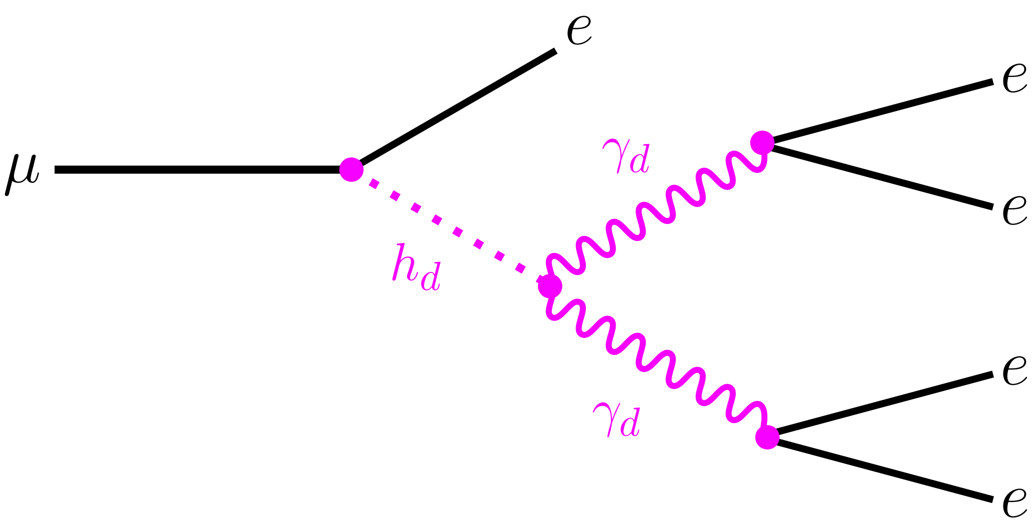
$$\downarrow$$

$$y_{ij} \simeq \frac{Cv}{\Lambda}$$

$$\mathscr{L} \supset -m_{\ell_i} \bar{\ell}_{Li} \ell_{Ri} \left(1 + \frac{h}{v} \right) - y_{ij} \bar{\ell}_{Li} \ell_{Rj} h_d \left(1 + \frac{h}{v} \right) + \text{h.c.},$$

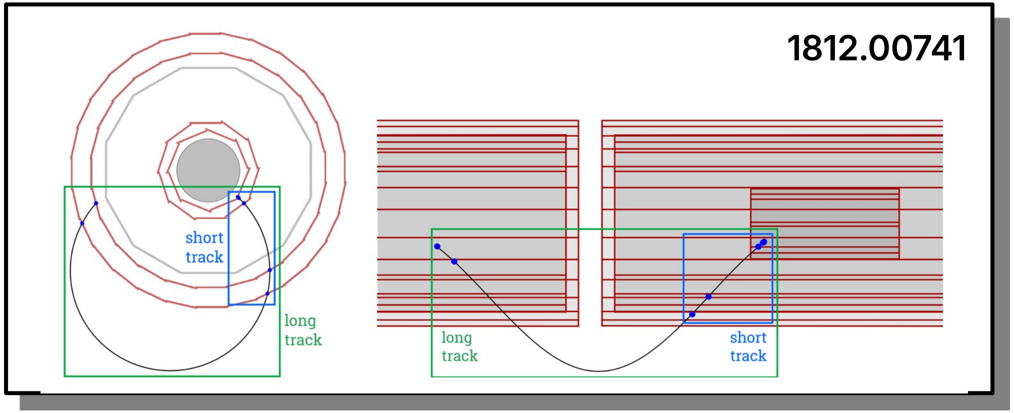
μ → eeeee

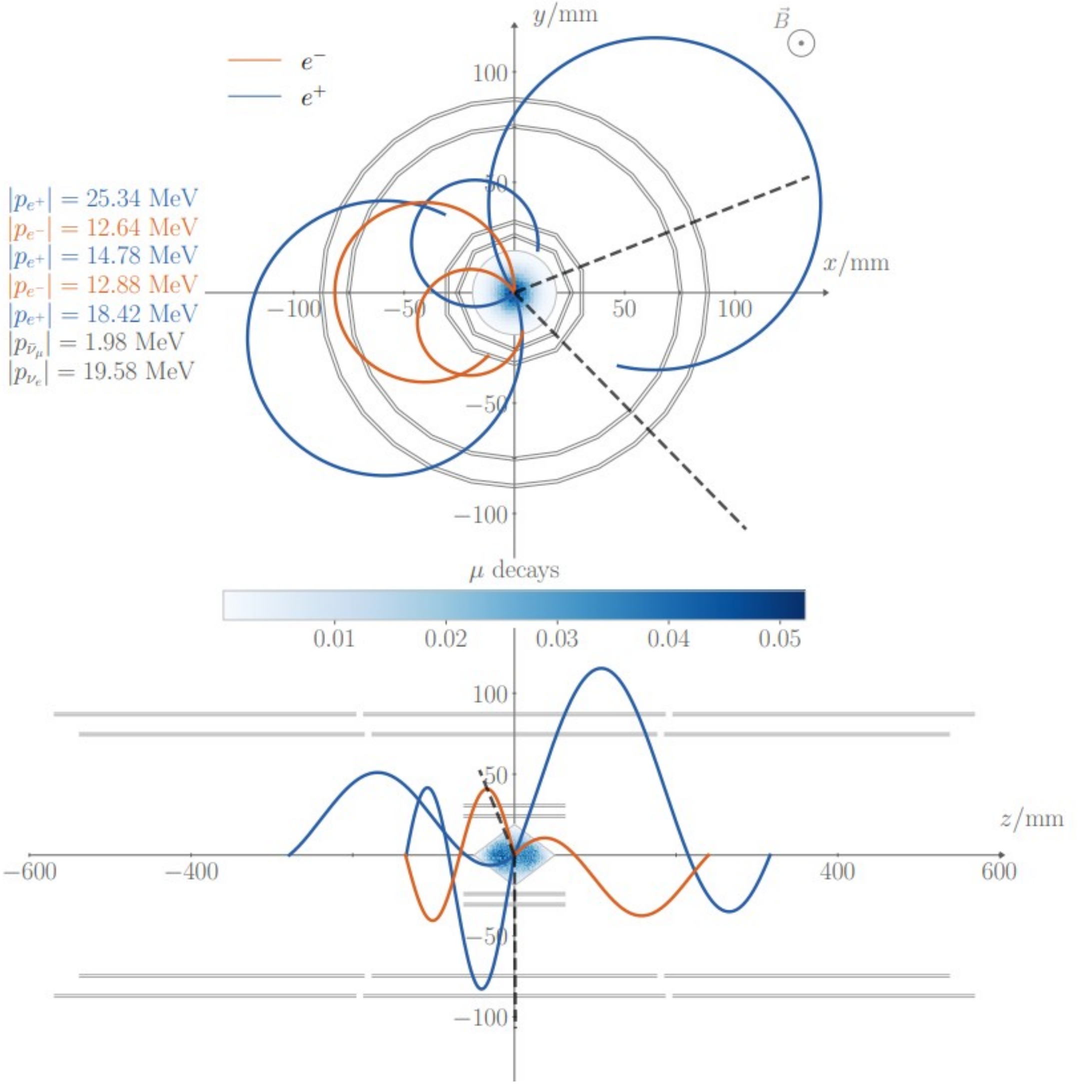
- Leads to cascade decay



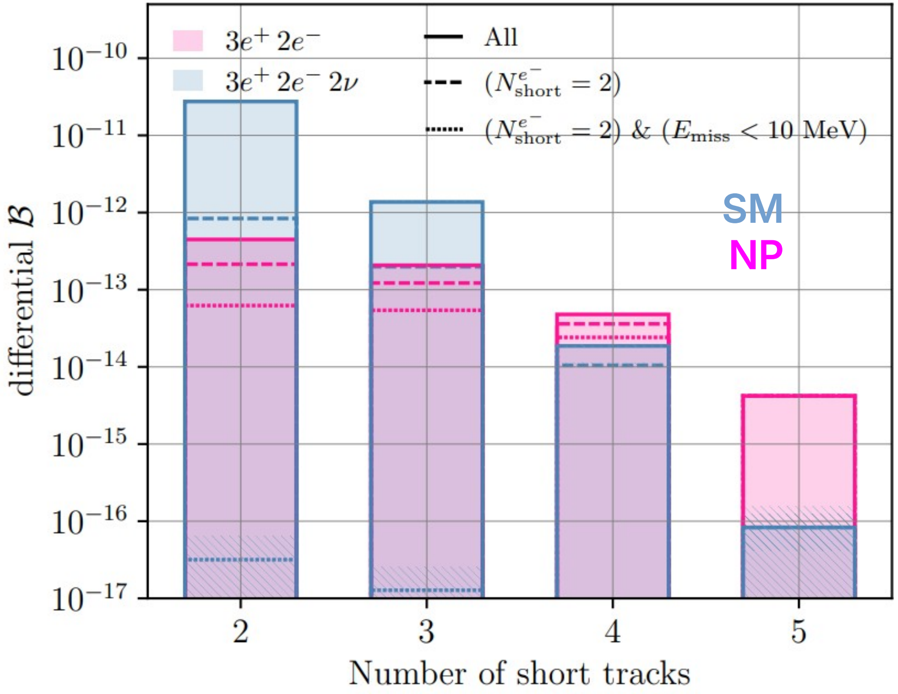
Signatures

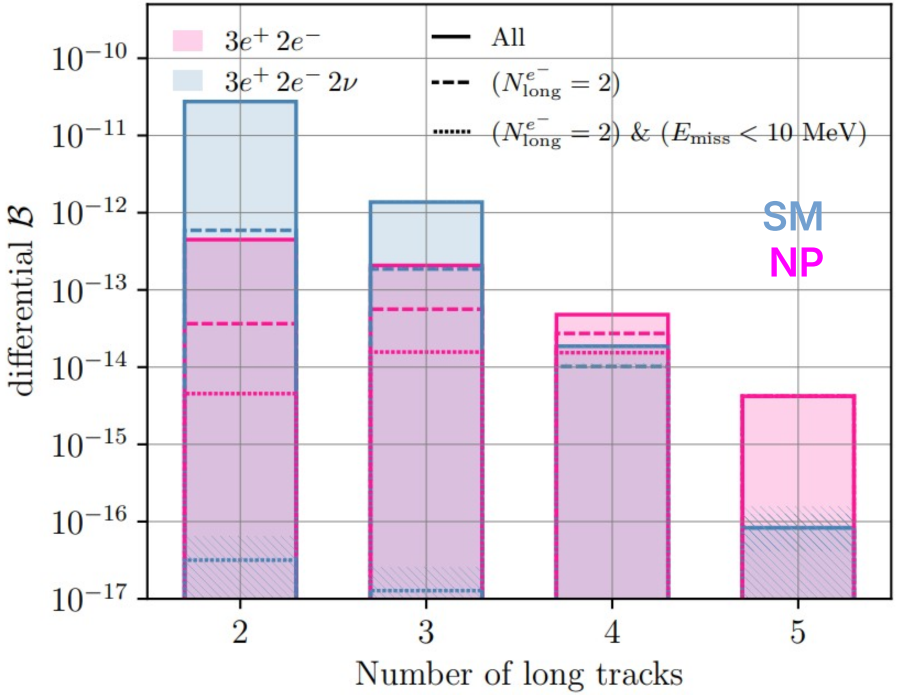
- Momentum of tracks must be reconstructed from energy deposits or ‘hits’ in the layers of the detectors.
- 4 hits = short track
- 6+ hits = long track



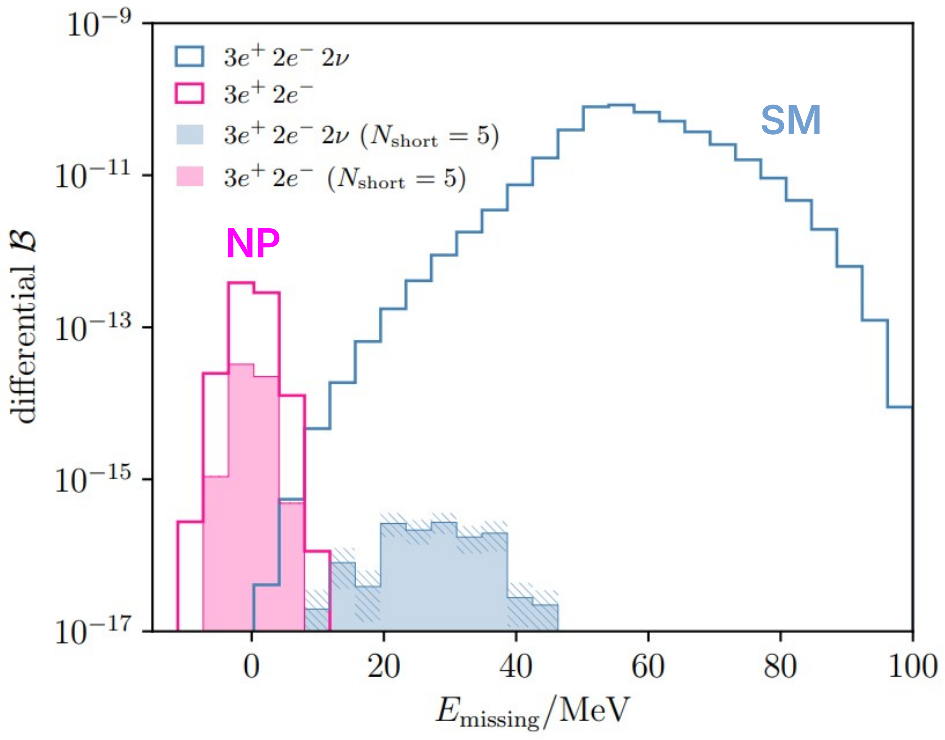


Signatures

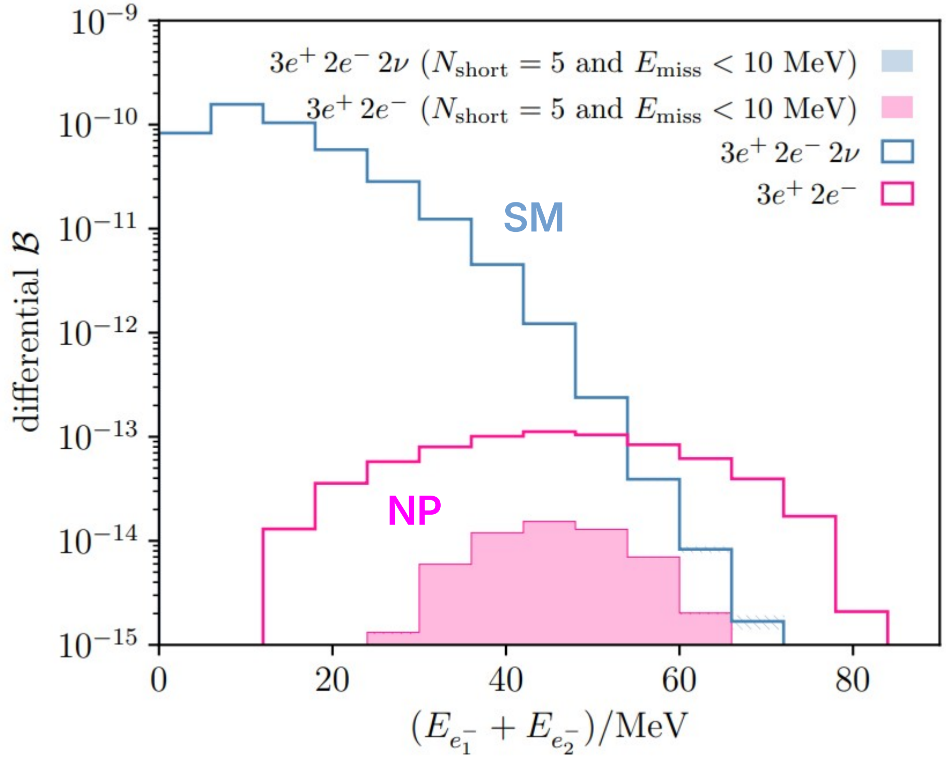

$3e^+ 2e^-$
$3e^+ 2e^- 2\nu$
All
$(N^{e^-}_{\rm short} = 2)$
$(N^{e^-}_{\rm short} = 2)$ & $(E_{\rm miss} < 10$ MeV$)$
SM
NP
differential $\mathcal{B}$
Number of short tracks


$3e^+ 2e^-$
$3e^+ 2e^- 2\nu$
All
$(N^{e^-}_{\rm long} = 2)$
$(N^{e^-}_{\rm long} = 2)$ & $(E_{\rm miss} < 10$ MeV$)$
SM
NP
differential $\mathcal{B}$
Number of long tracks

Signatures



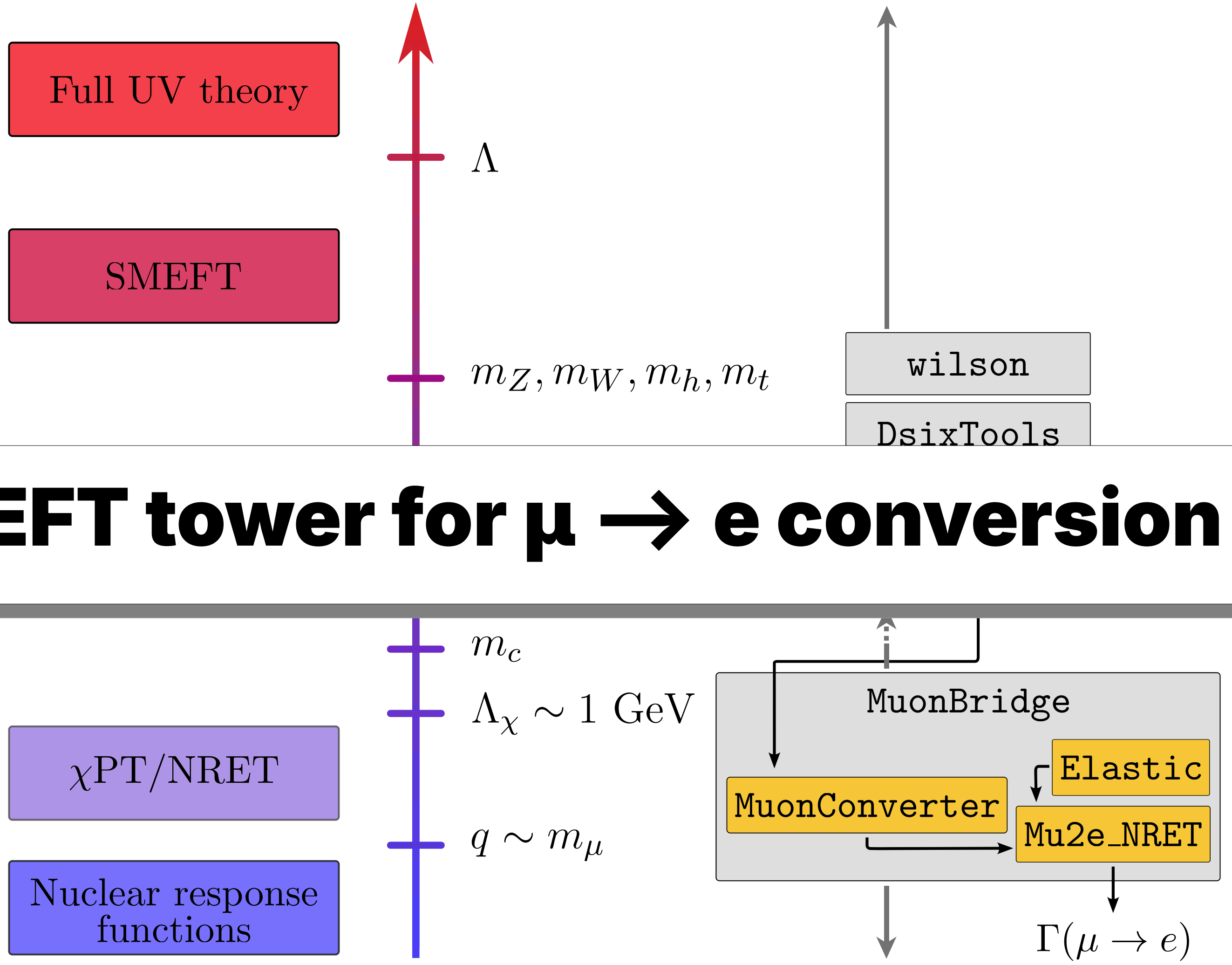
$3e^+ 2e^- 2\nu$
$3e^+ 2e^-$
$3e^+ 2e^- 2\nu$ ($N_{\text{short}} = 5$)
$3e^+ 2e^-$ ($N_{\text{short}} = 5$)
SM
NP
differential $\mathcal{B}$
E_{missing}/MeV



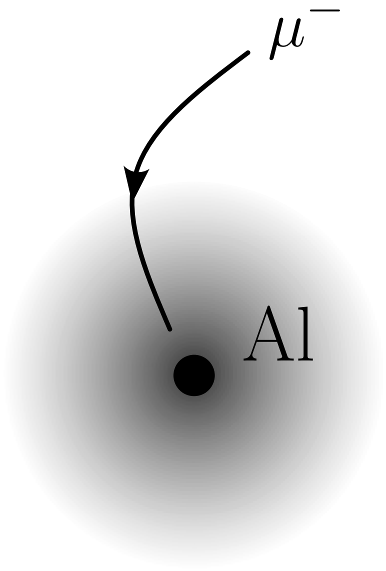
$3e^+ 2e^- 2\nu$ ($N_{\text{short}} = 5$ and $E_{\text{miss}} < 10$ MeV)
$3e^+ 2e^-$ ($N_{\text{short}} = 5$ and $E_{\text{miss}} < 10$ MeV)
$3e^+ 2e^- 2\nu$
$3e^+ 2e^-$
SM
NP
differential $\mathcal{B}$
$(E_{e_1^-} + E_{e_2^-})$/MeV

EFT tower for μ → e conversion

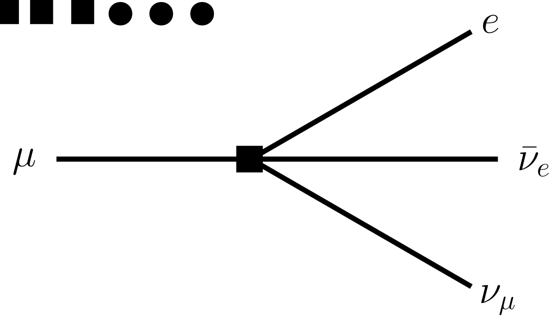
2406.13818



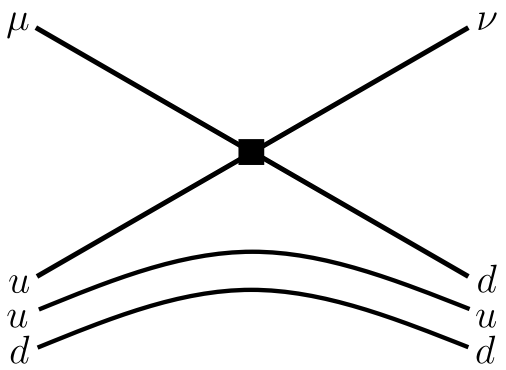
A trapped muon can…



1. Decay in orbit



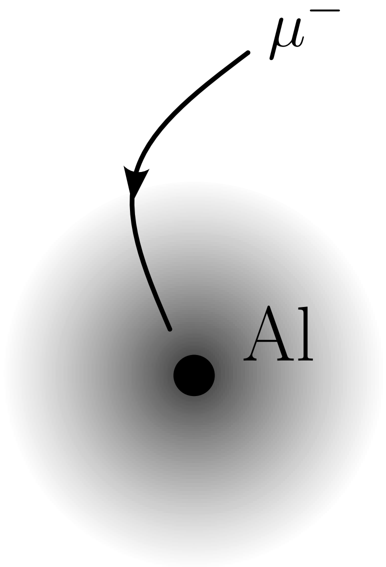
2. Be captured by the nucleus

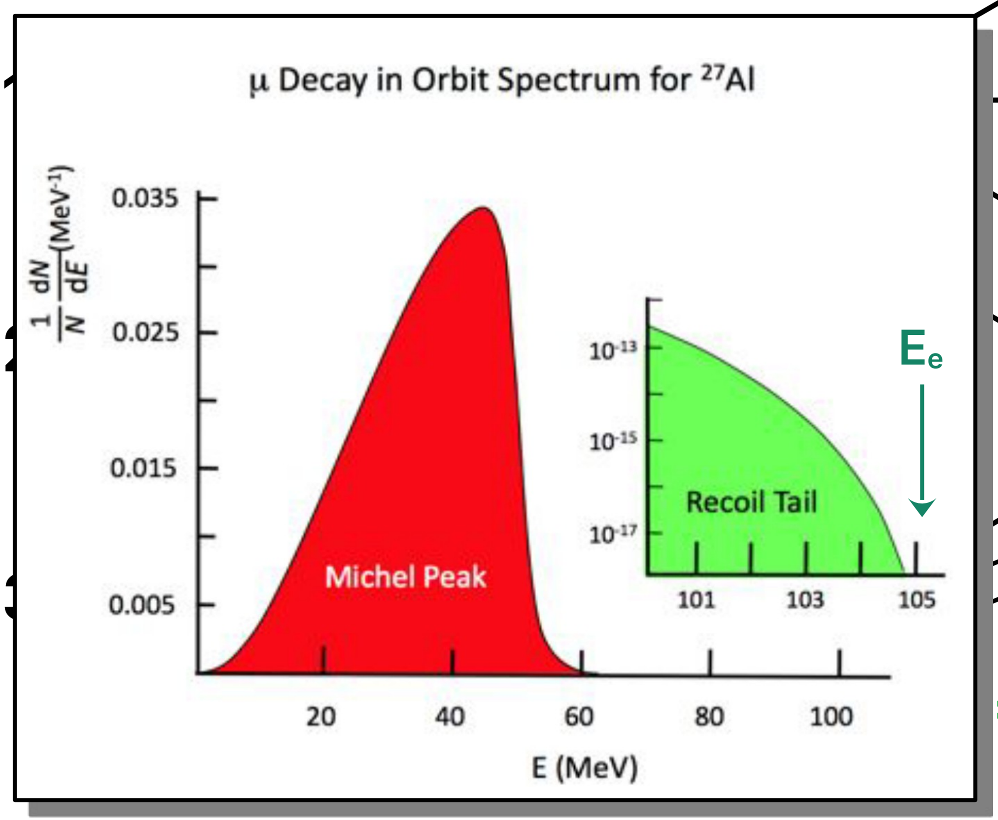


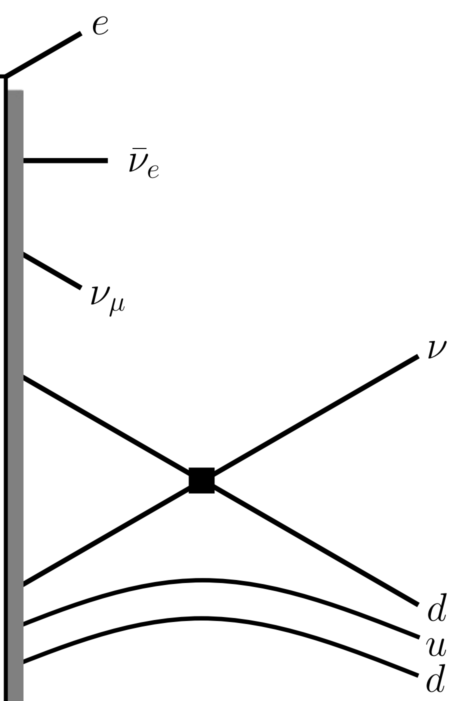
3. Convert to an electron

   a) Mono-energetic electron signal ($E_e = m_\mu$)

A trapped muon can…

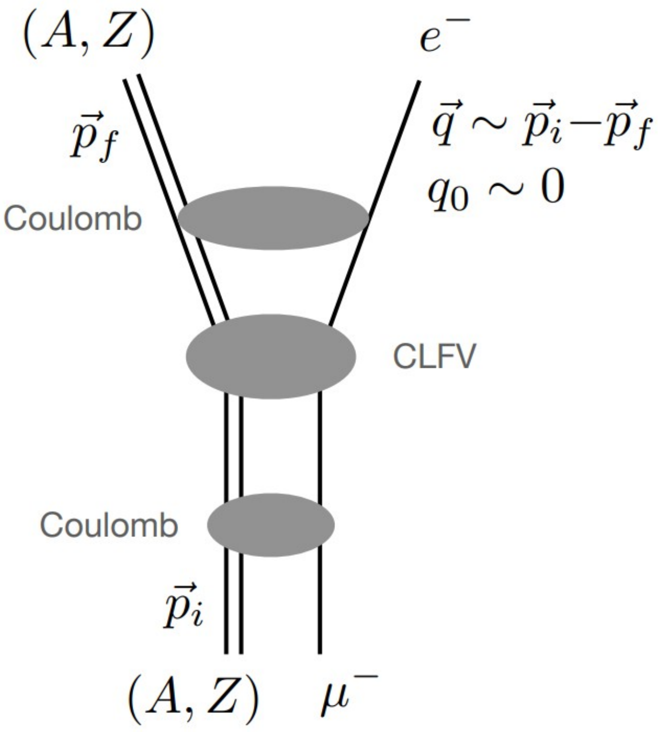






$= m_\mu$)

(A,Z) μ^- → (A,Z) e^-



2208.07945

arXiv:2208.07945v3 [nucl-th] 1 Apr 2023

Nuclear-level Effective Theory of $\mu \to e$ Conversion: Formalism and Applications

W. C. Haxton,[1,2] Evan Rule,[1] Ken McElvain,[1] and Michael J. Ramsey-Musolf[3,4]

[1] *Department of Physics, University of California, Berkeley, California 94720, USA*
[2] *Lawrence Berkeley National Laboratory, Berkeley, California 94720, USA*
[3] *Tsung-Dao Lee Institute and School of Physics and Astronomy, Shanghai Jiao Tong University, 800 Dongchuan Road, Shanghai 200240, China*
[4] *Amherst Center for Fundamental Interactions, Department of Physics, University of Massachusetts, Amherst, Massachusetts 01003, USA*

(Dated: April 4, 2023)

Over the next decade new $\mu \to e$ conversion searches at Fermilab (Mu2e) and J-PARC (COMET, DeeMe) are expected to advance limits on charged lepton flavor violation (CLFV) by more than four orders of magnitude. By considering the consequence of P and CP on elastic $\mu \to e$ conversion and the structure of possible charge and current densities, we show that rates are governed by six nuclear responses and a single scale, q/m_N, where $q \approx m_\mu$ is the momentum transferred from the leptons to the nucleus. To relate this result to microscopic formulations of CLFV, we construct in nonrelativistic effective theory (NRET) the CLFV nucleon-level interaction, pointing out the relevance of the dimensionless scales $y = (\frac{qb}{2})^2 > |\vec{v}_N| > |\vec{v}_\mu| > |\vec{v}_T|$, where b is the nuclear size, $\vec{v}_N$ and $\vec{v}_\mu$ are the nucleon and muon intrinsic velocities, and $\vec{v}_T$ is the target recoil velocity. We discuss previous work, noting the lack of a systematic treatment of the various small parameters. Because the parameter y is not small, a proper calculation of $\mu \to e$ conversion requires a full multipole expansion of the nuclear response functions, an apparently daunting task with Coulomb-distorted electron partial waves. We demonstrate that the multipole expansion can be carried out to high precision by introducing a simplifying local momentum $q_{\rm eff}$ for the electron. Previous work has been limited to simple charge or spin interactions, thereby treating the nucleus effectively as a point particle. We show that such formulations are not compatible with the general form of the $\mu \to e$ conversion rate, failing to generate three of the six allowed nuclear response functions. The inclusion of the nucleon velocity $\vec{v}_N$ yields an NRET with 16 operators and a rate of the general form. Consequently, in the current discovery era for CLFV, it provides the most sensible starting point for experimental analysis, defining what can and cannot be determined about CLFV from the highly exclusive process of $\mu \to e$ conversion. Finally, we expand the NRET operator basis to account for the effects of $\vec{v}_\mu$, associated with the muon's lower component, generating corrections to the CLFV coefficients of the point-nucleus response functions. Using advanced shell-model methods, we compute $\mu \to e$ conversion rates for a series of experimental targets, deriving bounds on the coefficients of the CLFV operators. These calculations are the first to include a general basis of CLFV operators, full evaluation of the associated nuclear response functions, and an accurate treatment of electron and muon Coulomb effects. We discuss target selection as an experimental "knob" that can be turned to probe the microscopic origins of CLFV. We describe two types of coherence that enhance certain CLFV operators and selection rules that blind elastic $\mu \to e$ conversion to others. We discuss the matching of the NRET onto higher level effective field theories, such as those constructed at the light quark level, noting opportunities to build on existing work in direct detection of dark matter. We discuss the relation of $\mu \to e$ conversion to $\mu \to e + \gamma$ and $\mu \to 3e$, showing how MEG II and Mu3e results will complement those of Mu2e and COMET. Finally we describe a accompanying script – in Mathematica and Python versions – that can be used to compute $\mu \to e$ conversion rates in various nuclear targets for the full set of NRET operators.

I. INTRODUCTION

Muon-to-electron conversion, in which a muon bound to a nucleus converts to a mono-energetic outgoing electron, occurs at an observable level only if there are new sources of flavor violation, beyond those responsible for neutrino mixing [1–4]. It has yond the standard model [5–7]. This has motivated a series of experimental advances that, in sum, have improved limits on $\mu \to e$ conversion rates by ≈12 orders of magnitude over the past 75 years [8].

The experimental attributes of $\mu \to e$ conversion are quite attractive. Intense muon beams exist, with rates on target of $\approx 10^{11}$/s expected in the

The rate

- **The conversion rate:** $\mathcal{O}(y)$

$$\Gamma(\mu \to e) = \frac{1}{2\pi} \frac{q_{\rm eff}^2}{1 + q/M_T} \left|\phi_{1s}^{Z_{\rm eff}}(\vec{0})\right|^2 \sum_{\tau,\tau'} \Big[R_{MM}^{\tau\tau'} W_{MM}^{\tau\tau'}(q_{\rm eff}) + R_{\Sigma'\Sigma'}^{\tau\tau'} W_{\Sigma'\Sigma'}^{\tau\tau'}(q_{\rm eff}) + R_{\Sigma''\Sigma''}^{\tau\tau'} W_{\Sigma''\Sigma''}^{\tau\tau'}(q_{\rm eff}) \Big],$$

$$R_{MM}^{\tau\tau'} = c_1^\tau c_1^{\tau'*} + c_{11}^\tau c_{11}^{\tau'*},$$

$$R_{\Sigma'\Sigma'}^{\tau\tau'} = c_4^\tau c_4^{\tau'*} + c_9^\tau c_9^{\tau'*},$$

$$R_{\Sigma''\Sigma''}^{\tau\tau'} = \left(c_4^\tau - c_6^\tau\right)\left(c_4^{\tau'} - c_6^{\tau'}\right)^* + c_{10}^\tau c_{10}^{\tau'*},$$

The rate

- The conversion rate: $\mathcal{O}(\vec{v}_N)$

*All nuclear responses allowed by symmetries are generated

$$\omega = \frac{G_F^2}{\pi} \frac{q_{\rm eff}^2}{1+\frac{q}{M_T}} |\phi_{1s}^{Z_{\rm eff}}(\vec{0})|^2 \sum_{\tau=0,1} \sum_{\tau'=0,1} \Bigg\{ \left[\tilde{R}_{MM}^{\tau\tau'} W_{MM}^{\tau\tau'}(q_{\rm eff}) + \tilde{R}_{\Sigma''\Sigma''}^{\tau\tau'} W_{\Sigma''\Sigma''}^{\tau\tau'}(q_{\rm eff}) + \tilde{R}_{\Sigma'\Sigma'}^{\tau\tau'} W_{\Sigma'\Sigma'}^{\tau\tau'}(q_{\rm eff}) \right]$$
$$+ \frac{q_{\rm eff}^2}{m_N^2} \left[\tilde{R}_{\Phi''\Phi''}^{\tau\tau'} W_{\Phi''\Phi''}^{\tau\tau'}(q_{\rm eff}) + \tilde{R}_{\tilde{\Phi}'\tilde{\Phi}'}^{\tau\tau'} W_{\tilde{\Phi}'\tilde{\Phi}'}^{\tau\tau'}(q_{\rm eff}) + \tilde{R}_{\Delta\Delta}^{\tau\tau'} W_{\Delta\Delta}^{\tau\tau'}(q_{\rm eff}) \right]$$
$$- \frac{2q_{\rm eff}}{m_N} \left[\tilde{R}_{\Phi''M}^{\tau\tau'} W_{\Phi''M}^{\tau\tau'}(q_{\rm eff}) + \tilde{R}_{\Delta\Sigma'}^{\tau\tau'} W_{\Delta\Sigma'}^{\tau\tau'}(q_{\rm eff}) \right] \Bigg\} \quad (59)$$

$$R_{MM}^{\tau\tau'} = c_1^\tau c_1^{\tau'*} + c_{11}^\tau c_{11}^{\tau'*},$$
$$R_{\Sigma'\Sigma'}^{\tau\tau'} = c_4^\tau c_4^{\tau'*} + c_9^\tau c_9^{\tau'*},$$
$$R_{\Sigma''\Sigma''}^{\tau\tau'} = \left(c_4^\tau - c_6^\tau\right)\left(c_4^{\tau'} - c_6^{\tau'}\right)^* + c_{10}^\tau c_{10}^{\tau'*},$$

$$\tilde{R}_{\Phi''\Phi''}^{\tau\tau'} = \tilde{c}_3^\tau \tilde{c}_3^{\tau'*} + (\tilde{c}_{12}^\tau - \tilde{c}_{15}^\tau)(\tilde{c}_{12}^{\tau'*} - \tilde{c}_{15}^{\tau'*})$$
$$\tilde{R}_{\Phi''M}^{\tau\tau'} = \mathrm{Re}[\tilde{c}_3^\tau \tilde{c}_1^{\tau'*} - (\tilde{c}_{12}^\tau - \tilde{c}_{15}^\tau)\, \tilde{c}_{11}^{\tau'*}]$$
$$\tilde{R}_{\Delta\Delta}^{\tau\tau'} = \tilde{c}_5^\tau \tilde{c}_5^{\tau'*} + \tilde{c}_8^\tau \tilde{c}_8^{\tau'*}$$
$$\tilde{R}_{\Delta\Sigma'}^{\tau\tau'} = \mathrm{Re}[\tilde{c}_5^\tau \tilde{c}_4^{\tau'*} + \tilde{c}_8^\tau \tilde{c}_9^{\tau'*}]$$
$$\tilde{R}_{\tilde{\Phi}'\tilde{\Phi}'}^{\tau\tau'} = \tilde{c}_{12}^\tau \tilde{c}_{12}^{\tau'*} + \tilde{c}_{13}^\tau \tilde{c}_{13}^{\tau'*}$$

Nuclear response hierarchy

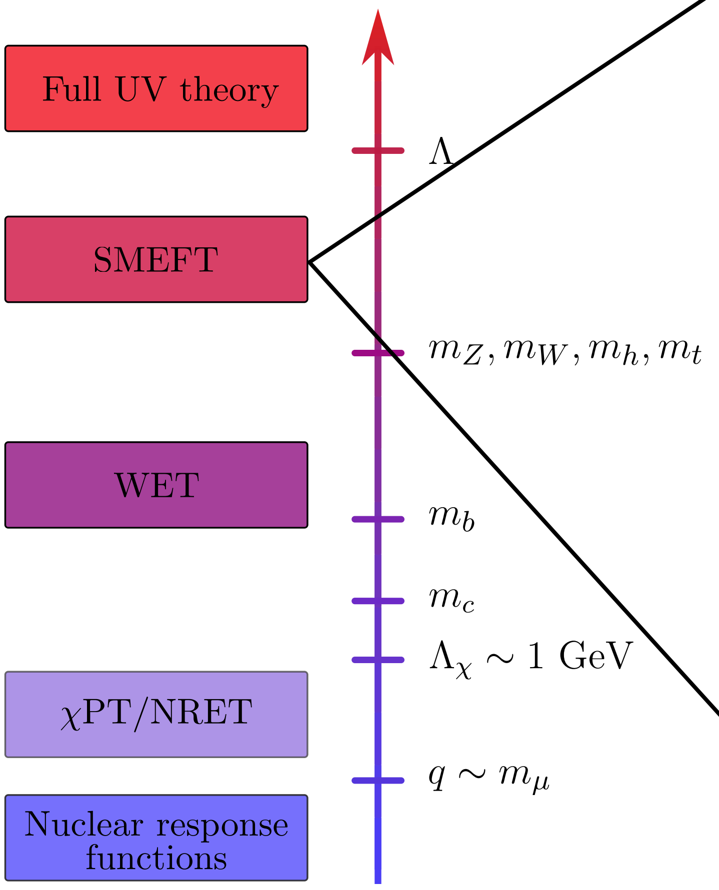
∗Can become semi-coherent in some nuclei with half-filled shells (e.g. Al, Cu)

$$W_{MM}^{00} \sim \mathcal{O}(A^2) \gg \frac{q_{\text{eff}}}{m_N} W_{M\Phi''}^{00} \gg \left\{ W_{\Sigma'\Sigma'}^{00}, W_{\Sigma''\Sigma''}^{00}, \frac{q_{\text{eff}}^2}{m_N^2} W_{\Phi''\Phi''}^{00} \right\} \gg \left\{ \frac{q_{\text{eff}}^2}{m_N^2} W_{\Delta\Delta}^{00}, \frac{q_{\text{eff}}^2}{m_N^2} W_{\tilde{\Phi}'\tilde{\Phi}'}^{00} \right\}$$

Where does the UV physics sit?

$$\Gamma \propto \sum R \times W, \qquad R(c_i)$$

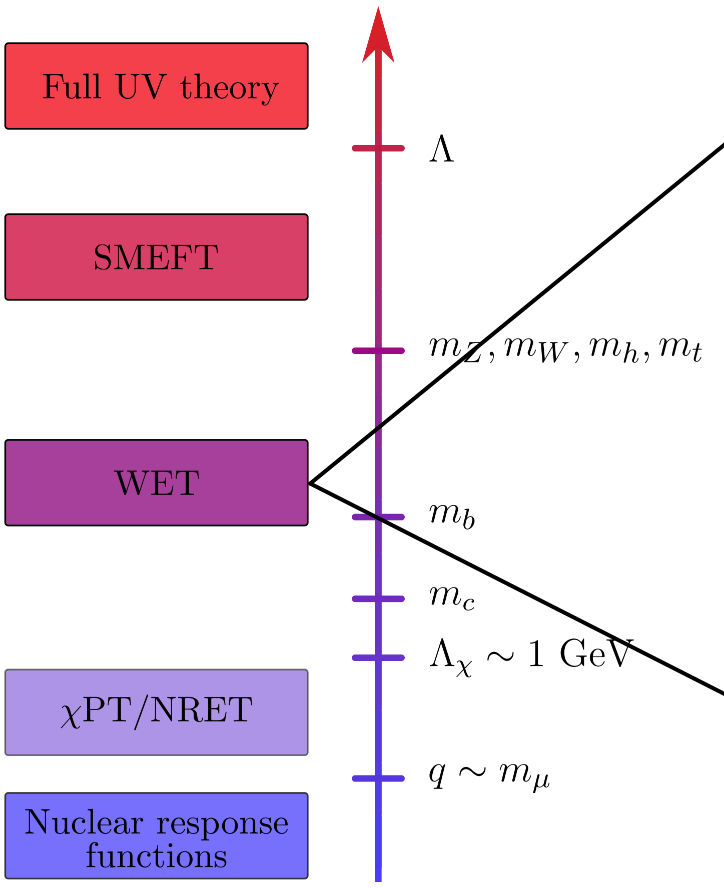
The tower



SMEFT

$$
\begin{aligned}
Q_{e\varphi}: &\quad (\varphi^\dagger\varphi)(\bar{L}_2 e_R \varphi), \quad (\varphi^\dagger\varphi)(\bar{\ell}_1 \mu_R \varphi), \\
Q_{eW}: &\quad (\bar{\ell}_2 \sigma^{\mu\nu} e_R)\tau^I \varphi W^I_{\mu\nu}, \quad (\bar{\ell}_1 \sigma^{\mu\nu} \mu_R)\tau^I \varphi W^I_{\mu\nu}, \\
Q_{eB}: &\quad (\bar{\ell}_2 \sigma^{\mu\nu} e_R)\varphi B_{\mu\nu}, \quad (\bar{\ell}_1 \sigma^{\mu\nu} \mu_R)\varphi B_{\mu\nu}, \\
Q^{(1)}_{\varphi\ell}: &\quad (\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar{\ell}_2 \gamma^\mu \ell_1), \\
Q^{(3)}_{\varphi\ell}: &\quad (\varphi^\dagger i \overleftrightarrow{D}^I_\mu \varphi)(\bar{\ell}_2 \gamma^\mu \tau^I \ell_1), \\
Q_{\varphi e}: &\quad (\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar{\mu}_R \gamma^\mu \tau^I e_R), \\
Q^{(1)}_{\ell q}: &\quad (\bar{\ell}_2 \gamma^\mu \ell_1)(\bar{q} \gamma^\mu q), \\
Q^{(3)}_{\ell q}: &\quad (\bar{\ell}_2 \gamma^\mu \tau^I \ell_1)(\bar{q} \gamma^\mu \tau^I q), \\
Q_{eu}: &\quad (\bar{\mu}_R \gamma^\mu e_R)(\bar{u}_R \gamma^\mu u_R), \\
Q_{ed}: &\quad (\bar{\mu}_R \gamma^\mu e_R)(\bar{d}_R \gamma^\mu d_R), \\
Q_{\ell u}: &\quad (\bar{\ell}_2 \gamma^\mu \ell_1)(\bar{u}_R \gamma^\mu u_R), \\
Q_{\ell d}: &\quad (\bar{\ell}_2 \gamma^\mu \ell_1)(\bar{d}_R \gamma^\mu d_R), \\
Q_{qe}: &\quad (\bar{q} \gamma^\mu q)(\bar{\mu}_R \gamma^\mu e_R), \\
Q_{\ell edq}: &\quad (\bar{\ell}_2 e_R)(\bar{d}_R q), \quad (\bar{\ell}_1 \mu_R)(\bar{d}_R q), \\
Q^{(1)}_{\ell equ}: &\quad (\bar{\ell}^j_2 e_R)\varepsilon_{jk}(\bar{q}^k u), \quad (\bar{\ell}^j_1 \mu_R)\varepsilon_{jk}(\bar{q}^k u), \\
Q^{(3)}_{\ell equ}: &\quad (\bar{\ell}^j_2 \sigma_{\mu\nu} e_R)\varepsilon_{jk}(\bar{q}^k \sigma^{\mu\nu} u), \quad (\bar{\ell}^j_1 \sigma_{\mu\nu} \mu_R)\varepsilon_{jk}(\bar{q}^k \sigma^{\mu\nu} u),
\end{aligned}
$$

The tower



WET

$$\mathcal{Q}_1^{(5)} = \frac{e}{8\pi^2}(\bar{e}\sigma^{\alpha\beta}\mu)F_{\alpha\beta}\,, \qquad \mathcal{Q}_2^{(5)} = \frac{e}{8\pi^2}(\bar{e}\sigma^{\alpha\beta}i\gamma_5\mu)F_{\alpha\beta}\,,$$

$$\mathcal{Q}_{1,q}^{(6)} = (\bar{e}\gamma_\alpha\mu)(\bar{q}\gamma^\alpha q)\,, \qquad \mathcal{Q}_{2,q}^{(6)} = (\bar{e}\gamma_\alpha\gamma_5\mu)(\bar{q}\gamma^\alpha q)\,,$$

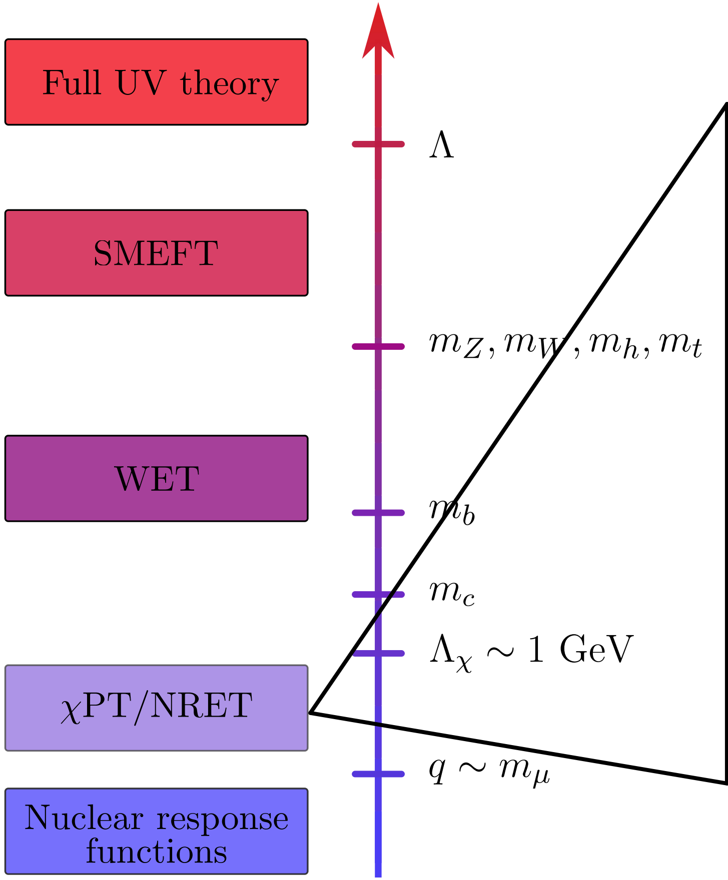
$$\mathcal{Q}_{3,q}^{(6)} = (\bar{e}\gamma_\alpha\mu)(\bar{q}\gamma^\alpha\gamma_5 q)\,, \qquad \mathcal{Q}_{4,q}^{(6)} = (\bar{e}\gamma_\alpha\gamma_5\mu)(\bar{q}\gamma^\alpha\gamma_5 q)\,.$$

$$\mathcal{Q}_{5,q}^{(6)} = (\bar{e}\mu)(\bar{q}q)\,, \qquad \mathcal{Q}_{6,q}^{(6)} = (\bar{e}i\gamma_5\mu)(\bar{q}q)\,,$$

$$\mathcal{Q}_{7,q}^{(6)} = (\bar{e}\mu)(\bar{q}i\gamma_5 q)\,, \qquad \mathcal{Q}_{8,q}^{(6)} = (\bar{e}i\gamma_5\mu)(\bar{q}i\gamma_5 q)\,,$$

$$\mathcal{Q}_{9,q}^{(6)} = (\bar{e}\sigma^{\alpha\beta}\mu)(\bar{q}\sigma_{\alpha\beta}q)\,, \qquad \mathcal{Q}_{10,q}^{(6)} = (\bar{e}i\sigma^{\alpha\beta}\gamma_5\mu)(\bar{q}\sigma_{\alpha\beta}q)\,.$$

The tower



NRET

$$\mathcal{O}_1 = 1_L\, 1_N, \qquad \mathcal{O}_2' = 1_L\, i\hat{q}\cdot\vec{v}_N,$$
$$\mathcal{O}_3 = 1_L\, i\hat{q}\cdot[\vec{v}_N\times\vec{\sigma}_N], \qquad \mathcal{O}_4 = \vec{\sigma}_L\cdot\vec{\sigma}_N,$$
$$\mathcal{O}_5 = \vec{\sigma}_L\cdot(i\hat{q}\times\vec{v}_N), \qquad \mathcal{O}_6 = i\hat{q}\cdot\vec{\sigma}_L\, i\hat{q}\cdot\vec{\sigma}_N,$$
$$\mathcal{O}_7 = 1_L\, \vec{v}_N\cdot\vec{\sigma}_N, \qquad \mathcal{O}_8 = \vec{\sigma}_L\cdot\vec{v}_N,$$
$$\mathcal{O}_9 = \vec{\sigma}_L\cdot(i\hat{q}\times\vec{\sigma}_N), \qquad \mathcal{O}_{10} = 1_L\, i\hat{q}\cdot\vec{\sigma}_N,$$
$$\mathcal{O}_{11} = i\hat{q}\cdot\vec{\sigma}_L\, 1_N, \qquad \mathcal{O}_{12} = \vec{\sigma}_L\cdot[\vec{v}_N\times\vec{\sigma}_N],$$
$$\mathcal{O}_{13}' = \vec{\sigma}_L\cdot(i\hat{q}\times[\vec{v}_N\times\vec{\sigma}_N]), \qquad \mathcal{O}_{14} = i\hat{q}\cdot\vec{\sigma}_L\, \vec{v}_N\cdot\vec{\sigma}_N,$$
$$\mathcal{O}_{15} = i\hat{q}\cdot\vec{\sigma}_L\, i\hat{q}\cdot[\vec{v}_N\times\vec{\sigma}_N], \qquad \mathcal{O}_{16}' = i\hat{q}\cdot\vec{\sigma}_L\, i\hat{q}\cdot\vec{v}_N.$$

Matching

- Non-perturbative matching between WET and NRET (hadronization)
- Parameterize with nuclear form factors

$$\langle N'|\bar{q}\gamma^\mu q|N\rangle = \bar{u}'_N\Big[F_1^{q/N}(q^2)\gamma^\mu - \frac{i}{2m_N}F_2^{q/N}(q^2)\sigma^{\mu\nu}q_\nu\Big]u_N\,,$$

$$\langle N'|\bar{q}\gamma^\mu\gamma_5 q|N\rangle = \bar{u}'_N\Big[F_A^{q/N}(q^2)\gamma^\mu\gamma_5 - \frac{1}{2m_N}F_{P'}^{q/N}(q^2)\gamma_5 q^\mu\Big]u_N\,,$$

$$\langle N'|m_q\bar{q}q|N\rangle = F_S^{q/N}(q^2)\,\bar{u}'_N u_N\,,$$

$$\langle N'|m_q\bar{q}i\gamma_5 q|N\rangle = F_P^{q/N}(q^2)\,\bar{u}'_N i\gamma_5 u_N\,,$$

$$\langle N'|\frac{\alpha_s}{12\pi}G^{a\mu\nu}G^a_{\mu\nu}|N\rangle = F_G^N(q^2)\,\bar{u}'_N u_N\,,$$

$$\langle N'|\frac{\alpha_s}{8\pi}G^{a\mu\nu}\tilde{G}^a_{\mu\nu}|N\rangle = F_{\tilde{G}}^N(q^2)\,\bar{u}'_N i\gamma_5 u_N\,,$$

$$\langle N'|\bar{q}\sigma^{\mu\nu}q|N\rangle = \bar{u}'_N\Big[\hat{F}_{T,0}^{q/N}(q^2)\,\sigma^{\mu\nu} - \frac{i}{2m_N}\gamma^{[\mu}q^{\nu]}\hat{F}_{T,1}^{q/N}(q^2) - \frac{i}{m_N^2}q^{[\mu}k_{12}^{\nu]}\hat{F}_{T,2}^{q/N}(q^2)\Big]u_N\,,$$

$$\langle N'|\frac{\alpha}{12\pi}F^{\mu\nu}F_{\mu\nu}|N\rangle = F_\gamma^N(q^2)\,\bar{u}'_N u_N\,,$$

$$\langle N'|\frac{\alpha}{8\pi}F^{\mu\nu}\tilde{F}_{\mu\nu}|N\rangle = F_{\tilde{\gamma}}^N(q^2)\,\bar{u}'_N i\gamma_5 u_N\,.$$

EFT tower

- Non-perturbative matching between WET and NRET (hadronization)
- Matching expressions

$$c_1^N = -\frac{\alpha}{\pi q}\hat{\mathcal{C}}_1^{(5)}\sum_q Q_q F_1^{q/N} + \sum_q \hat{\mathcal{C}}_{1,q}^{(6)} F_1^{q/N} + \sum_q \frac{1}{m_q}\hat{\mathcal{C}}_{5,q}^{(6)} F_S^{q/N}$$
$$- \frac{q}{m_N}\sum_q \hat{\mathcal{C}}_{9,q}^{(6)}\left(\hat{F}_{T,0}^{q/N} - \hat{F}_{T,1}^{q/N} + 4\hat{F}_{T,2}^{q/N}\right)$$
$$+ \hat{\mathcal{C}}_1^{(7)} F_G^N + \hat{\mathcal{C}}_5^{(7)} F_\gamma^N + (q + m_+)\sum_q \hat{\mathcal{C}}_{9,q}^{(7)} F_1^{q/N}$$
$$- \frac{q^2}{2m_N}\sum_q \hat{\mathcal{C}}_{13,q}^{(7)}\left[\hat{F}_{T,0}^{q/N} - \hat{F}_{T,1}^{q/N} + \left(4 + \frac{q^2}{m_N^2}\right)\hat{F}_{T,2}^{q/N}\right],$$

$$c_2^N = i\left[\sum_q \hat{\mathcal{C}}_{1,q}^{(6)} F_1^{q/N} + m_+\sum_q \hat{\mathcal{C}}_{9,q}^{(7)} F_1^{q/N} + \frac{q^2}{2m_N}\sum_q \hat{\mathcal{C}}_{13,q}^{(7)}\left(\hat{F}_{T,1}^{q/N} - 4\hat{F}_{T,2}^{q/N}\right)\right],$$

$$\vdots$$

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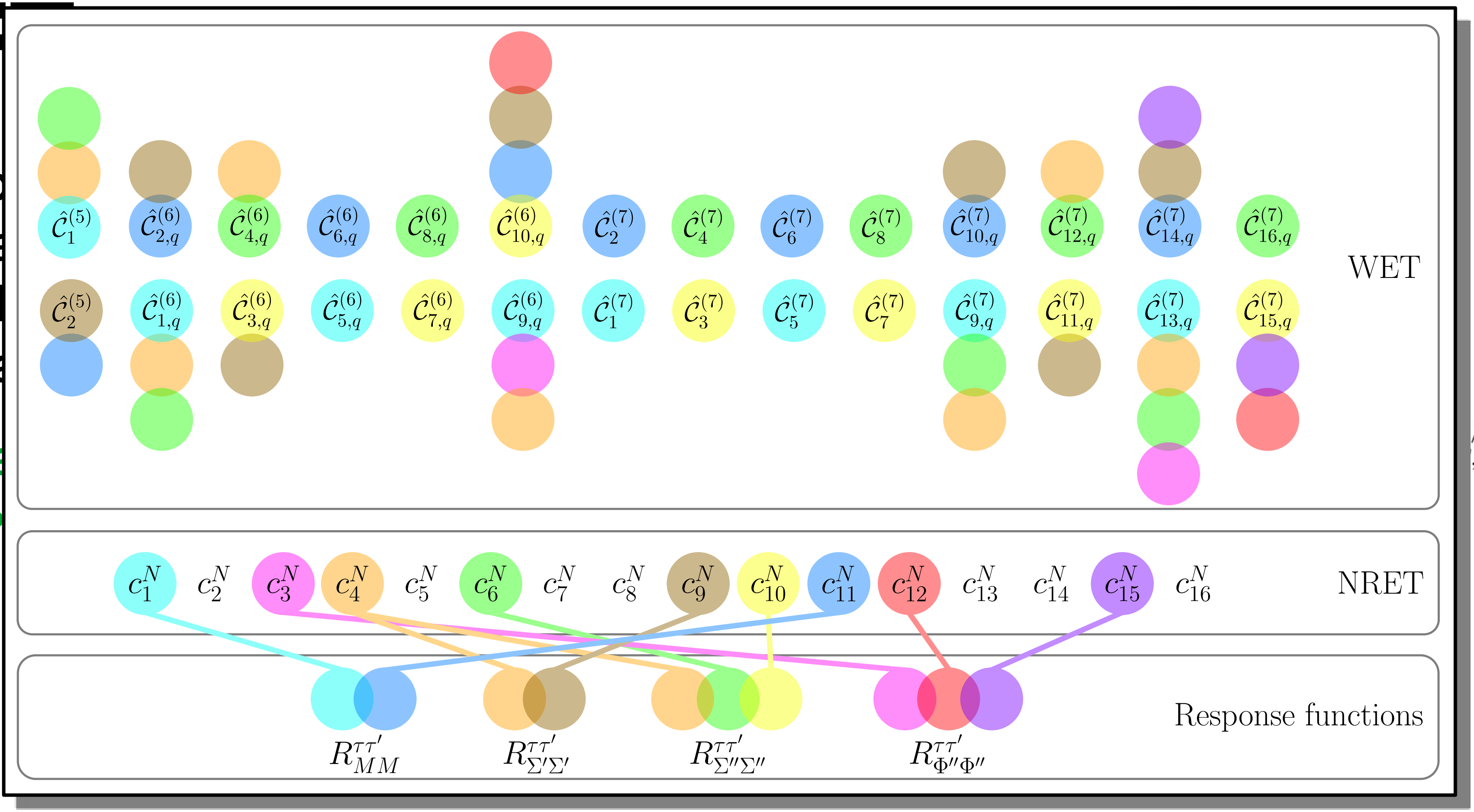


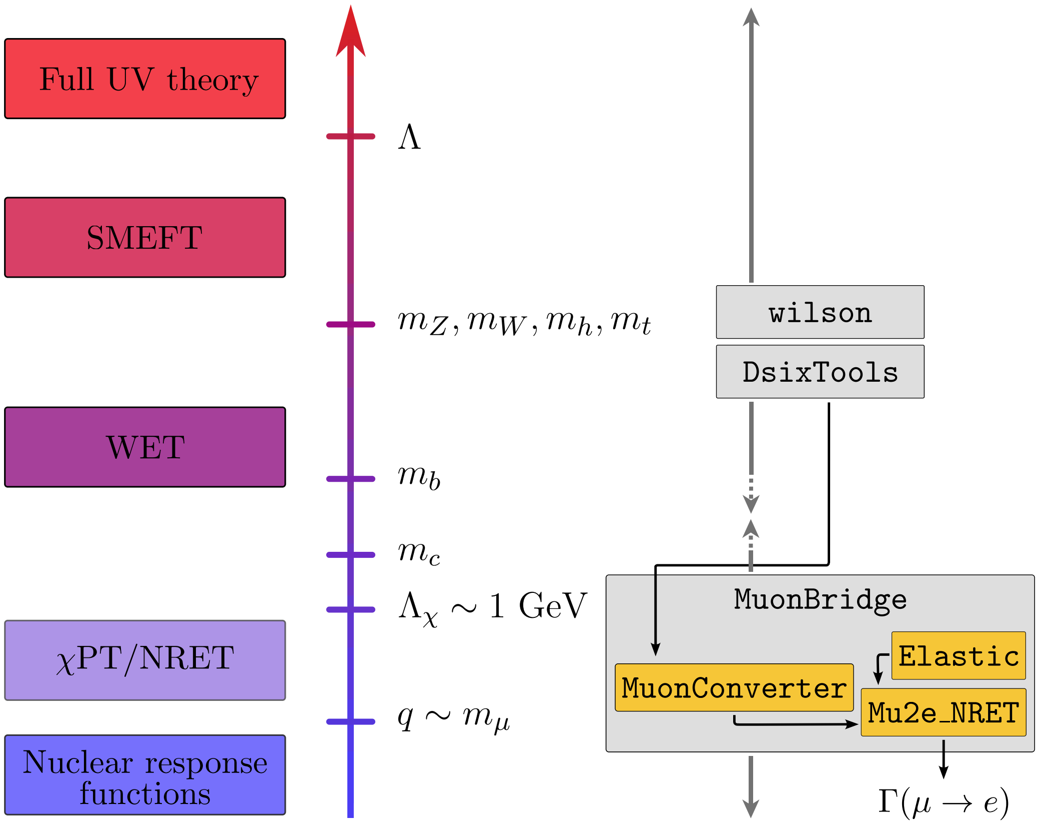
$/N \,,2 \Big) \Big],$

MuonBridge

https://github.com/Berkeley-Electroweak-Physics/MuonBridge

Three components:

1. Elastic - one body density matrices
2. Mu2e_NRET – Computes μ→e conversion rate
3. MuonConverter – matches WET to NRET and facilitates interface with existing EFT software



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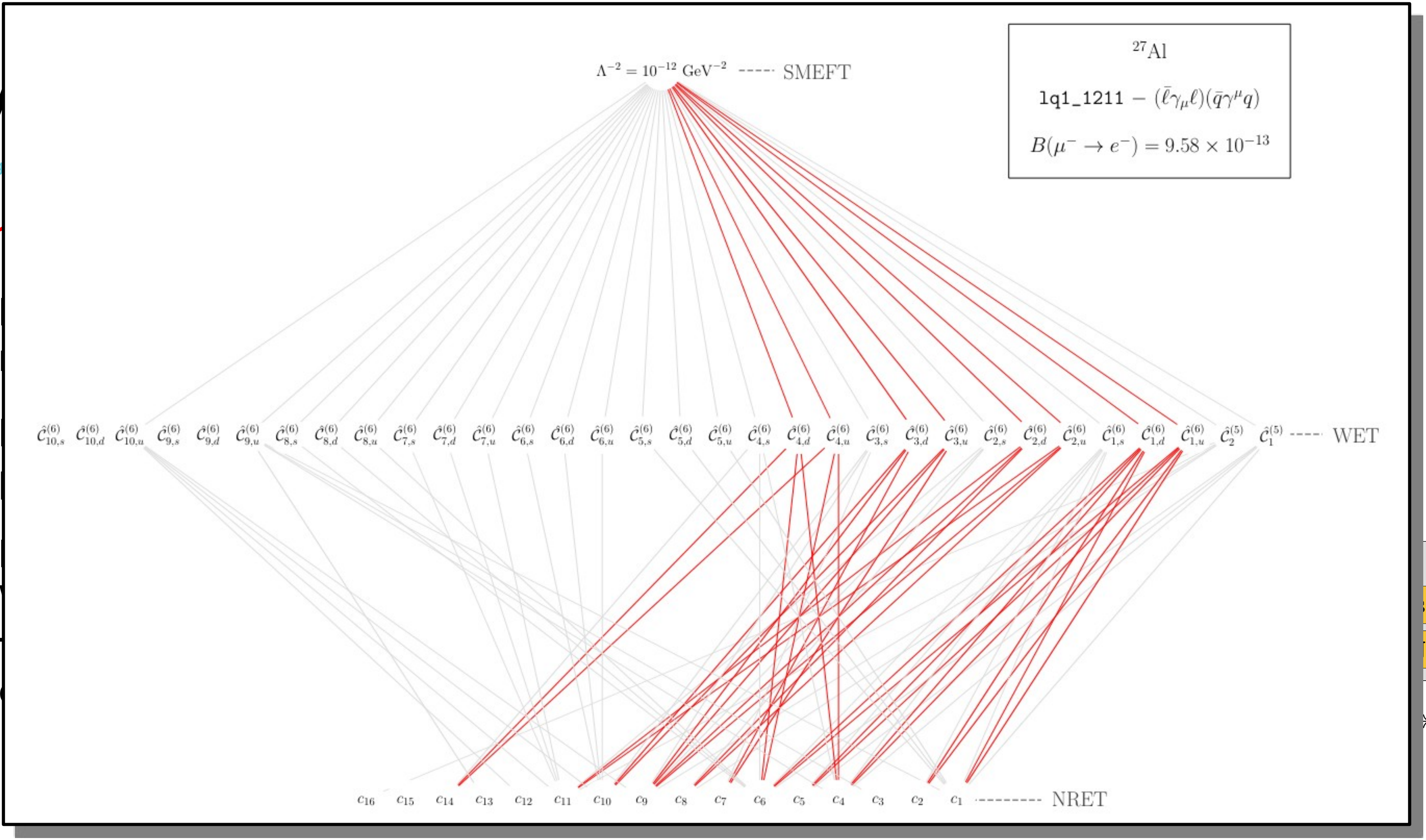

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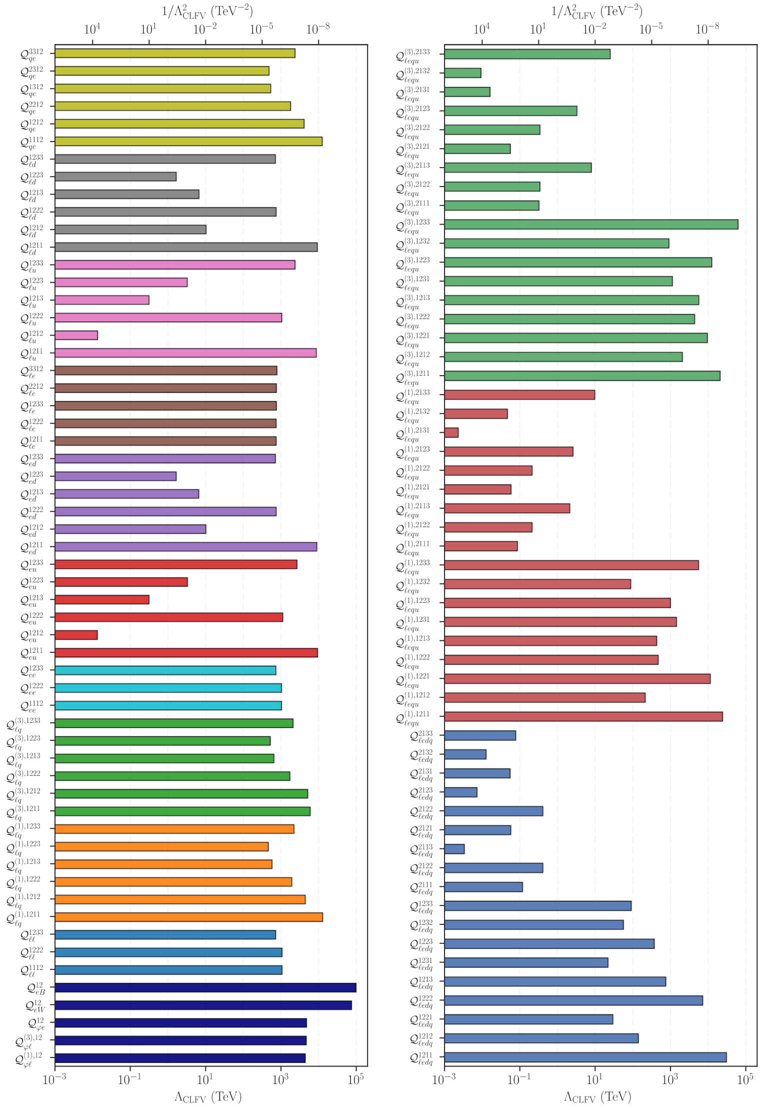
RET

$e)$

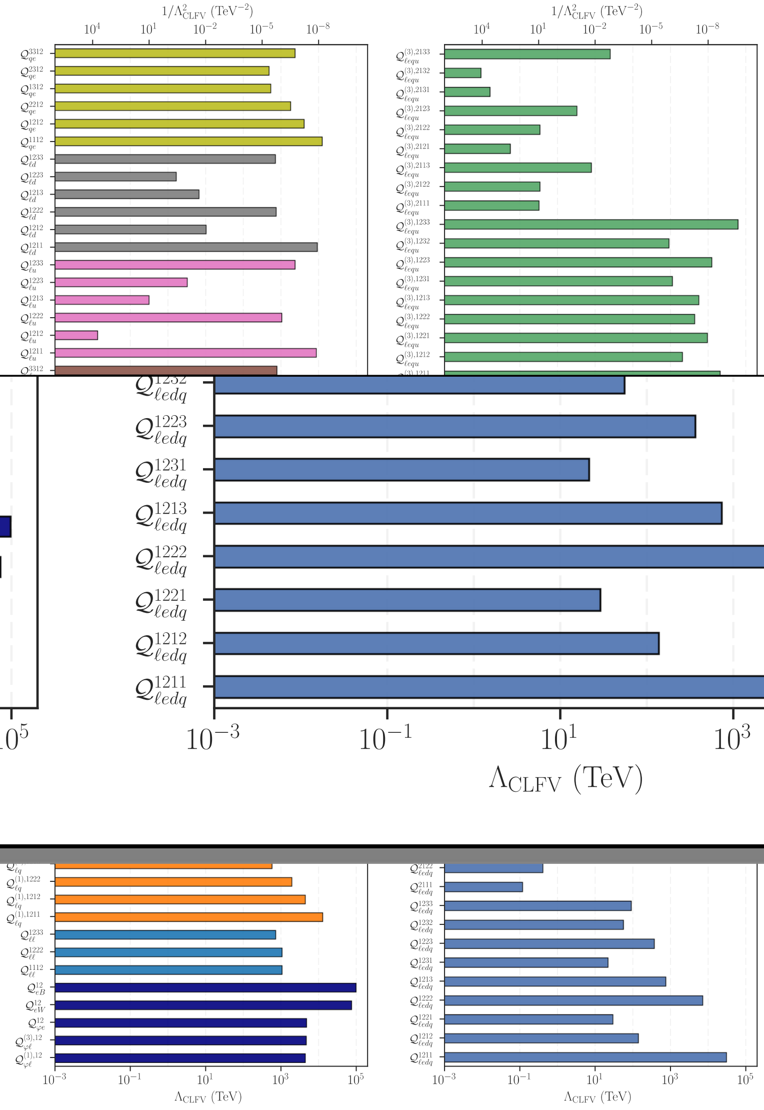
Bottom-up

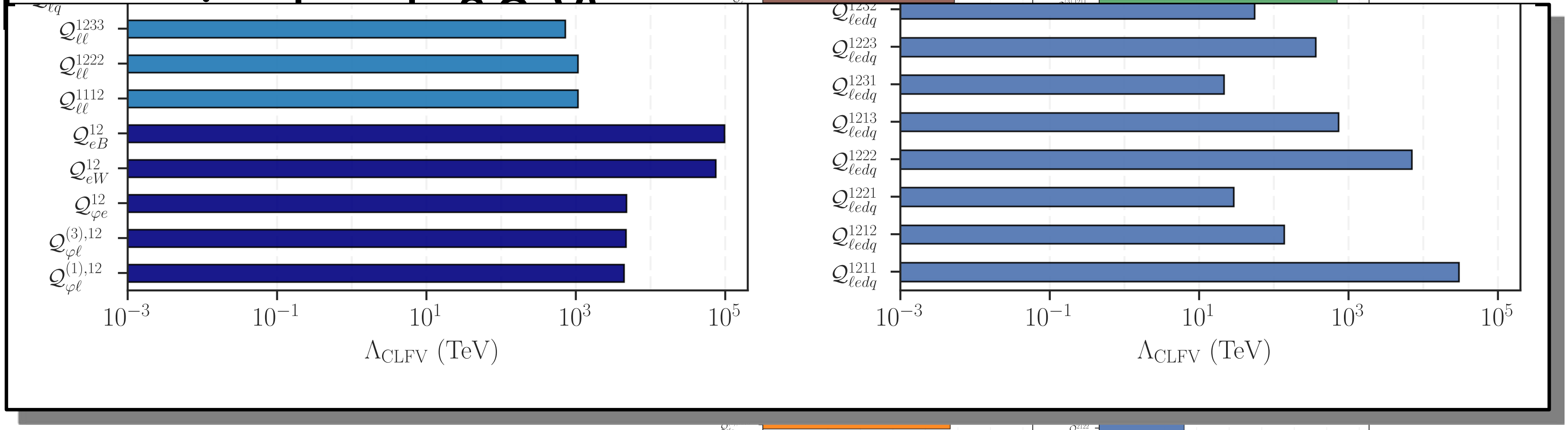
- Single dim-6 SMEFT operator bounds (with one-loop running down to 2 GeV)



Bottom-up

- Single dim-6 SMEFT operator bounds (with one-





Top-down

- Consider the following leptoquark model

$$\mathcal{L} \supset -y^{RL}_{2\,ij}\,\bar{u}^i_R R^a_2 \epsilon^{ab} L^{j,b}_L + y^{LR}_{2\,ij}\,\bar{e}^i_R R^{a\,*}_2 Q^{j,a}_L + \text{h.c.},$$

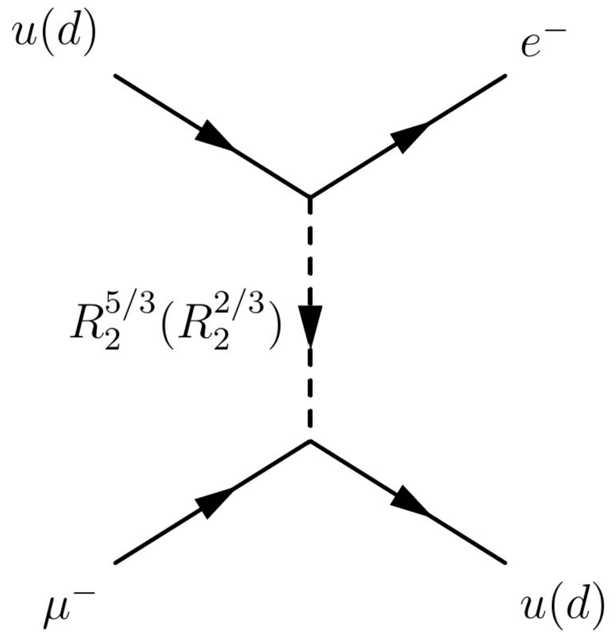
Match onto SMEFT @ Λ = m_{LQ}:

$$C_{\substack{\ell u\\12ii}} = -\frac{1}{2m^2_{LQ}} y^{RL}_{2\,i2} y^{RL*}_{2\,i1},$$

$$C_{\substack{qe\\ii12}} = -\frac{1}{2m^2_{LQ}} y^{LR*}_{2\,2i} y^{LR}_{2\,1i},$$

$$C^{(1)}_{\substack{\ell equ\\12ii}} = 2C^{(3)}_{\substack{\ell equ\\12ii}} = -\frac{1}{2m^2_{LQ}} y^{LR*}_{2\,2i} y^{RL*}_{2\,i1},$$

$$C^{(1)*}_{\substack{\ell equ\\21ii}} = 2C^{(3)*}_{\substack{\ell equ\\21ii}} = -\frac{1}{2m^2_{LQ}} y^{LR}_{2\,i2} y^{RL}_{2\,1i},$$

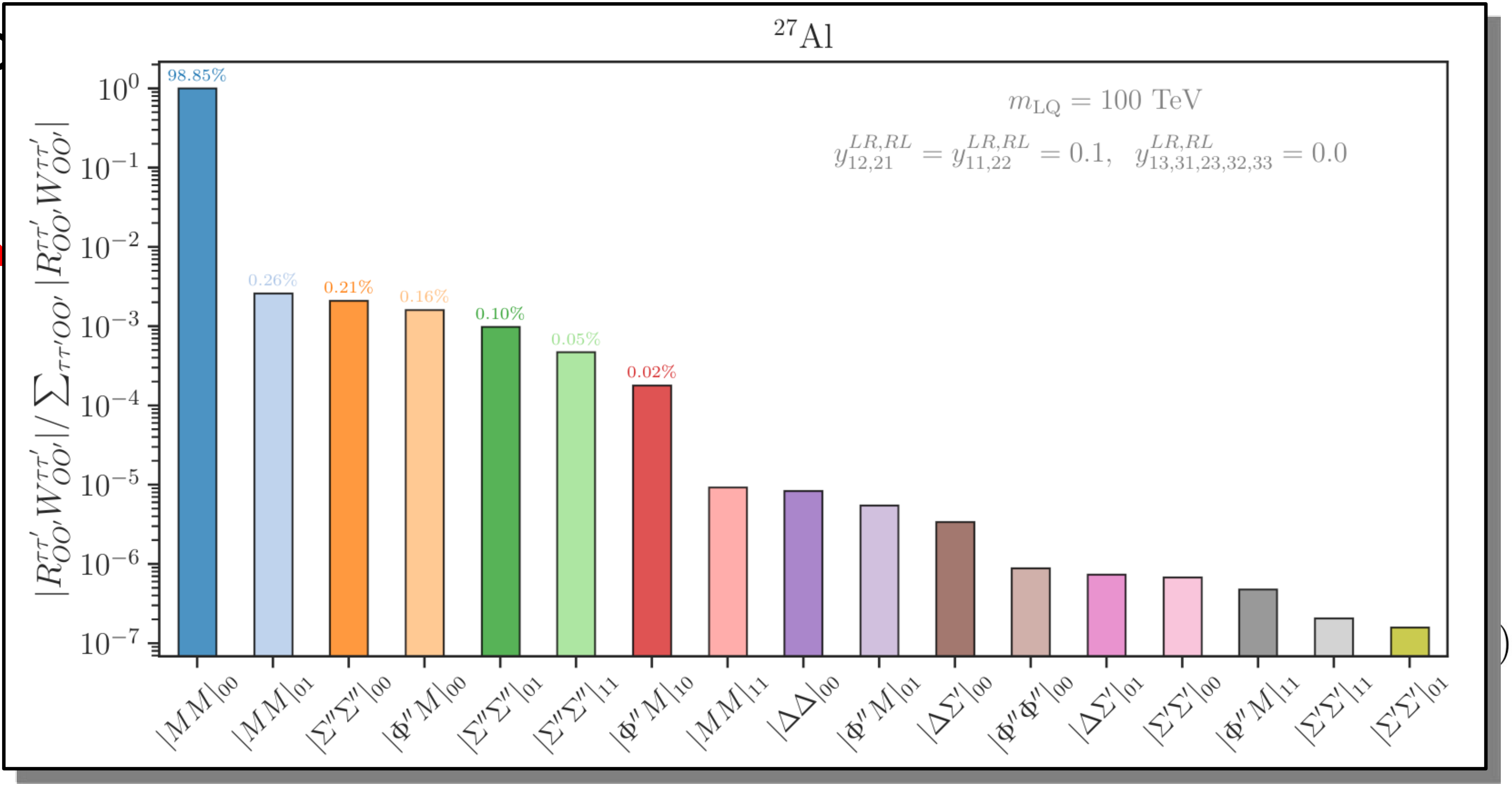


Top-down

- C

$\mathcal{L} \supset$

Match



- **Consider**

$\mathcal{L} \supset -y^{RL}_{2\,ij}\,\bar{u}^i_R$

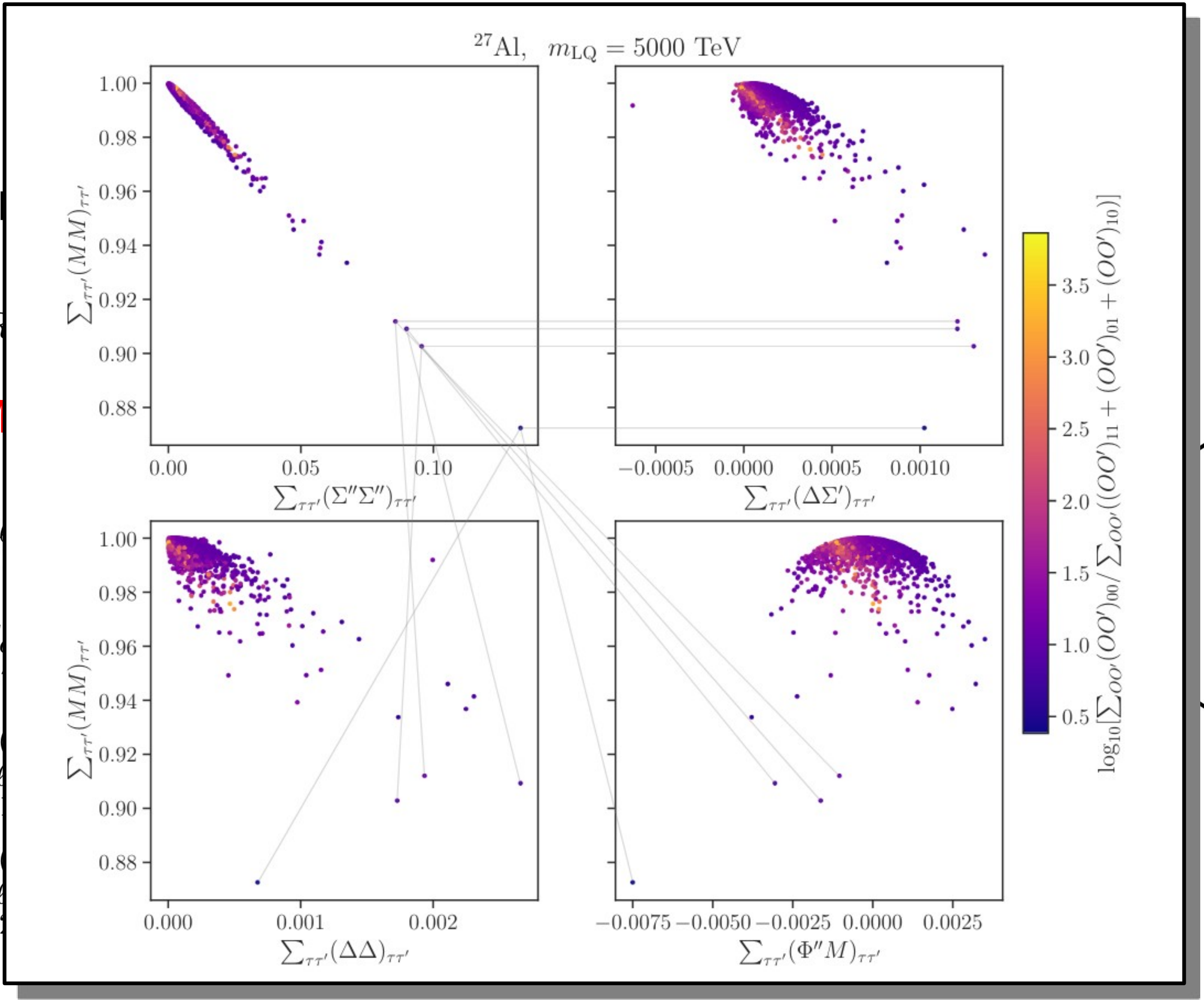
Match onto SM

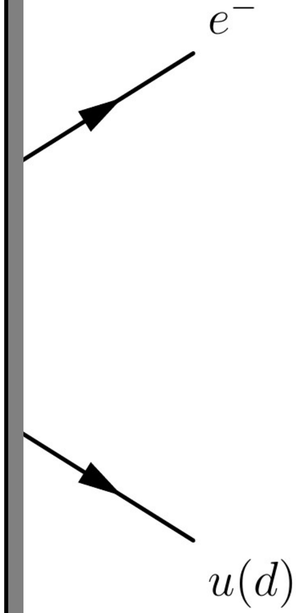
C

C

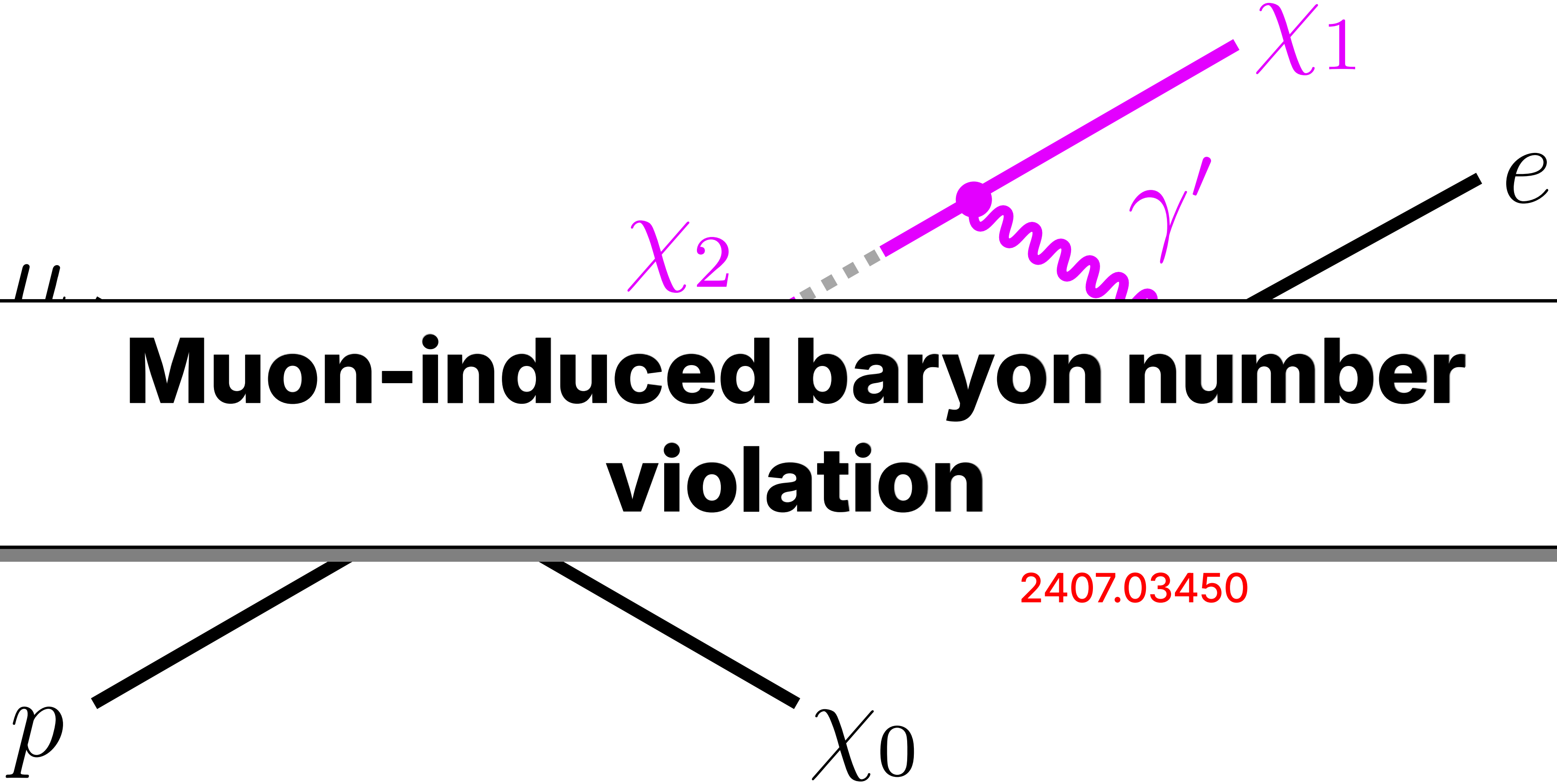
C

C





Muon-induced baryon number violation



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Widening the view

- Can the CLFV signal @ Mu2e stem from non-CLFV physics?
- NP scenarios where E_e>105 MeV?
  - Can we take advantage of the huge energy reservoir that is the nucleus?

$\mu\, ^{27}Al \rightarrow\, ^{26}Mg^{(*)} \chi_0 \chi_1 ee$

- Three dark states + $U(1)_D$ + SM portal via kinetic mixing

$$\mathcal{L}_{\mu p} = G_{\mu p}(\bar{p}\chi_2)(\bar{\mu}\chi_0) + \text{h.c.}$$

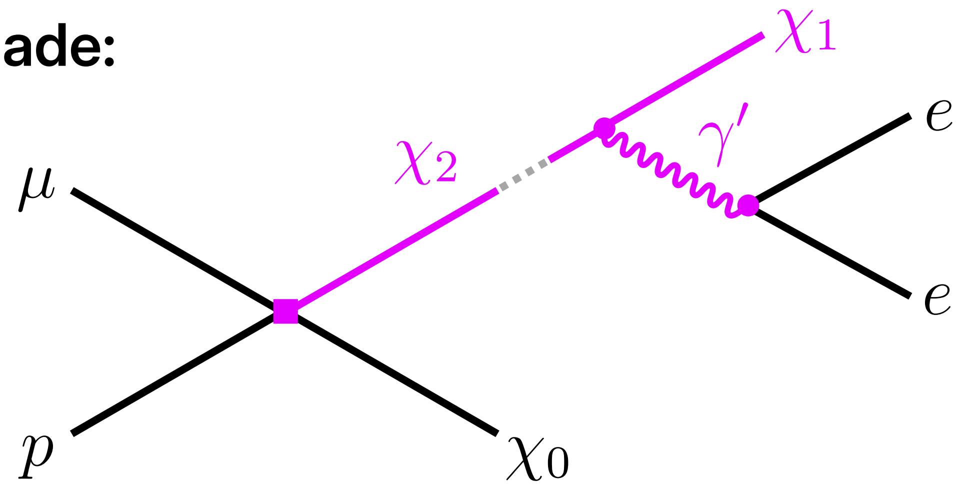
$$\mathcal{L}_{eA'} = e\varepsilon A'_\mu J^\mu_{\text{EM}}$$

$$\mathcal{L}_\chi = g_D\big(\bar{\chi}_2\gamma^\mu\chi_1\big)A'_\mu + \text{h.c.}$$

$\mu\, ^{27}Al \rightarrow\, ^{26}Mg^{(*)}\, \chi_0 \chi_1 ee$

Benchmark: $m_2 = 1030$ MeV, $m_1 = 900$ MeV, $m_0 = 0$, $m_{A'} = 20$ MeV, $G_{\mu p} = (300\ \text{TeV})^{-2}$, $\varepsilon = 10^{-4}$, $\alpha_D = 10^{-3}$

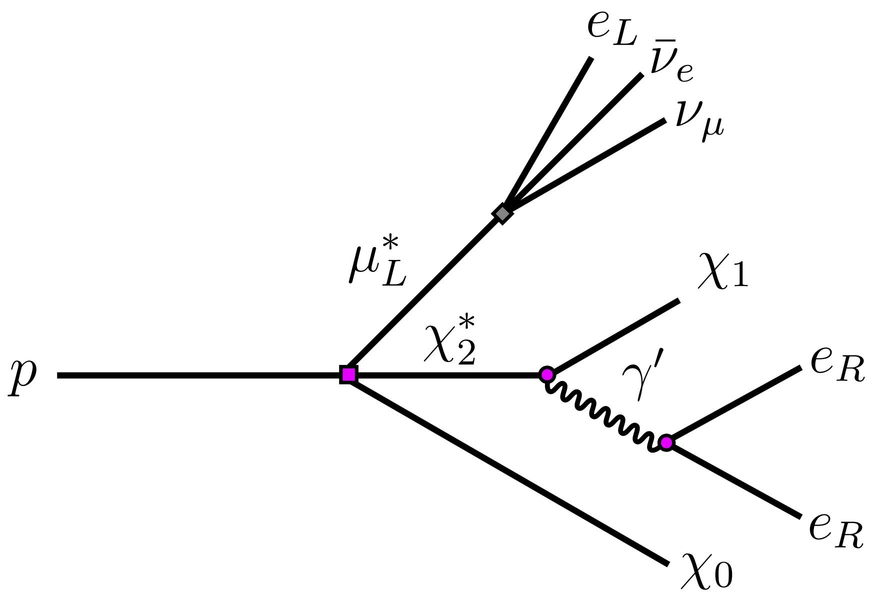
- **Leads to the cascade:**



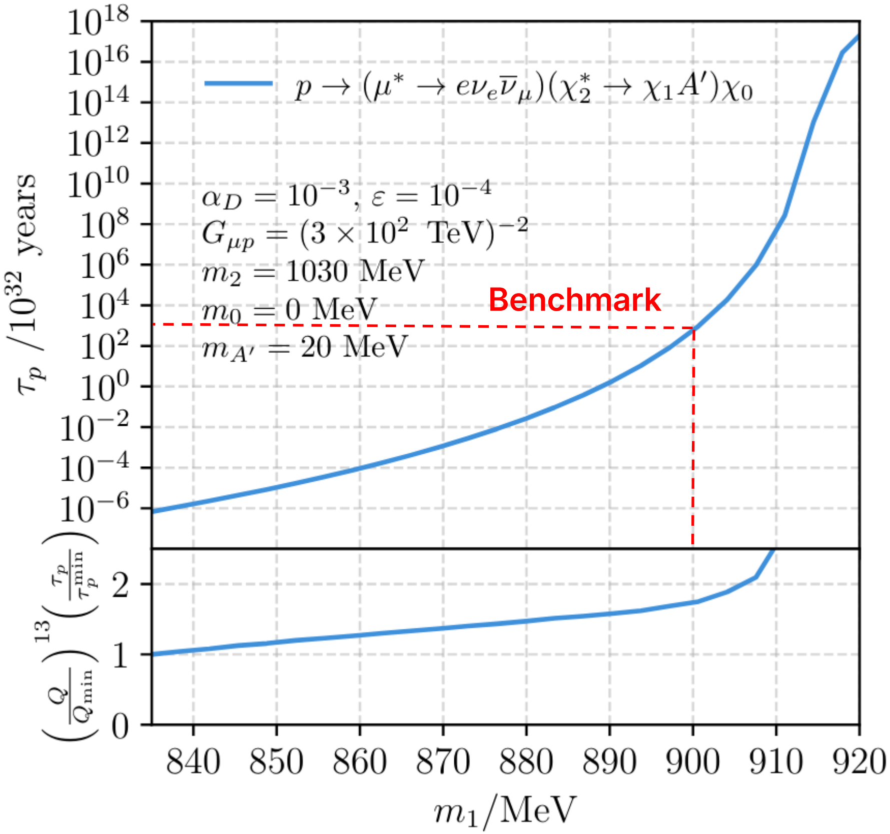
~4 × 10^{-3}

$$\Gamma(\mu\, ^{27}\text{Al} \to \chi_0 \chi_2\, ^{26}\text{Mg}) \simeq r_{\text{p.s.}} \frac{G_{\mu p}^2}{G_F^2} \Gamma(\mu\, ^{27}\text{Al} \to \nu_\mu n\, ^{26}\text{Mg}) \longrightarrow R \equiv \frac{\Gamma_{\text{exotic}}}{\Gamma_{\mu \text{Al}}} \sim 10^{-15}$$

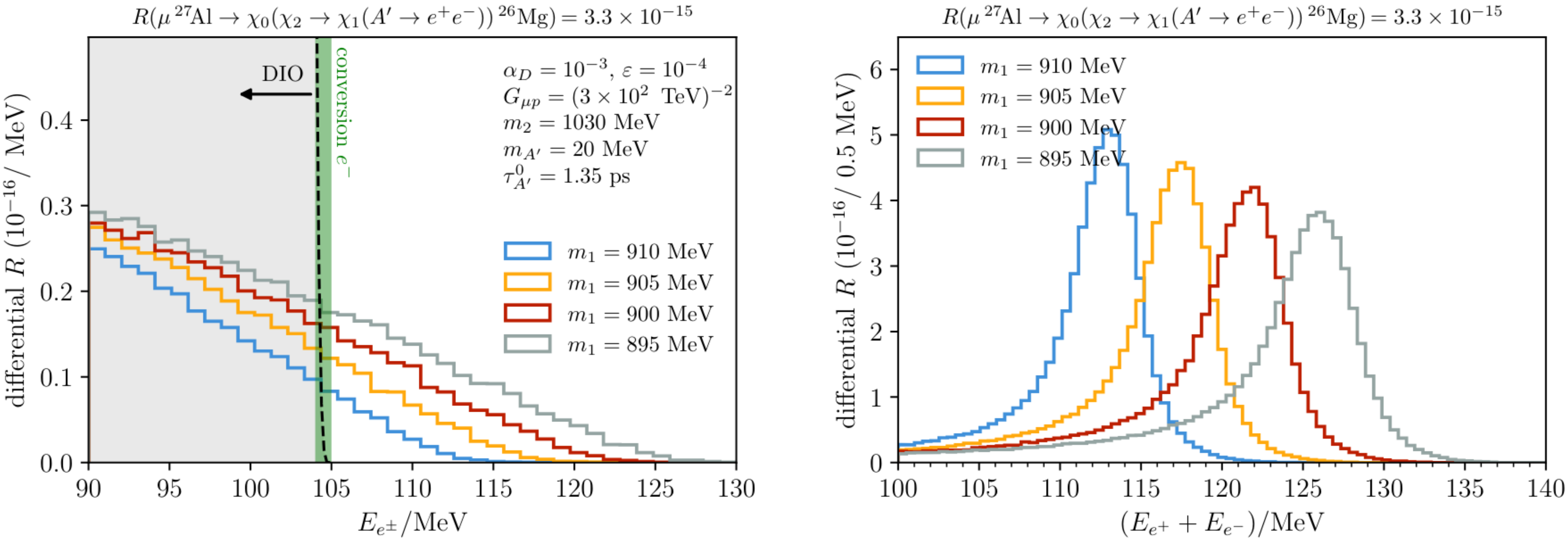
Tug-of-war with nucleon decay



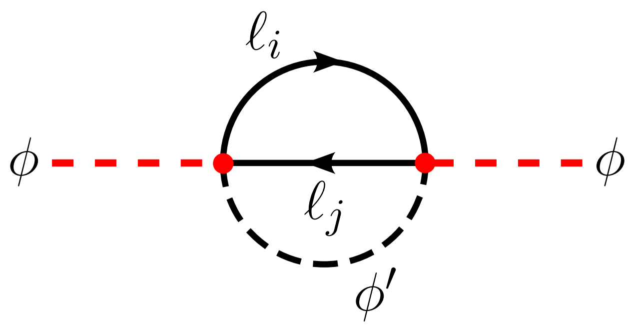
$$\Gamma_p \propto \frac{G_{\mu p}^2 G_F^2 \alpha_D Q^{13}}{(16\pi^2)^4 m_\mu^2 m_2^2}$$



Signature



$R(\mu\,^{27}\text{Al} \to \chi_0(\chi_2 \to \chi_1(A' \to e^+e^-))\,^{26}\text{Mg}) = 3.3 \times 10^{-15}$
DIO
conversion e^-
$\alpha_D = 10^{-3}$, $\varepsilon = 10^{-4}$
$G_{\mu p} = (3 \times 10^2\ \text{TeV})^{-2}$
$m_2 = 1030$ MeV
$m_{A'} = 20$ MeV
$\tau^0_{A'} = 1.35$ ps
$m_1 = 910$ MeV
$m_1 = 905$ MeV
$m_1 = 900$ MeV
$m_1 = 895$ MeV
differential R (10^{-16}/ MeV)
$E_{e^\pm}$/MeV
$R(\mu\,^{27}\text{Al} \to \chi_0(\chi_2 \to \chi_1(A' \to e^+e^-))\,^{26}\text{Mg}) = 3.3 \times 10^{-15}$
$m_1 = 910$ MeV
$m_1 = 905$ MeV
$m_1 = 900$ MeV
$m_1 = 895$ MeV
differential R (10^{-16}/ 0.5 MeV)
$(E_{e^+} + E_{e^-})$/MeV



Direct detection of ultralight DM via CLFV

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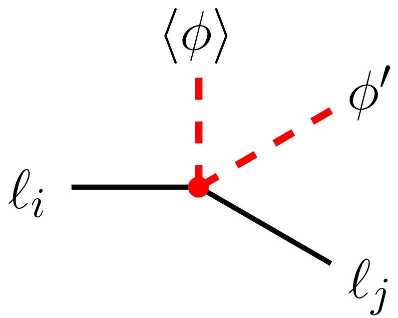
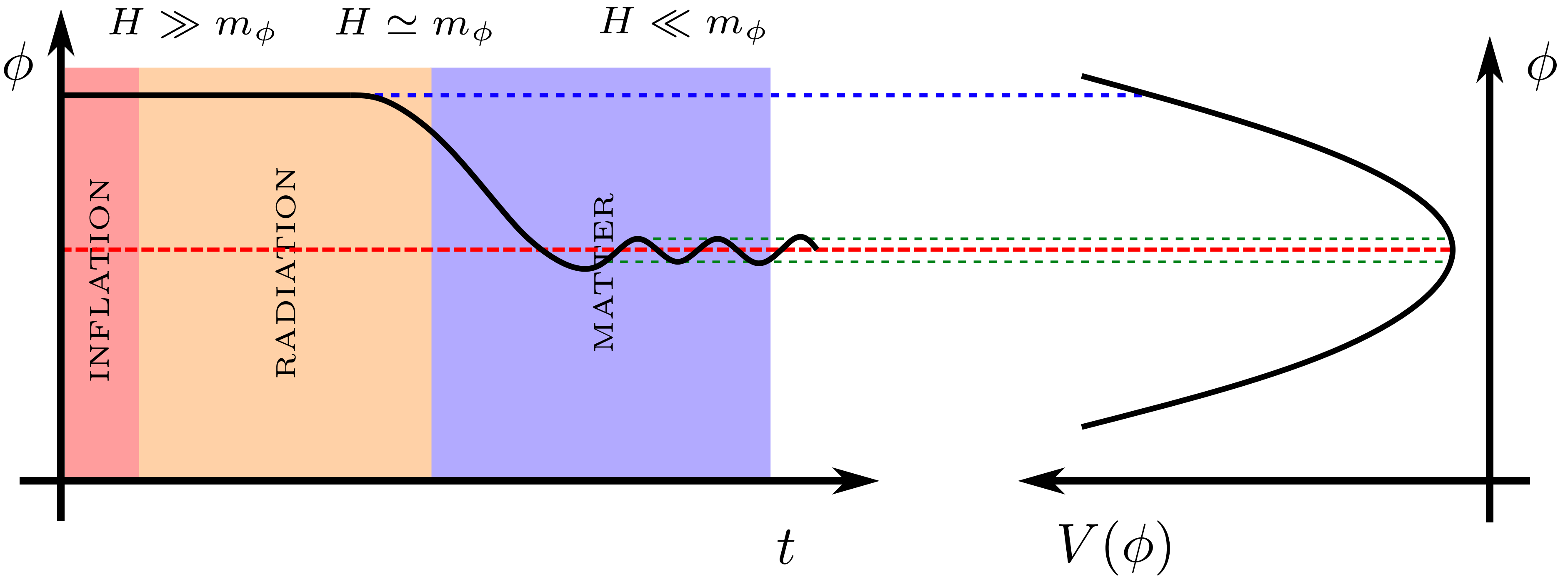


Standard lore - ULDM

EOM: $\int d^4x\sqrt{-g}\left(\frac{1}{2}(\partial_\mu\phi)^2 - \frac{1}{2}m_\phi^2\phi^2\right) \to \ddot{\phi} + 3H\dot{\phi} - \frac{\nabla^2\phi}{a^2} + m_\phi^2\phi = 0$



Standard lore - ULDM

For $m_\phi \lesssim \mathrm{eV}$ the de Broglie volume admits a huge occupation number

$$N \sim n\lambda_{\mathrm{dB}}^3 \sim \frac{\rho_\phi}{m_\phi}\left(\frac{1}{m_\phi v}\right)^3 \simeq 10^3 \left(\frac{1\ \mathrm{eV}}{m_\phi}\right)^4 \left(\frac{10^{-3}}{v}\right)^3 \left(\frac{\rho_\phi}{10^{-42}\ \mathrm{GeV}^4}\right)$$

ULDM can be accurately described as a classical wave

$$\phi_c(\mathbf{x}, t) = \phi_0(\mathbf{x}) \cos(m_\phi t + \delta)$$

$$\phi_0 = \frac{\sqrt{2\rho_\phi}}{m_\phi} \qquad \rho_\phi = 0.4\,\mathrm{GeV/cm}^3 \simeq 10^{-42}\mathrm{GeV}^4$$

Standard lore - ULDM

For $m_\phi \lesssim \mathrm{eV}$ the de Broglie volume admits a huge occupation number

$$N \sim n\lambda_{\mathrm{dB}}^3 \sim \ldots \left(\frac{\rho_\phi}{10^{-42}\ \mathrm{GeV}^4} \right)$$

ULDM ca... wave

Each mass has an associated "timescale"

$$\tau_\phi = \frac{2\pi}{m_\phi} \simeq 4\mathrm{s} \left(\frac{10^{-15}\mathrm{eV}}{m_\phi} \right)$$

$$\phi_0 = \frac{\sqrt{2\rho_\phi}}{m_\phi} \qquad \rho_\phi = 0.4\,\mathrm{GeV/cm^3} \simeq 10^{-42}\mathrm{GeV}^4$$

Why ULDM + CLFV?

CLFV experiments probe extremely high NP scales

Detecting a CLFV signal does not immediately imply DM is the source.

Detecting a *time-dependent* CLFV signal is a smoking gun signal of DM.

Minimal analysis can convert intensity frontier experiments into dark matter detectors

ULDM + CLFV

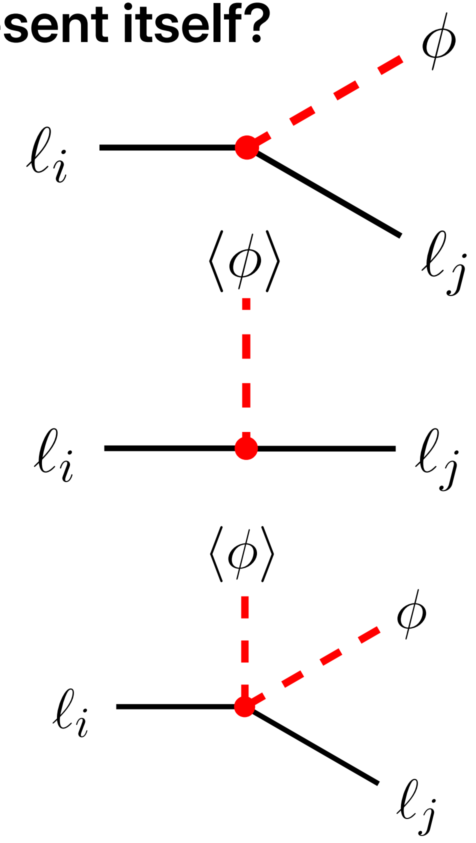
How and where does time-modulation present itself?

Two types of time-dependence:

- Manifestation in mass matrix

$$m_{ij} = \mathrm{diag}(m_e, m_\mu, m_\tau) + y_{ij}\phi_c$$

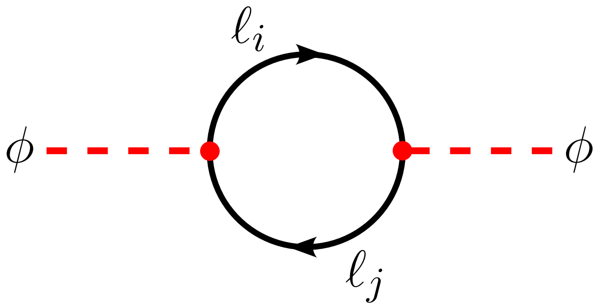
- Manifestation in decay/scattering rates
  - Inherently higher dimensional



Journey towards phenomenologically viable operator

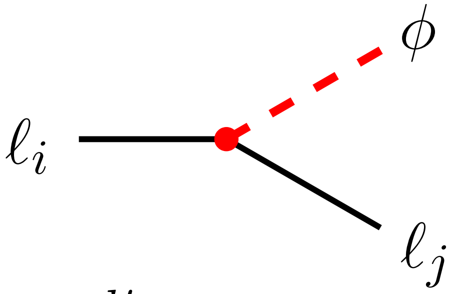
First guess: $\frac{C_{ij}}{\Lambda}\phi H\bar{L}_i\ell_j \to y_{ij}\phi\bar{\ell}_i\ell_j$

1. Fine-tuning

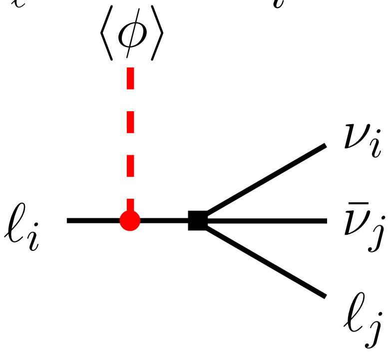


$\longrightarrow \delta m_\phi^2 \simeq \frac{\lambda^2}{4\pi^2}m_\ell^2$

2. No time-modulation



3. SM rate-modulation



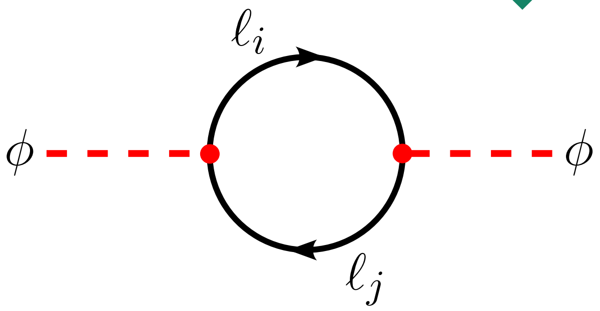
$|y_{\tau\ell}| \lesssim 10^{-7}$ (Belle II)

$|y_{\mu e}| \lesssim 10^{-10}$ (TWIST)

Journey towards phenomenologically viable operator

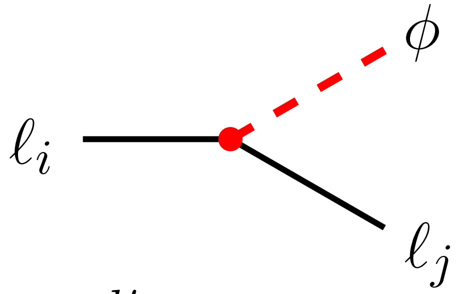
First guess: $\frac{C_{ij}}{\Lambda}\phi H \bar{L}_i \ell_j \rightarrow y_{ij}\phi\bar{\ell}_i\ell_j$

1. Fine-tuning "✓"

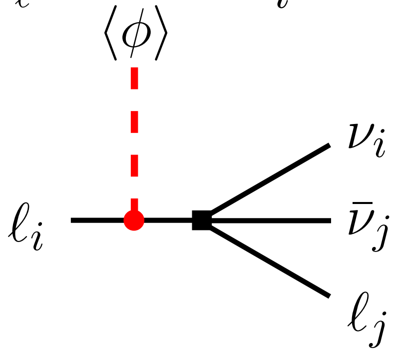


$\longrightarrow \delta m_\phi^2 \simeq \frac{\lambda^2}{4\pi^2} m_\ell^2$

2. No time-modulation X



3. SM rate-modulation X



$|y_{\tau\ell}| \lesssim 10^{-7}$ (Belle II)

$|y_{\mu e}| \lesssim 10^{-10}$ (TWIST)

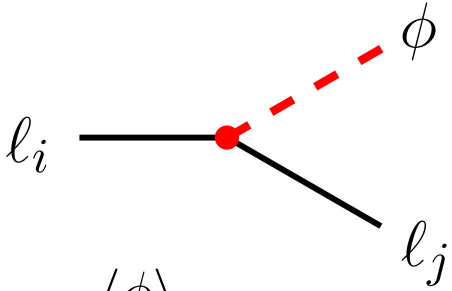
Journey towards phenomenologically viable operator

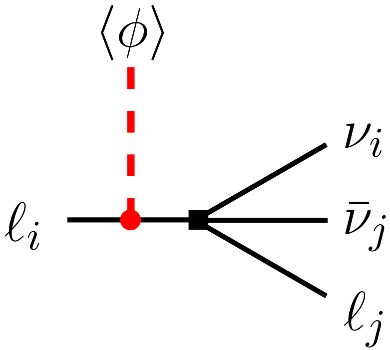
Second guess: $\frac{C_{ij}}{\Lambda}\partial_\mu\phi(\bar{\ell}_i\gamma^\mu\gamma^5\ell_j)$

1. Fine-tuning: ✓

2. Time-modulation: X

3. SM-modulation: X





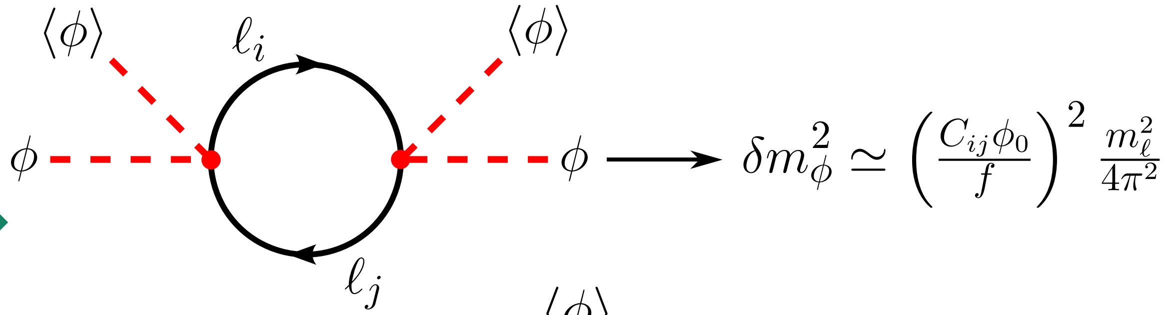
Journey towards phenomenologically viable operator

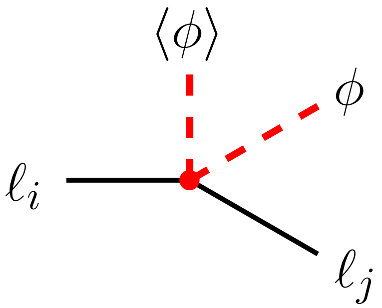
Third guess: $\frac{C_{ij}}{f}\phi^2(\bar{\ell}_i \ell_j)$

1. Fine-tuning: ✗

2. Time-modulation: ✓

3. SM-modulation: ✓

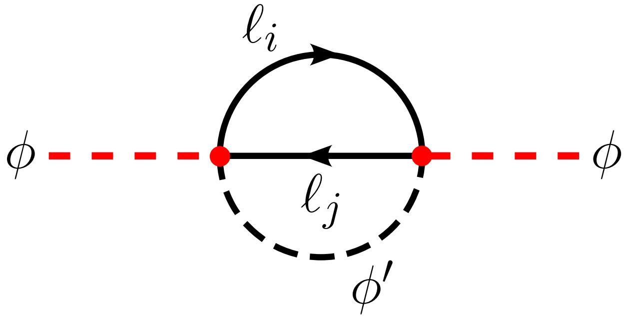


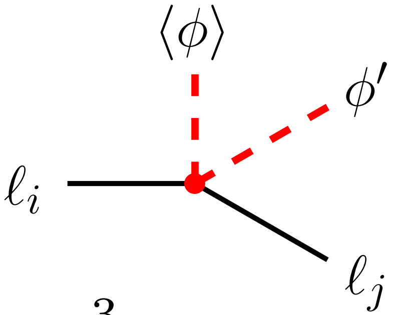


Journey towards phenomenologically viable operator

Fourth guess: $\frac{C_{ij}}{f}\phi\phi'(\bar{\ell}_i\ell_j)$ or $\frac{C_{ij}}{f^2}\phi\partial_\mu\phi'(\bar{\ell}_i\gamma^\mu\ell_j)$

1. Fine-tuning: "✓"/✓
2. Time-modulation: ✓
3. SM-modulation: ✓





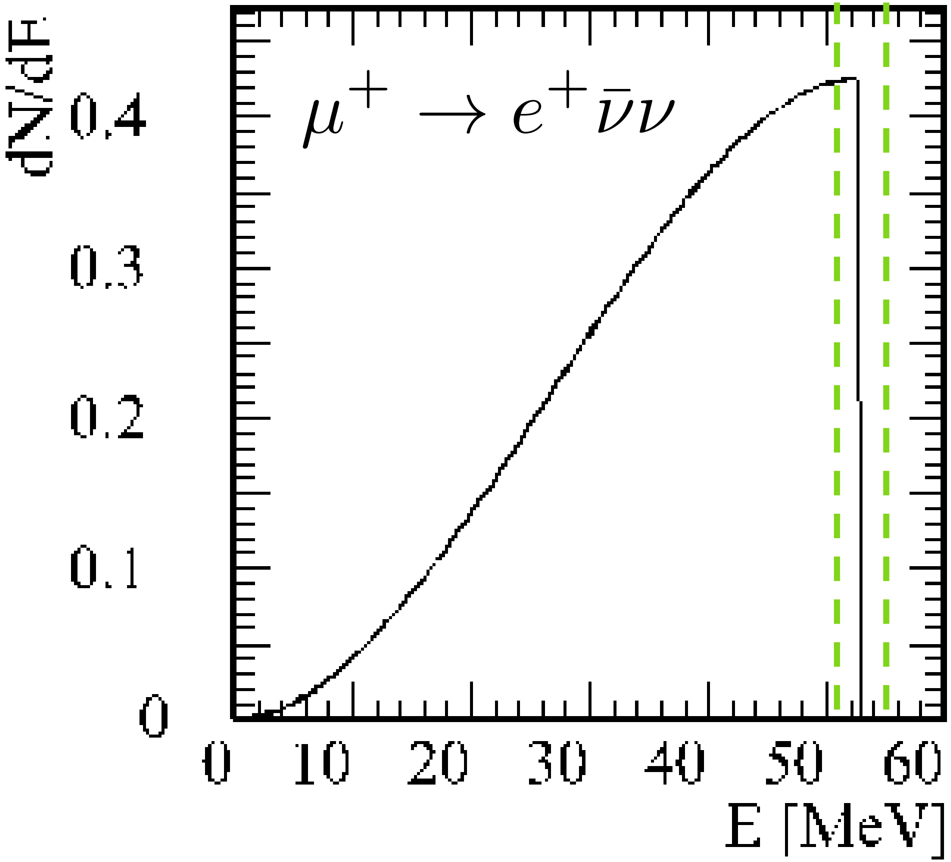
$$\mathcal{B}(\ell_i \to \ell_j\phi) = \frac{C_{ij}^2\phi_0^2}{64\pi f^4}\frac{m_{\ell_i}^3}{\Gamma_{\ell_i}}\cos^2(m_\phi t + \delta)$$

Sensitivity (Mu3e)

Consider, for example, Mu3e.

- Will run for T~300 days
- ~10^{15} muon decays
- ~10^{13} of which will lie in the final kinematic bin
- ~10^{8} muon decays/s

Michel spectrum



Sensitivity

- Sensitivity can essentially be determined by:

$$m_\phi, \quad T, \quad N_{\text{total}}$$

- Statistical uncertainty + systematic uncertainty maximally correlated across all time bins

$$\sigma_{\text{stat}} = \sqrt{N_{\text{bg}}/n_{\text{bin}}} \;\;, \quad \sigma_{\text{sys}} = \alpha N_{\text{bg}}/n_{\text{bin}}$$

$$\chi^2 = \sum_{k,p=1}^{n_{\text{bin}}} S_k C_{kp}^{-1} S_p = \frac{1}{N_{\text{bg},1}} \sum_{k=1}^{n_{\text{bin}}} S_k^2 - \frac{\alpha^2}{1+\alpha^2 n_{\text{bin}} N_{\text{bg},1}} \left(\sum_{k=1}^{n_{\text{bin}}} S_k \right)^2$$

Sensitivity

$$\chi^2 = \sum_{k,p=1}^{n_{\rm bin}} S_k C_{kp}^{-1} S_p = \frac{1}{N_{\rm bg,1}} \sum_{k=1}^{n_{\rm bin}} S_k^2 - \frac{\alpha^2}{1+\alpha^2 n_{\rm bin} N_{\rm bg,1}} \left(\sum_{k=1}^{n_{\rm bin}} S_k \right)^2$$

$$S_k = 2\mathcal{B}_{\rm sig} f_{\rm sig} \frac{N_{\rm tot}}{T} \int_{(k-1)\Delta t}^{k\Delta t} dt \cos^2(m_\phi t + \delta)$$

$$= \mathcal{B}_{\rm sig} f_{\rm sig} \frac{N_{\rm tot}}{T} \left[\Delta t + \frac{\sin(2k m_\phi \Delta t + 2\delta) - \sin(2(k-1) m_\phi \Delta t + 2\delta)}{2 m_\phi} \right]$$

Three interesting limits:

1. **Signal does not oscillate over the duration of the experiment**
2. **Signal oscillates but time bins cannot resolve the oscillations**
3. **Signal oscillates and time bins resolve the oscillations**

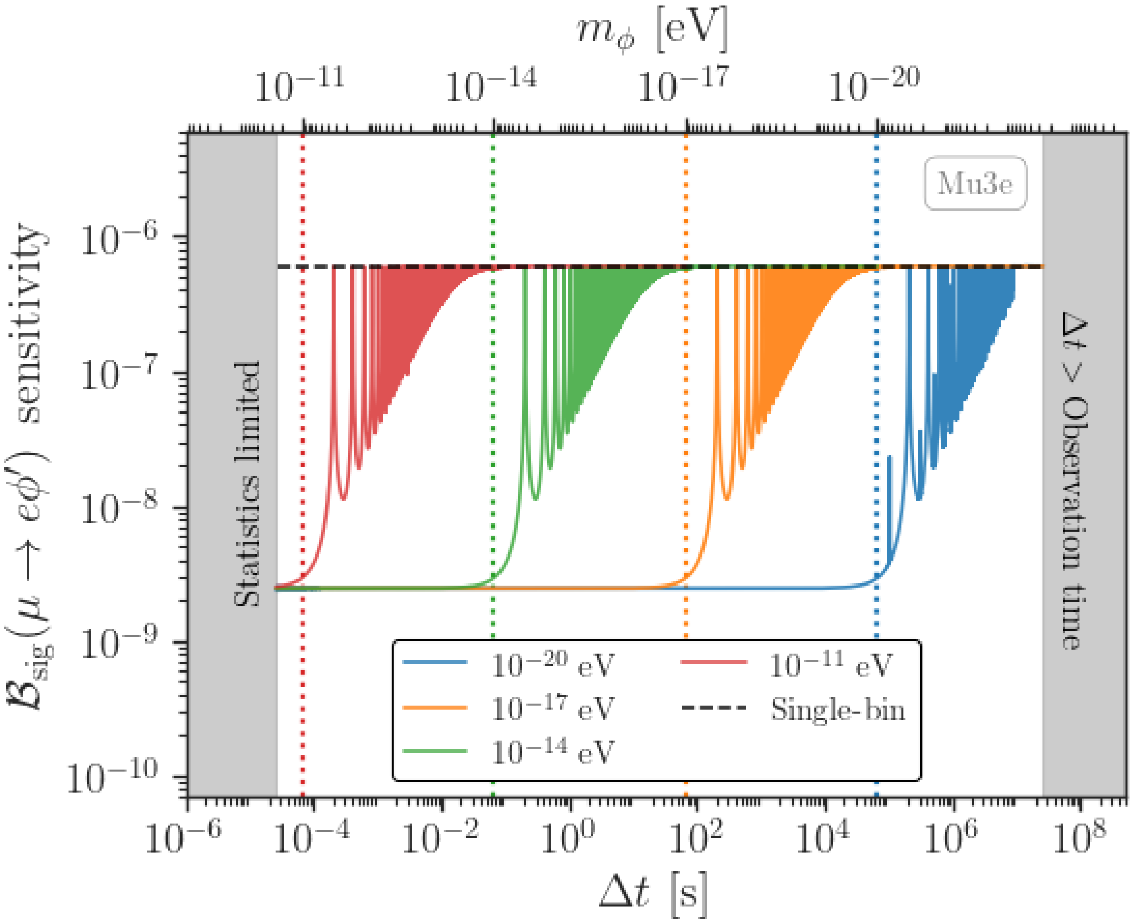
Systematics dominate

Statistics dominate

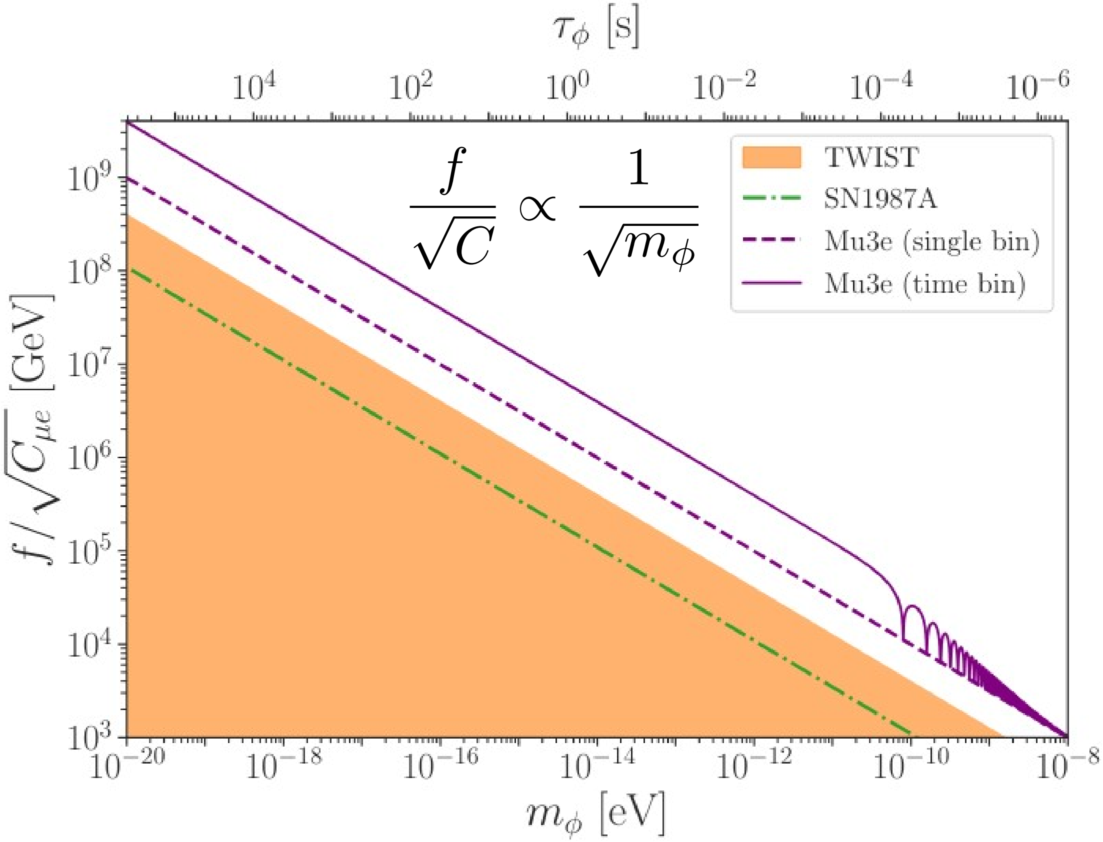
Sensitivity

Assumptions:
1) Asimov dataset
2) Continuous running
3) Time-independent systematics

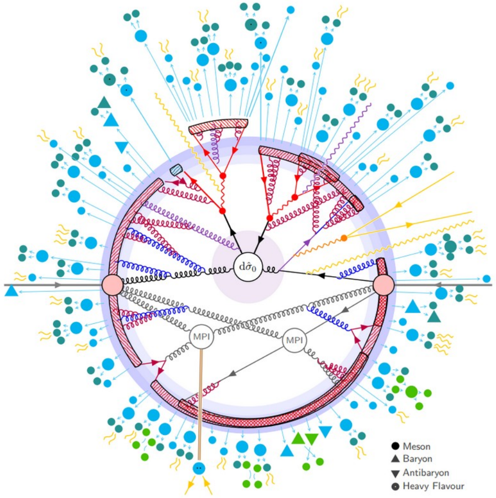
- **Notably, the sensitivity to ULDM mass is limited by statistics, not experimental time resolution**

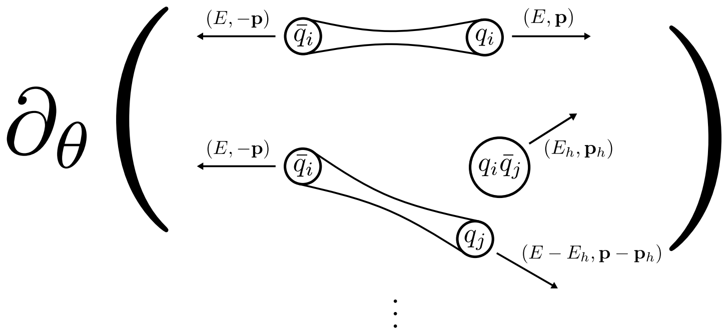


Reach



τ_ϕ [s]
10^4
10^2
10^0
10^{-2}
10^{-4}
10^{-6}
$\frac{f}{\sqrt{C}} \propto \frac{1}{\sqrt{m_\phi}}$
TWIST
SN1987A
Mu3e (single bin)
Mu3e (time bin)
$f\,/\sqrt{C_{\mu e}}$ [GeV]
10^9
10^8
10^7
10^6
10^5
10^4
10^3
10^{-20}
10^{-18}
10^{-16}
10^{-14}
10^{-12}
10^{-10}
10^{-8}
m_ϕ [eV]



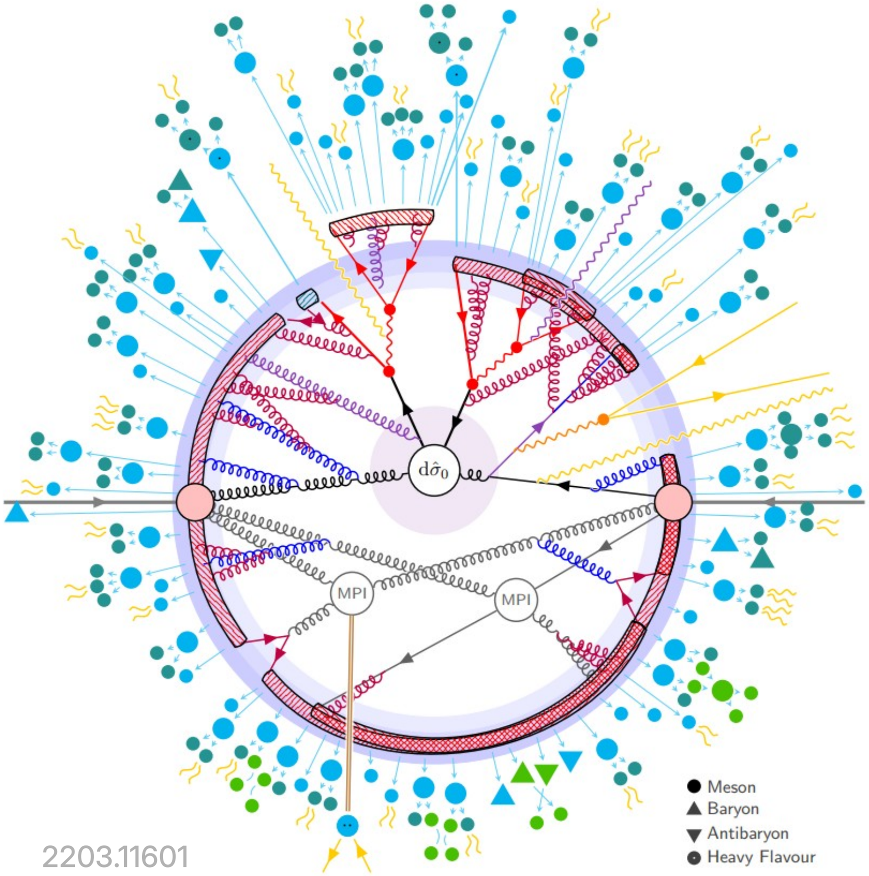


Part II:

Differentiable hadronization models

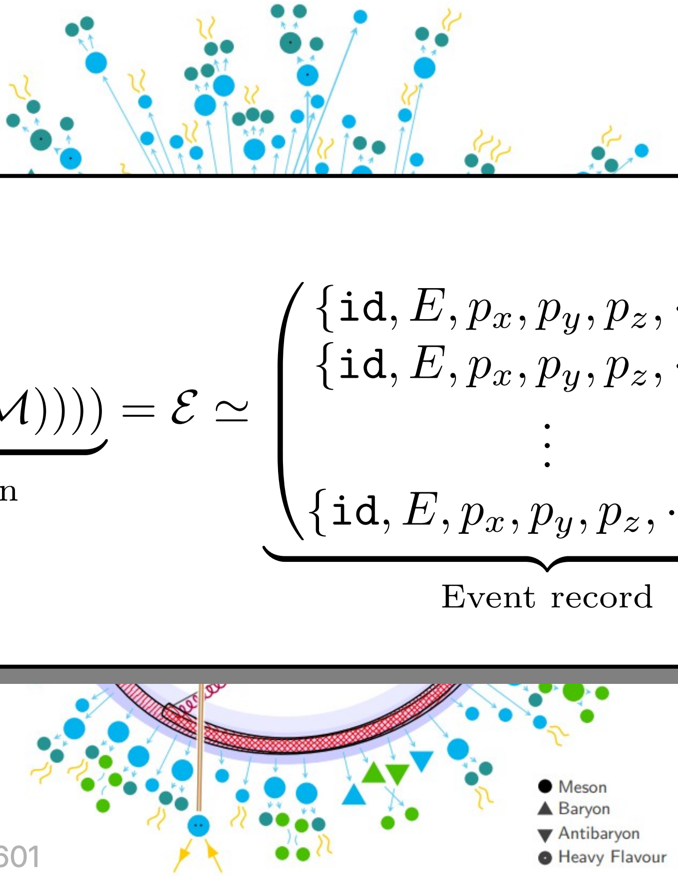
MLHAD

Monte Carlo Event generators



2203.11601

Monte Carlo Event generators



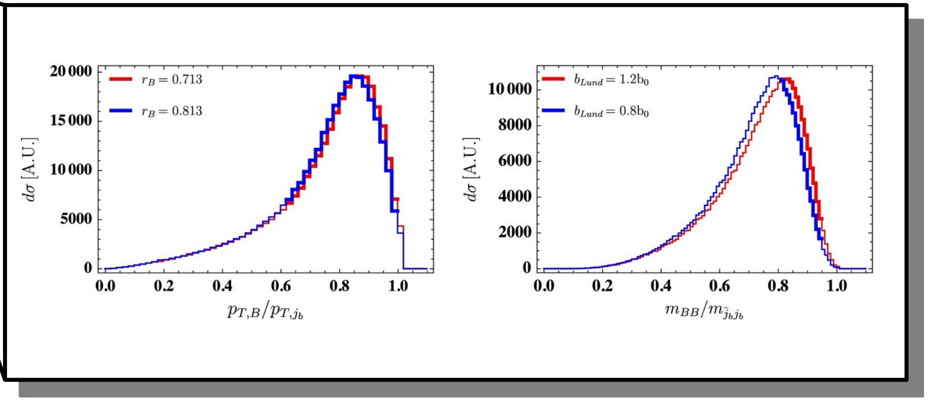
$$\mathcal{G} : \underbrace{\mathcal{S}(\mathcal{D}(\mathcal{H}(\mathcal{P}(\mathcal{M}))))}_{\text{Simulation}} = \mathcal{E} \simeq \underbrace{\begin{pmatrix} \{\texttt{id}, E, p_x, p_y, p_z, \cdots\}_1 \\ \{\texttt{id}, E, p_x, p_y, p_z, \cdots\}_2 \\ \vdots \\ \{\texttt{id}, E, p_x, p_y, p_z, \cdots\}_N \end{pmatrix}}_{\text{Event record}} .$$

2203.11601

Motivation

Precision (exclusive) measurements dependent on hadronization!

- Uncertainty reduction
  - Top quark mass measurement (r_b)
  - e^+e^- determination of α_s
- Mis-modeling
  - High-multiplicity events
  - Tuning discrepancies



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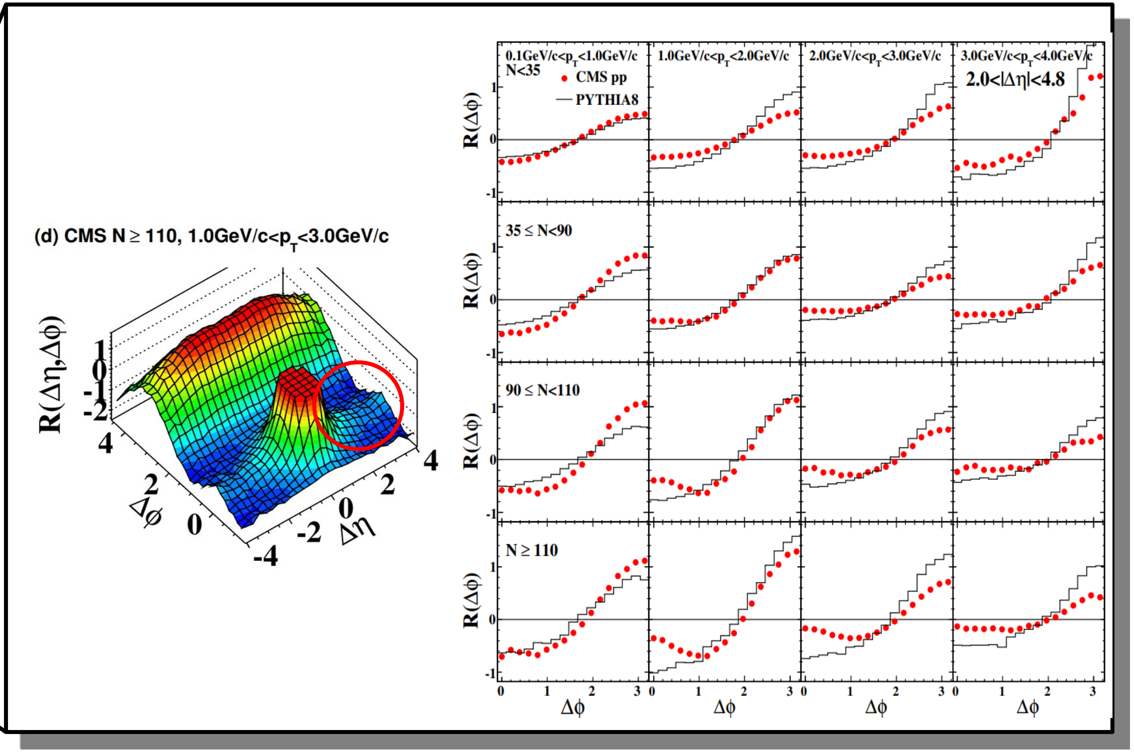

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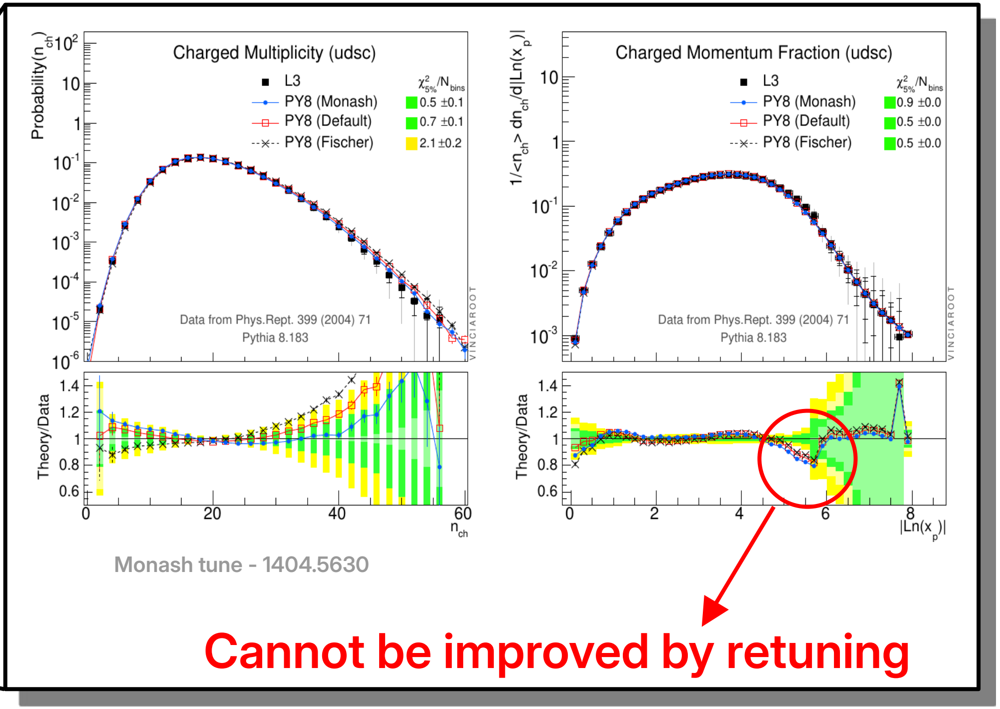
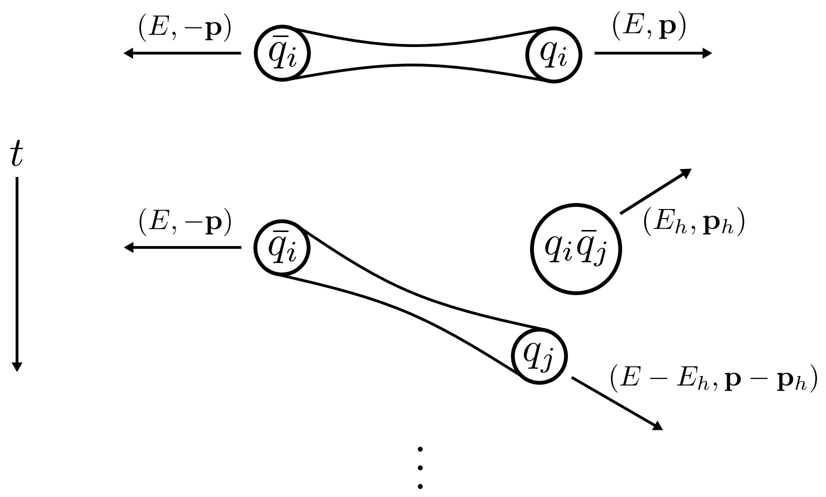


Phenomenological models

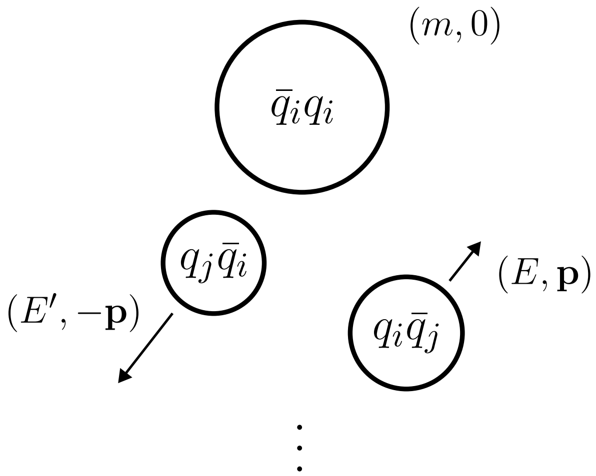
Lund string model

(used in Pythia)

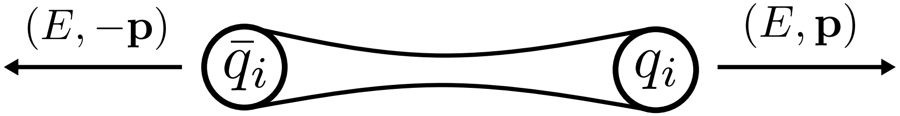


Cluster model

(used in Herwig, Sherpa)



The algorithm ($q\bar{q}$)



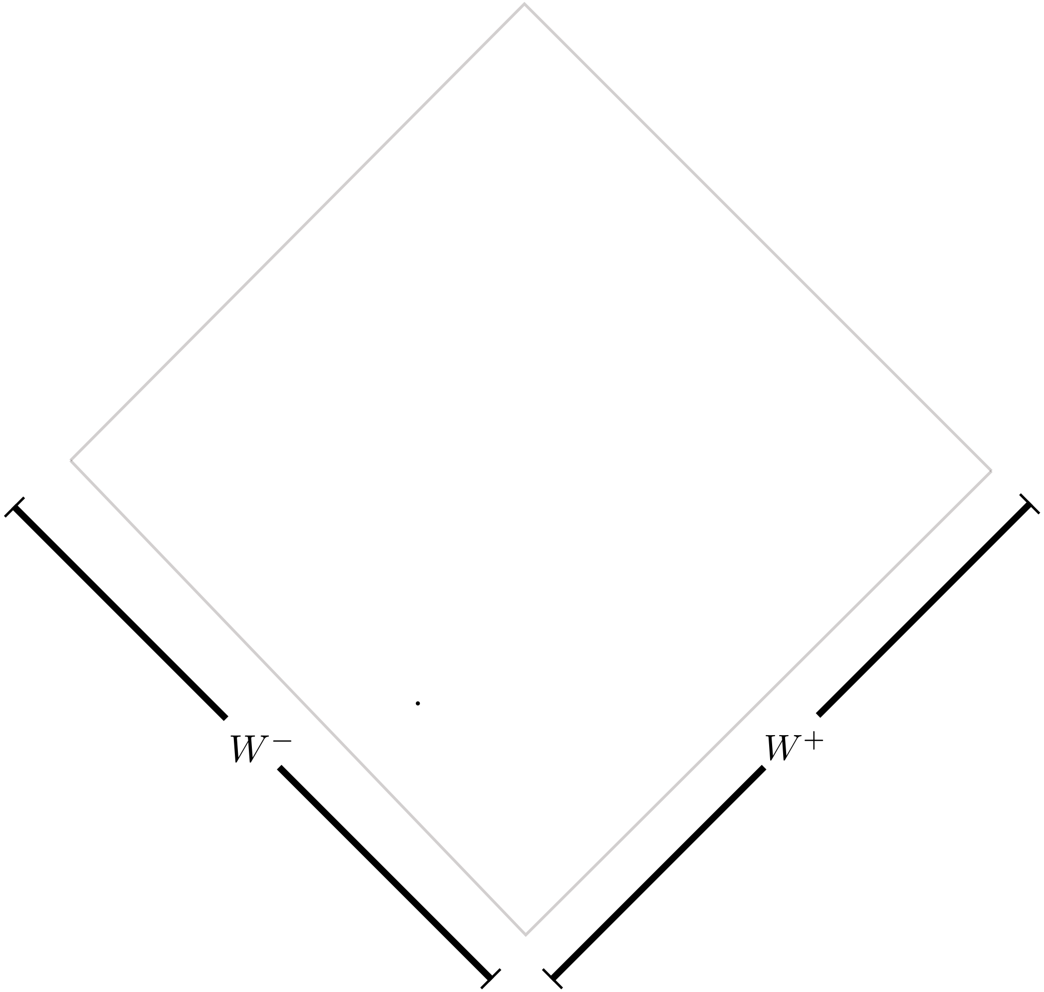
1) Randomly select one of the string ends
2) Sample new quark flavor
3) Sample transverse momentum of new quarks

$$\mathcal{P}(p_x, p_y; \sigma_{p_T}) = \frac{1}{\pi\sigma_{p_T}^2} \exp\left(-\frac{p_x^2+p_y^2}{\sigma_{p_T}^2}\right)$$

4) Sample longitudinal momentum fraction of new hadron

$$f(z) \propto \frac{(1-z)^a}{z} \exp\left(\frac{-bm_T^2}{z}\right), \quad z = \frac{p_z + E_h}{2E}$$

5) Repeat steps 1-4



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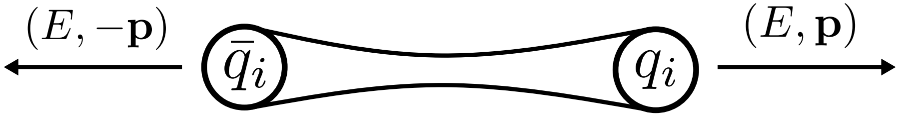

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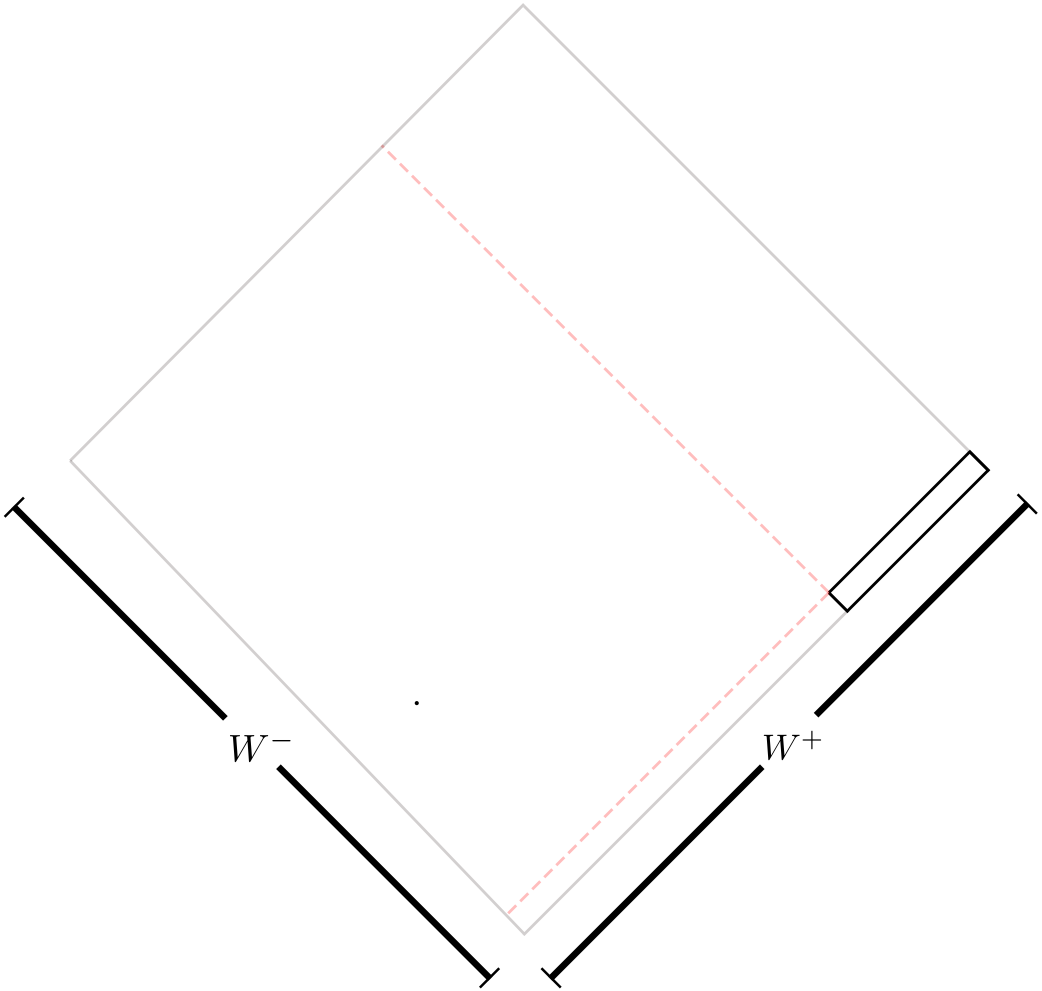

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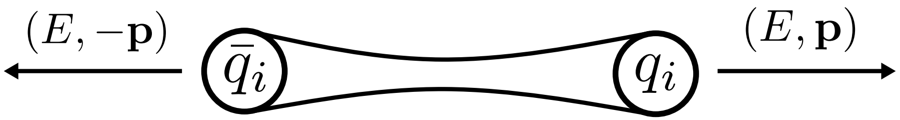

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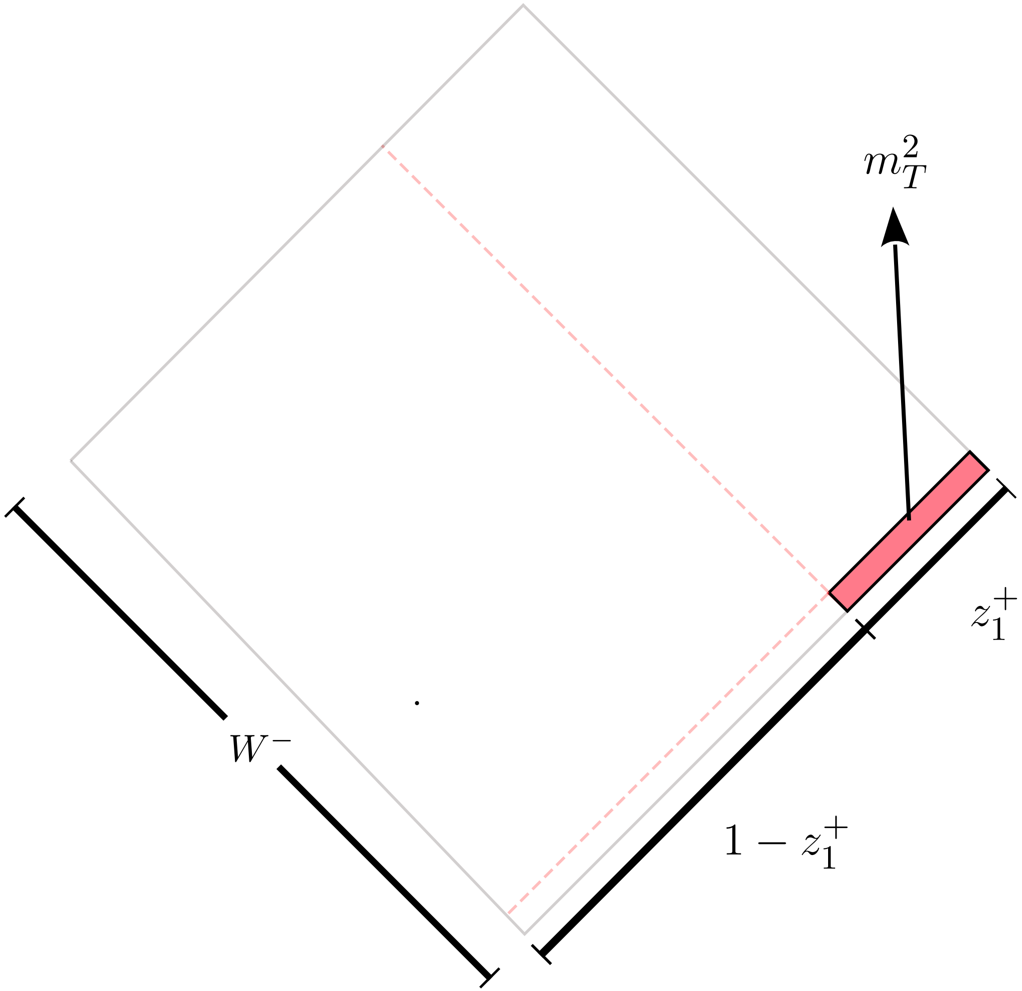

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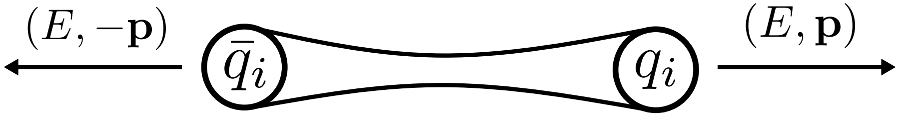

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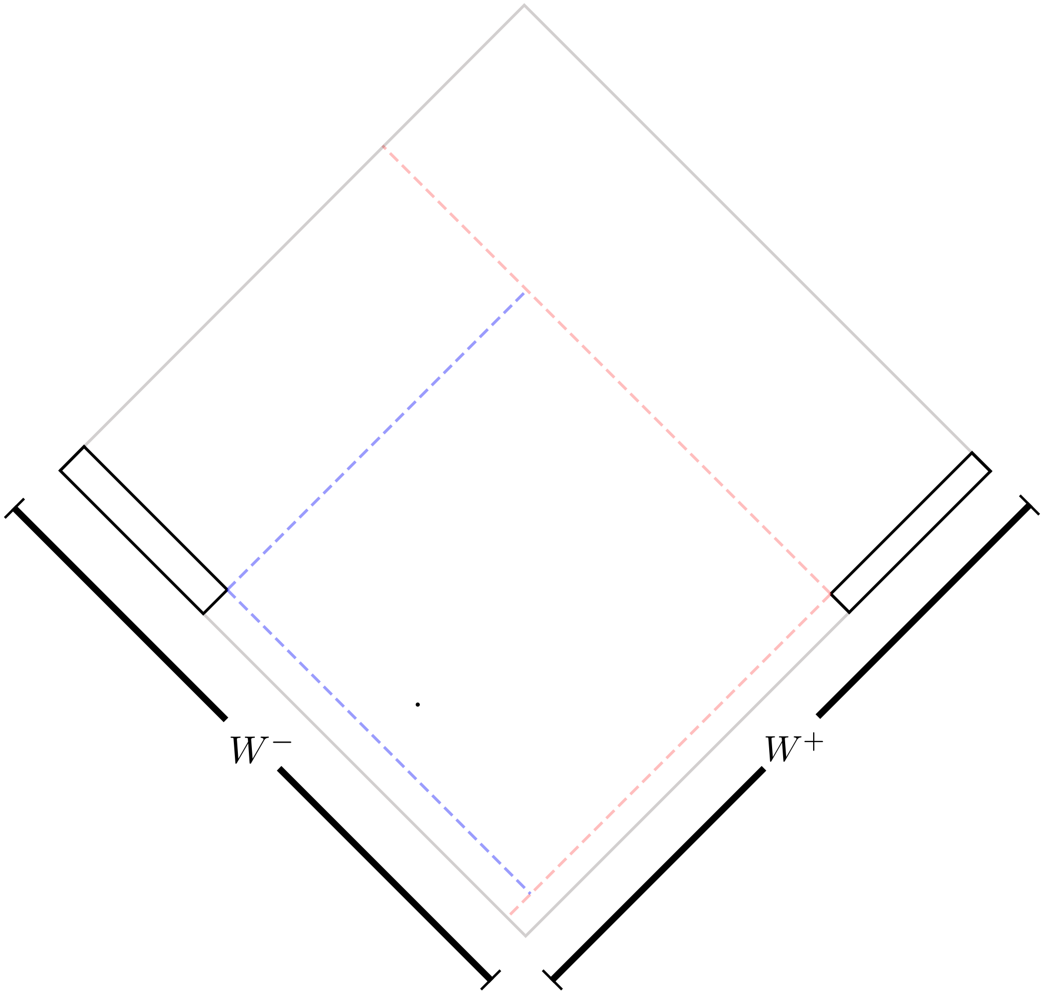

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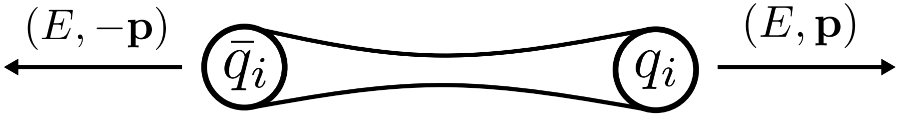

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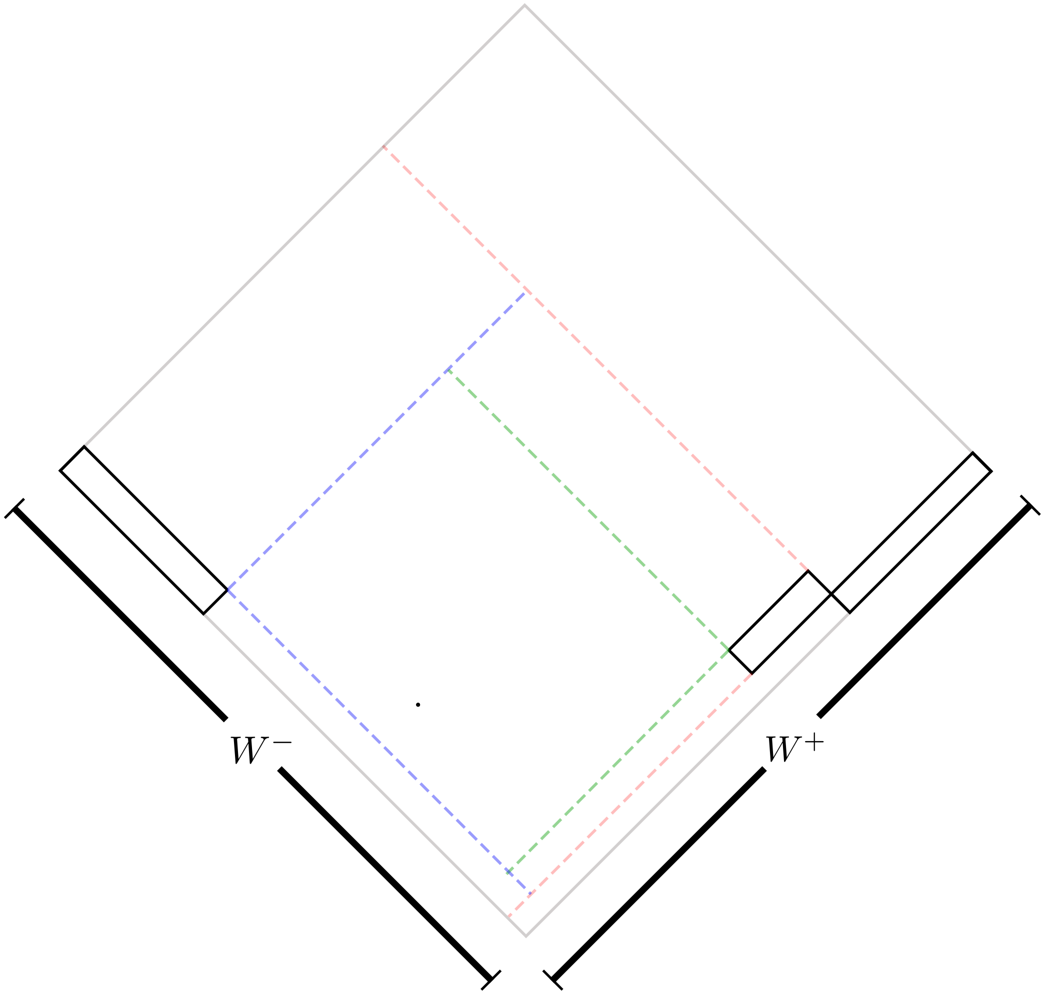

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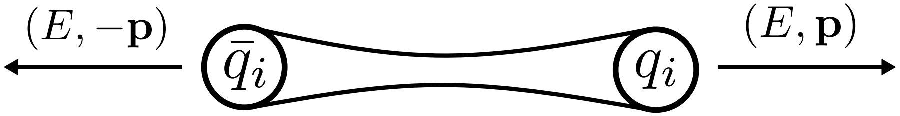

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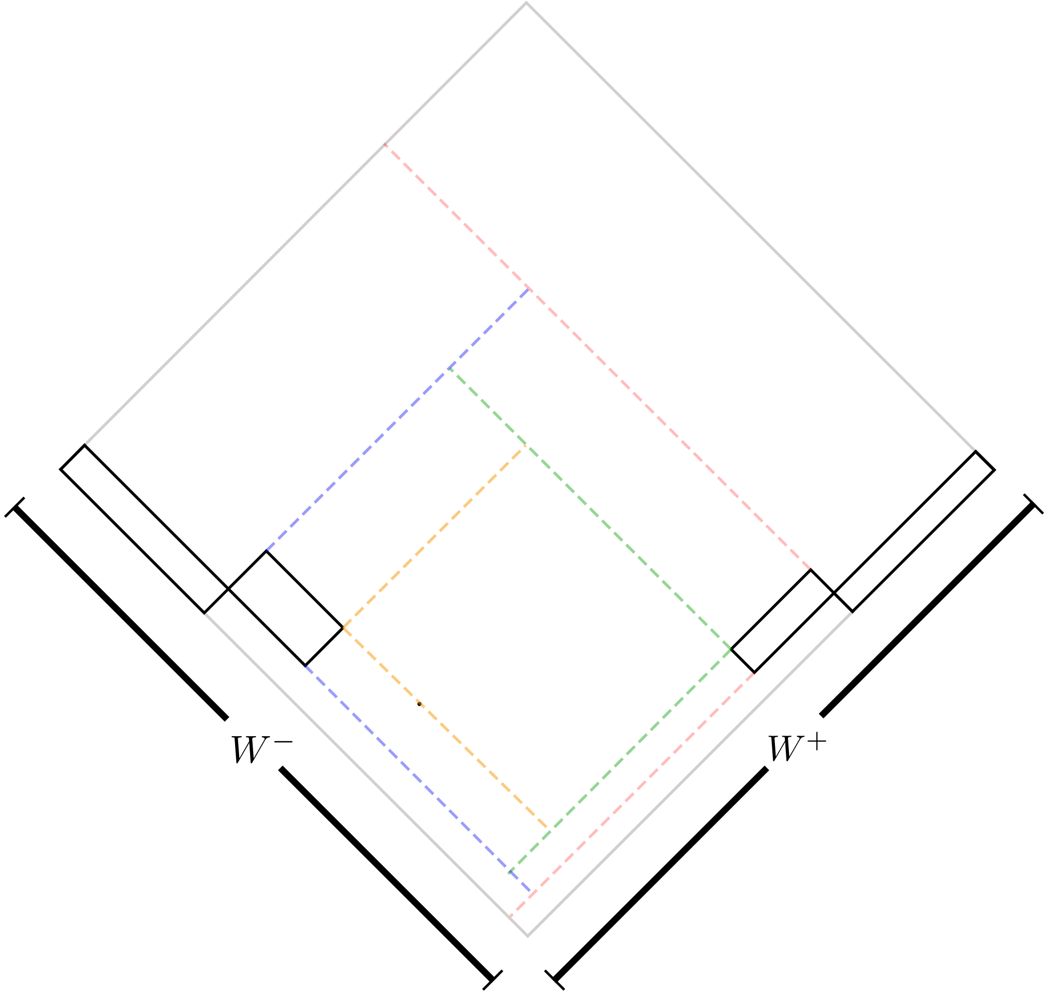

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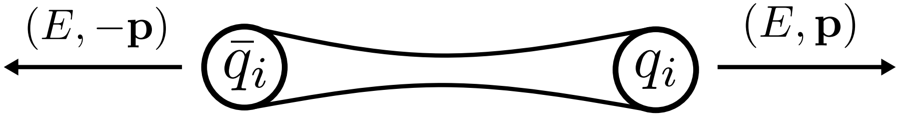

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$\mathcal{P}($

4) Samp
new h

$f(z) \propto \frac{\ldots}{z} \exp\left(\frac{\ldots}{z}\right), \quad z = \frac{\ldots}{2E}$

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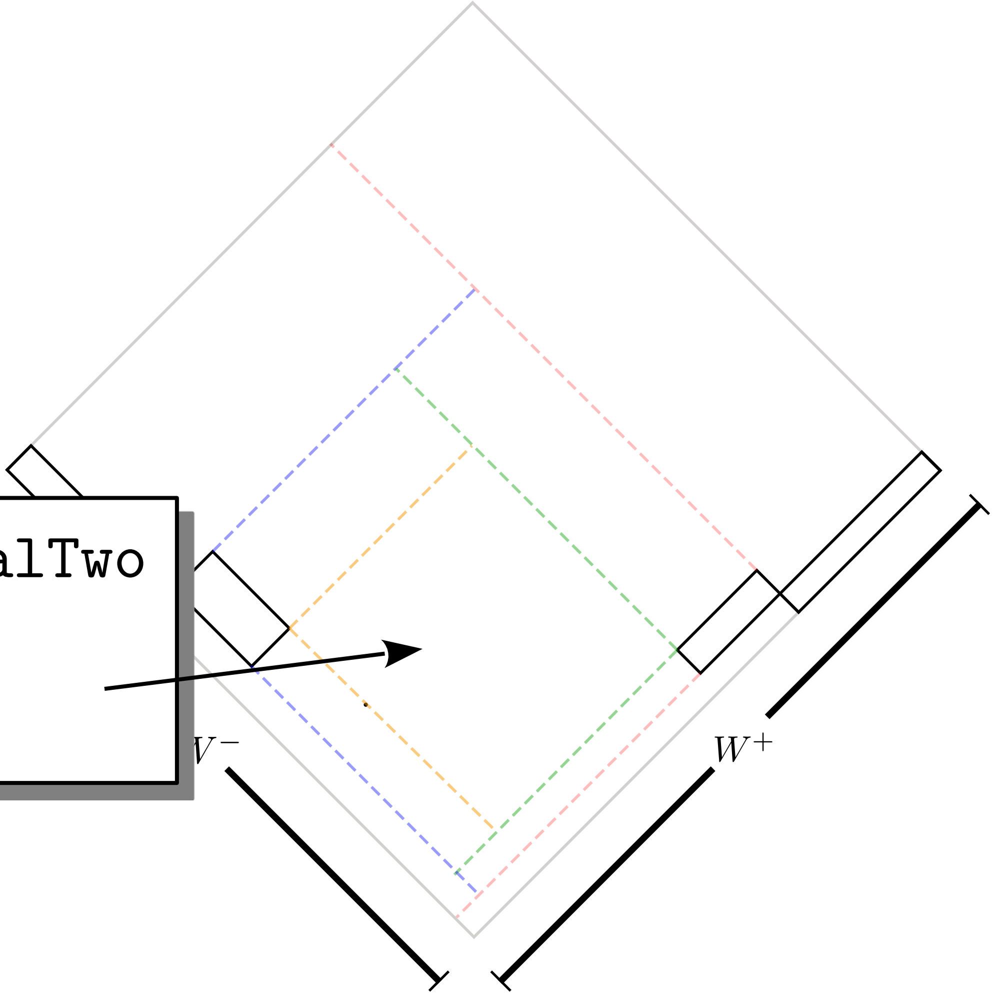
When E_{CM} goes below ~2 GeV, `finalTwo` is called. This can fail, when it does, the full string system is re-simulated from the beginning.



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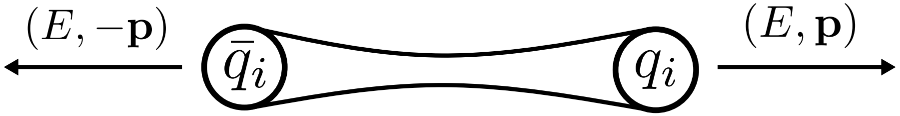

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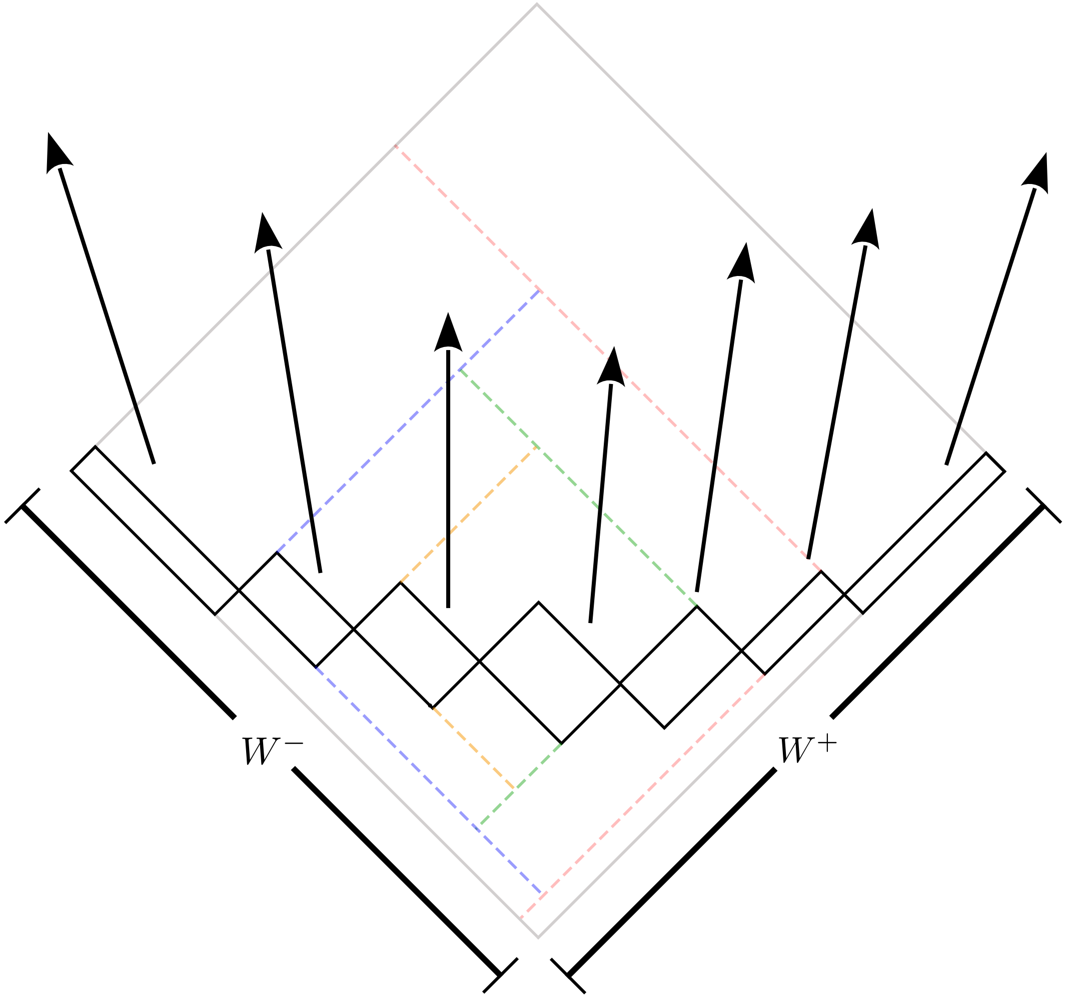
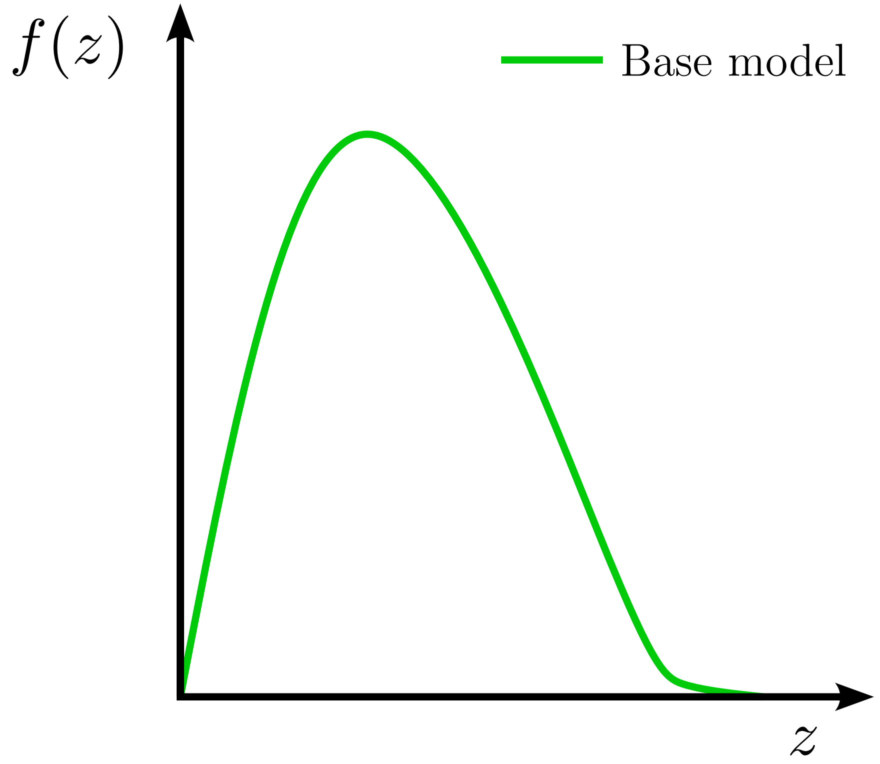


MLHAD efforts: big picture

Solving the “inverse problem of hadronization”



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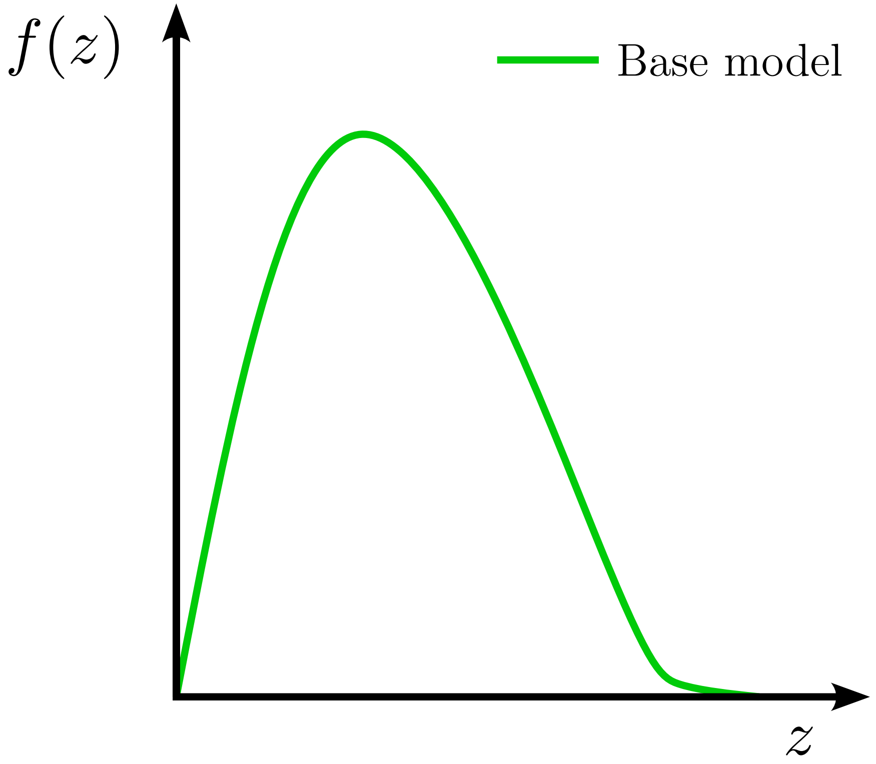

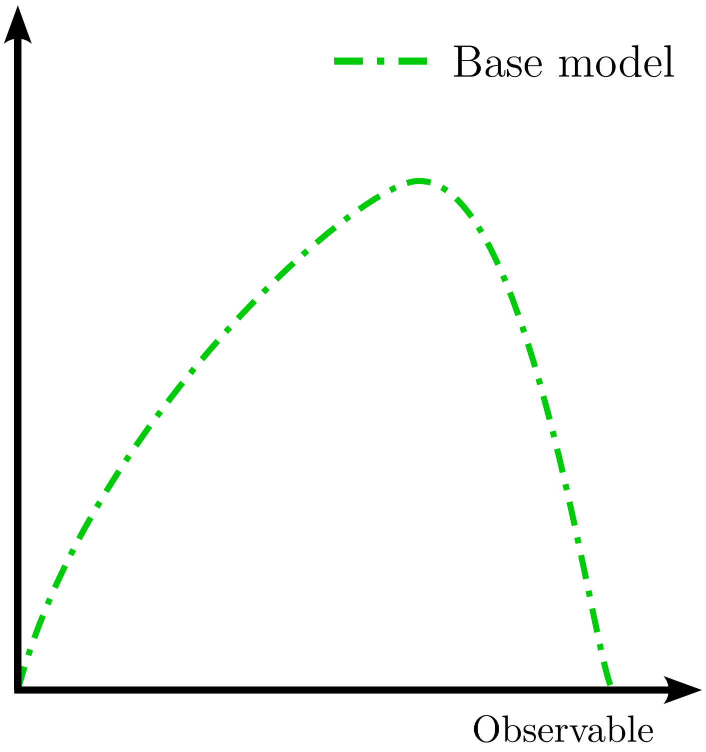

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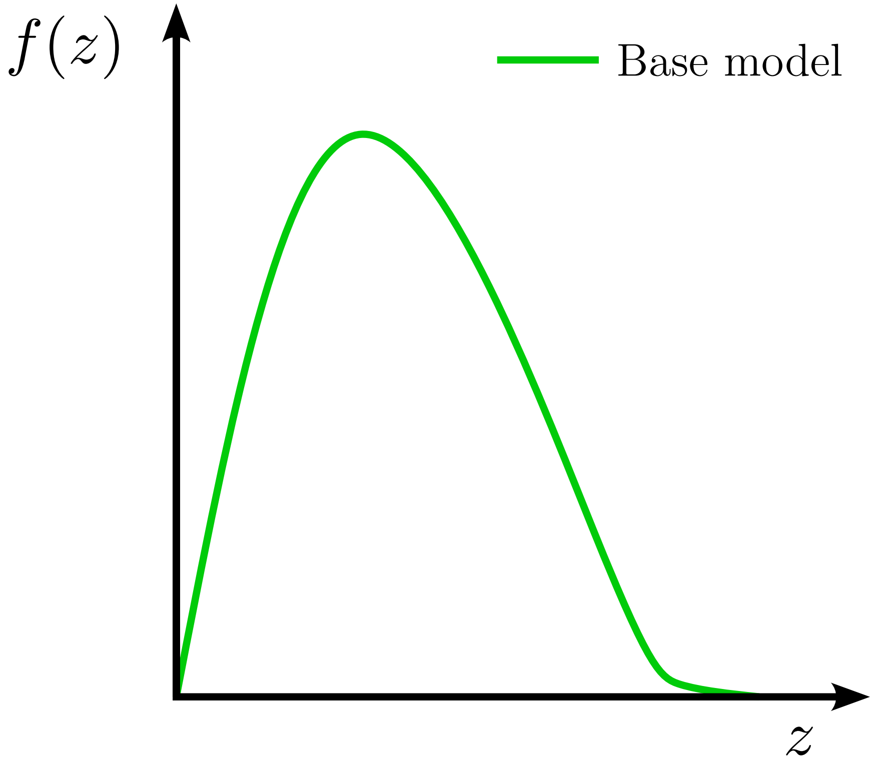

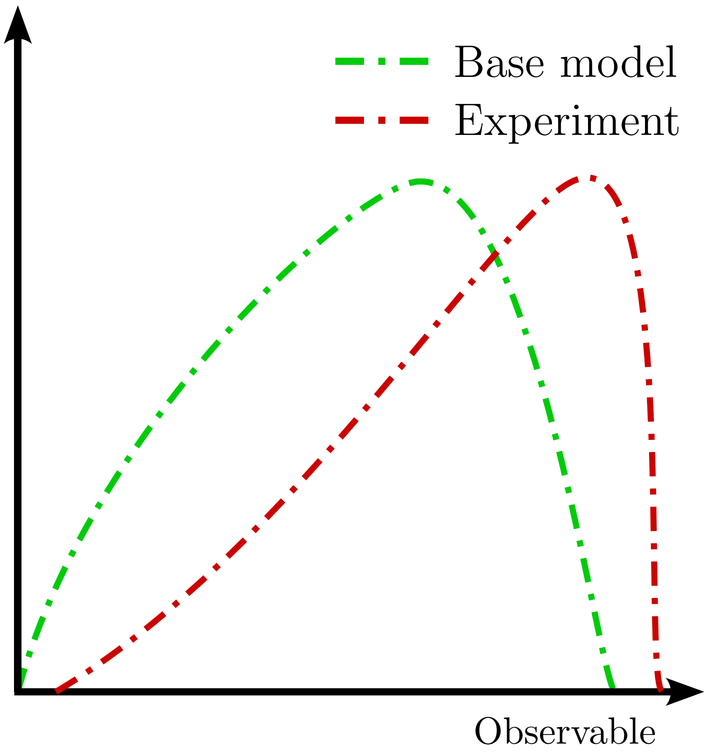

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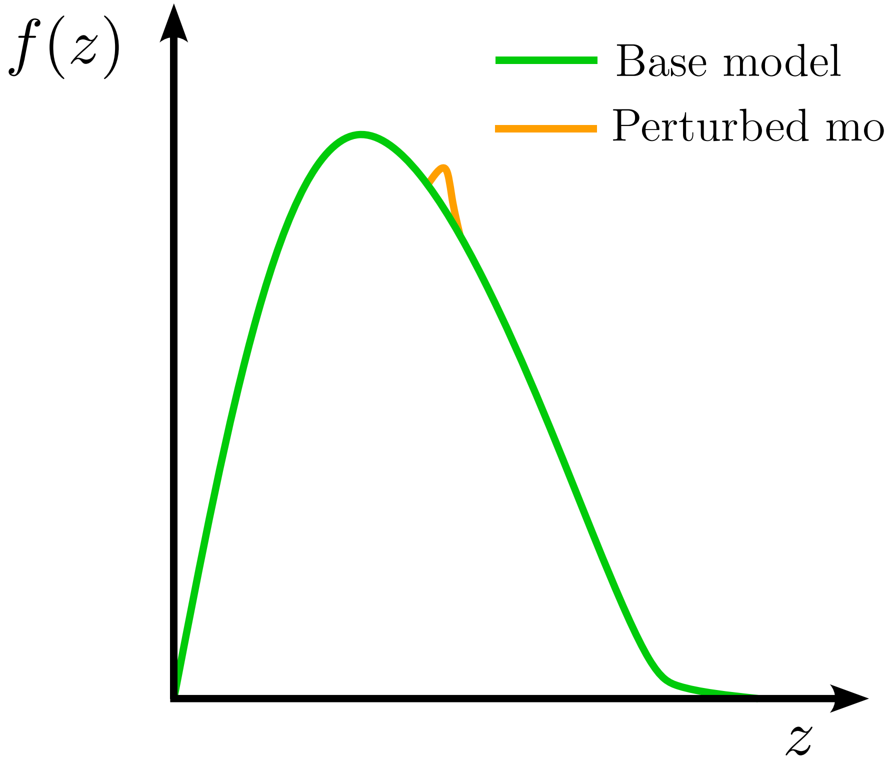

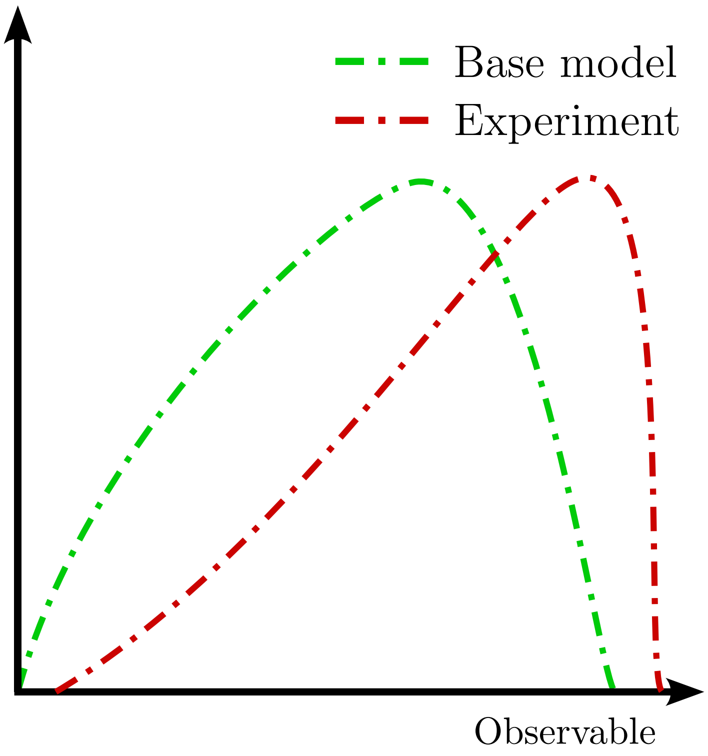

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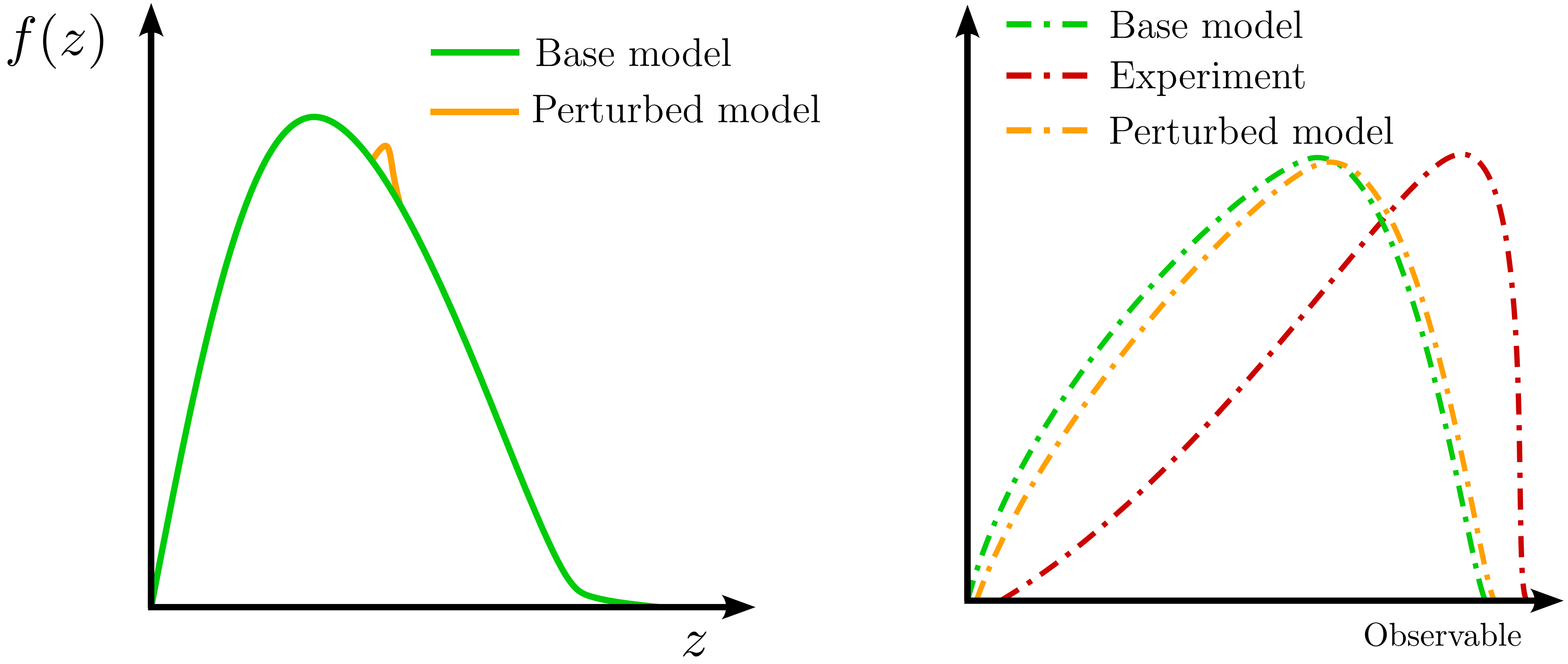

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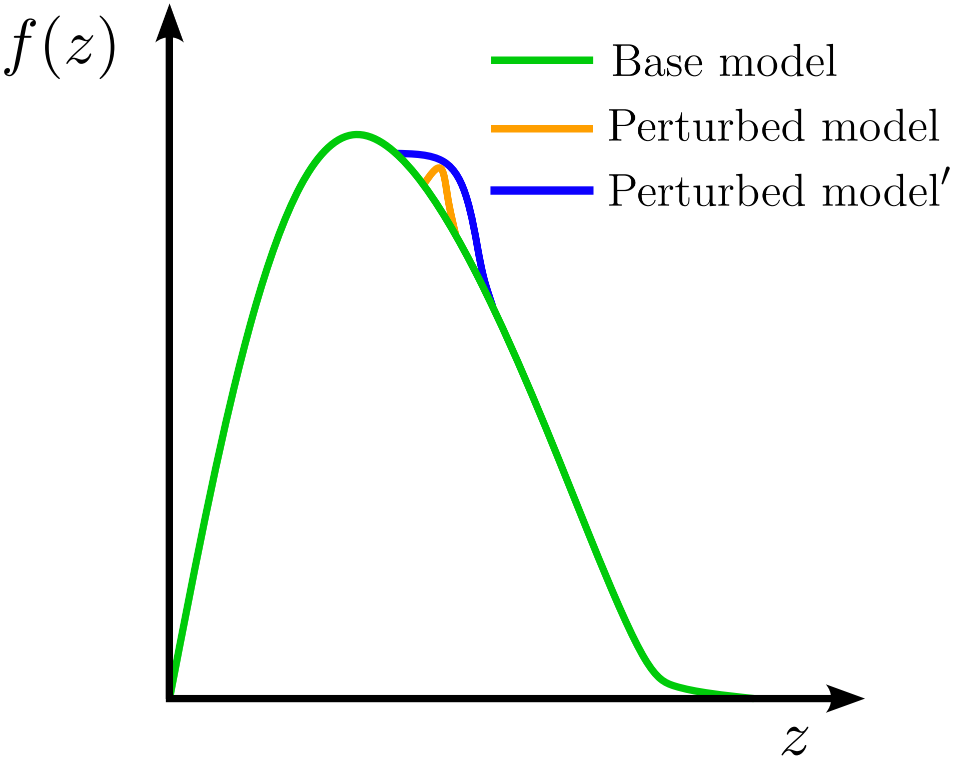

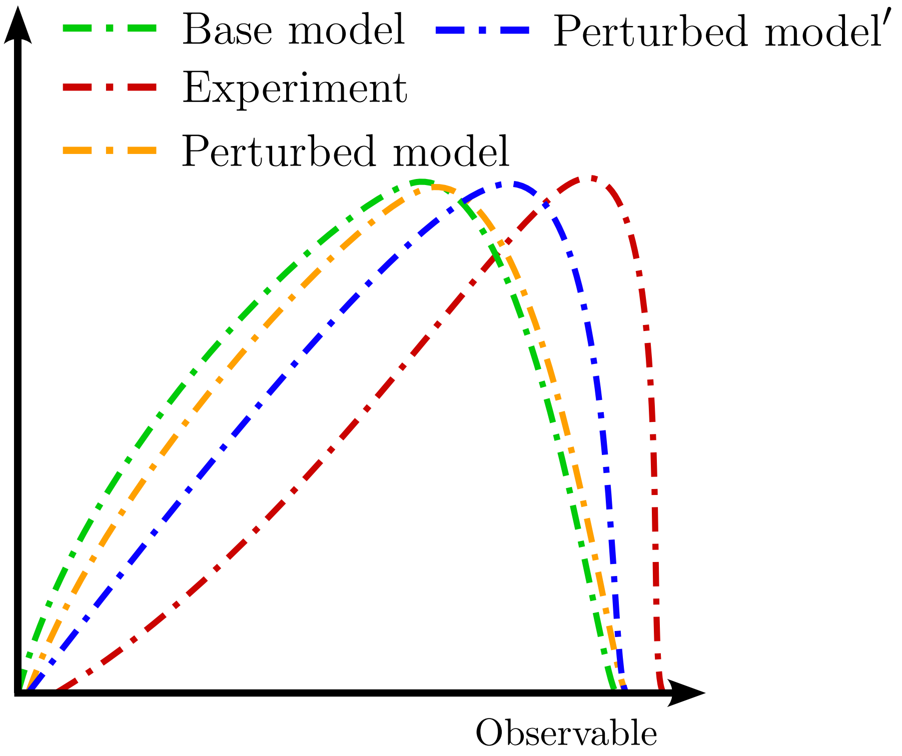

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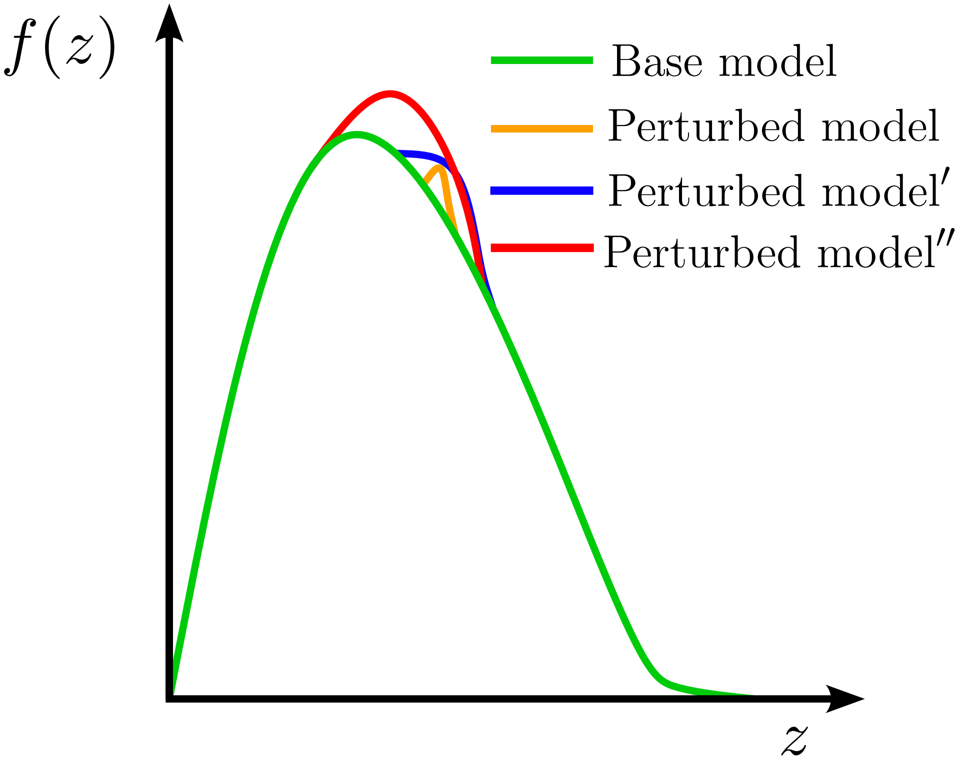

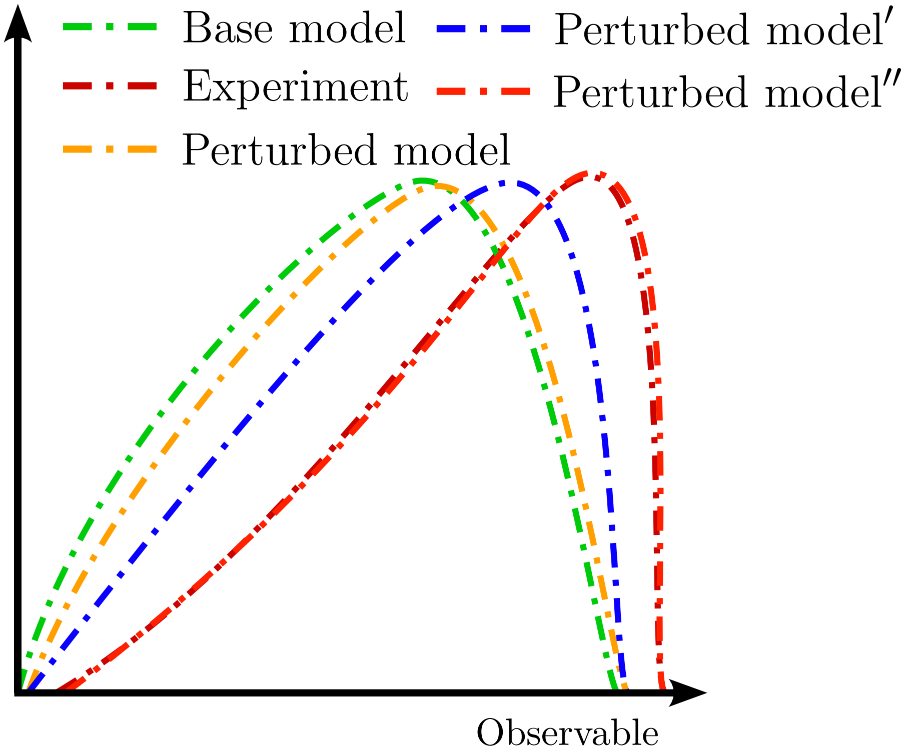

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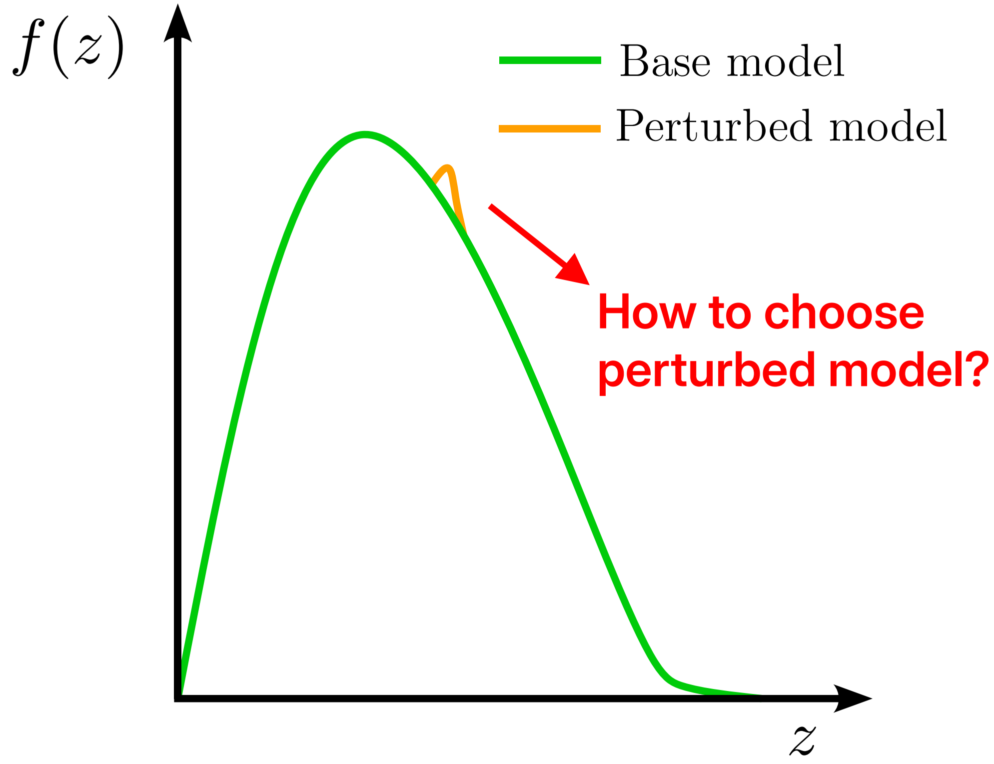

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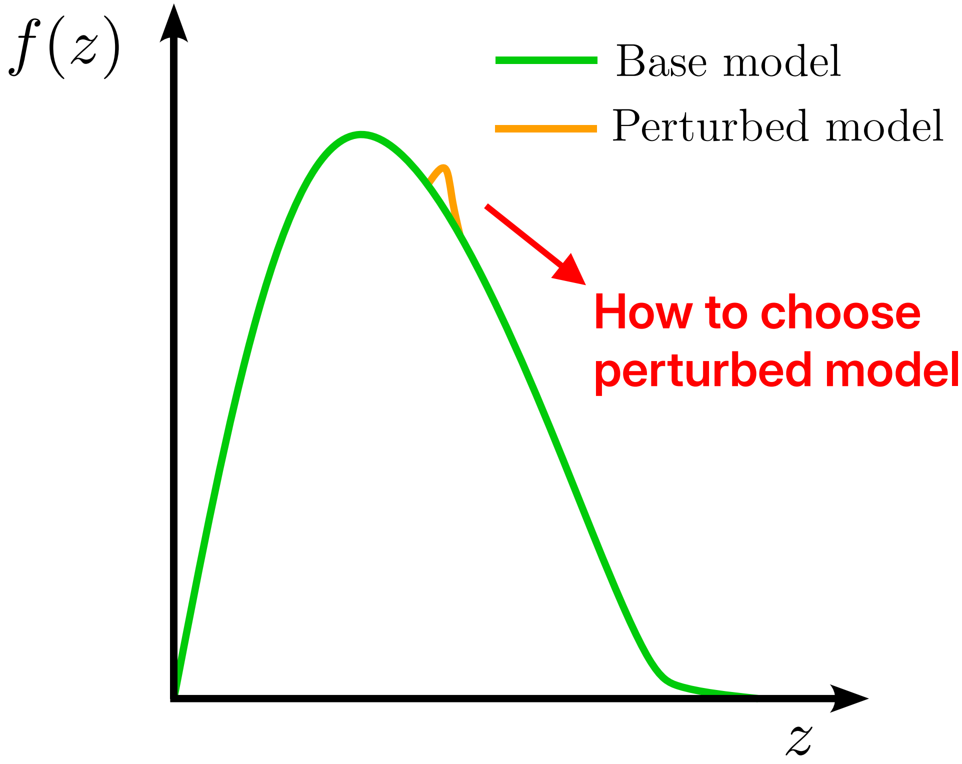


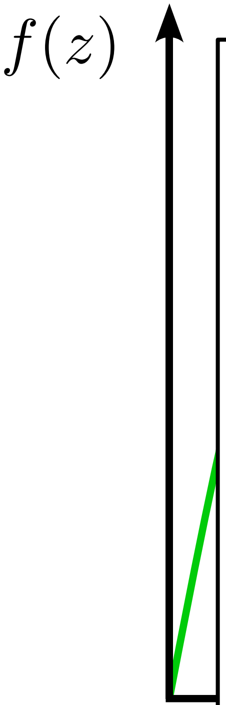
How to choose perturbed model?

~~ML~~ vs ML

- Vary Lund parameters (traditional "manual"/semi-"manual" tuning)
- ML-based (data-driven) fragmentation function
  - MAGIC, HOMER
- Hybrid – keep Lund fragmentation function, promote Lund parameters to differentiable objects
  - Rejection sampling with autodifferentiation (RSA)

MLHAD efforts: big picture

Solving the "inverse problem of hadronization"

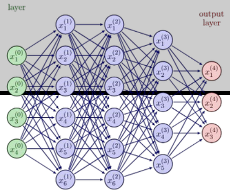


Machine learning

Form of universal function approximation.

Inputs → Model $\equiv f(\vec{x};\theta)$ → Prediction

Data ↕ Prediction



ML

- ML-based (data-driven) fragmentation function
  - MAGIC, HOMER

Lund
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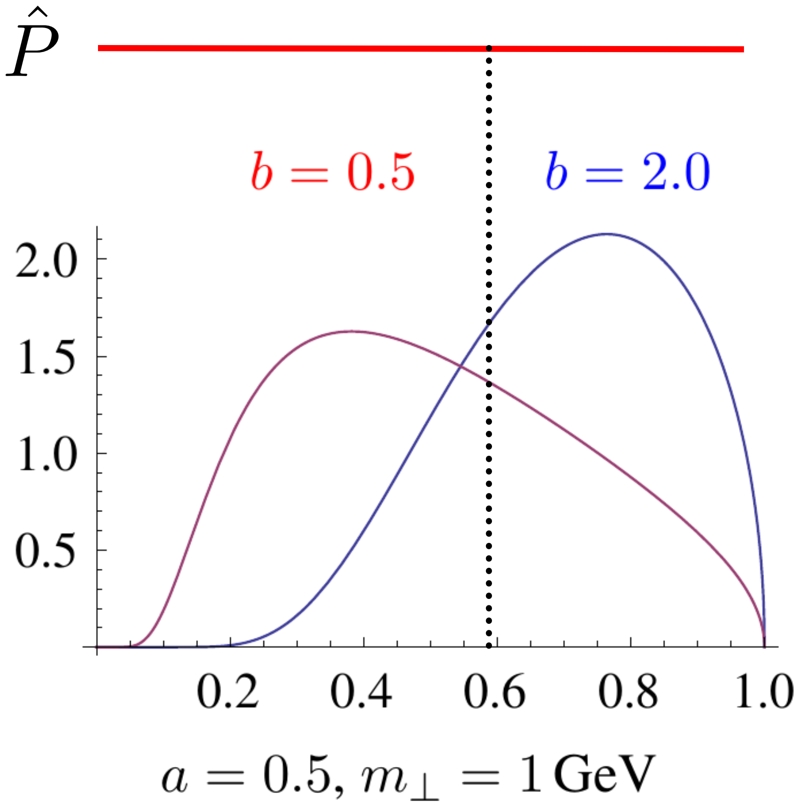
Kinematic reweighting (2308.13459)



Rejection sampling:

Acceptance probability: $P_{\mathrm{accept}} = \dfrac{p(z,\theta)}{\hat{P}}$

Rejection probability: $P_{\mathrm{reject}} = 1 - P_{\mathrm{accept}}$

$$w_{\mathrm{accept}} = \frac{p(z;\theta')}{p(z;\theta)}$$

$$w_{\mathrm{reject}} = \frac{\hat{P} - p(z;\theta')}{\hat{P} - p(z;\theta)}$$

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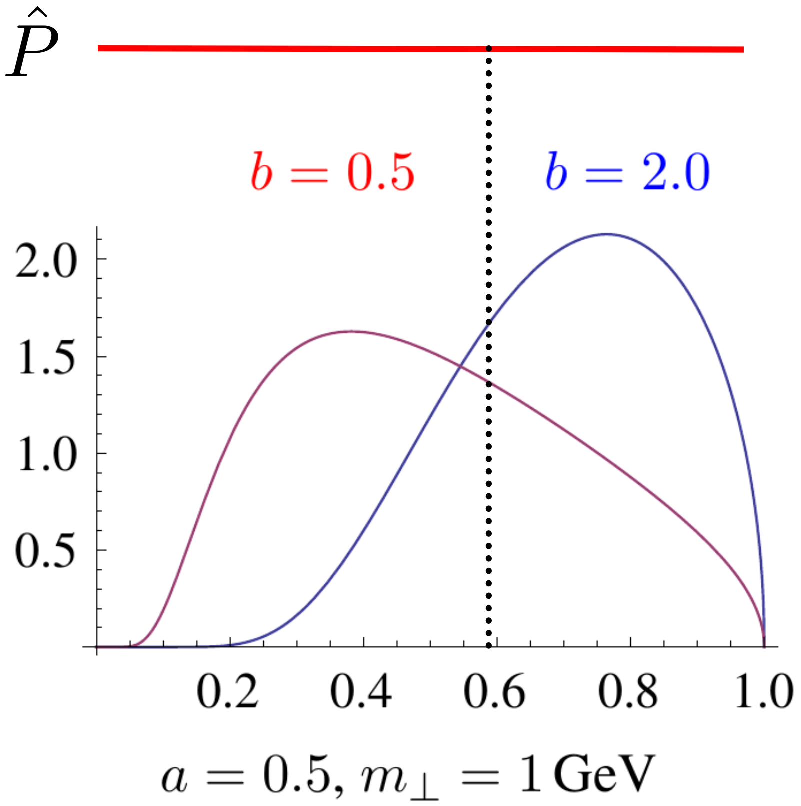

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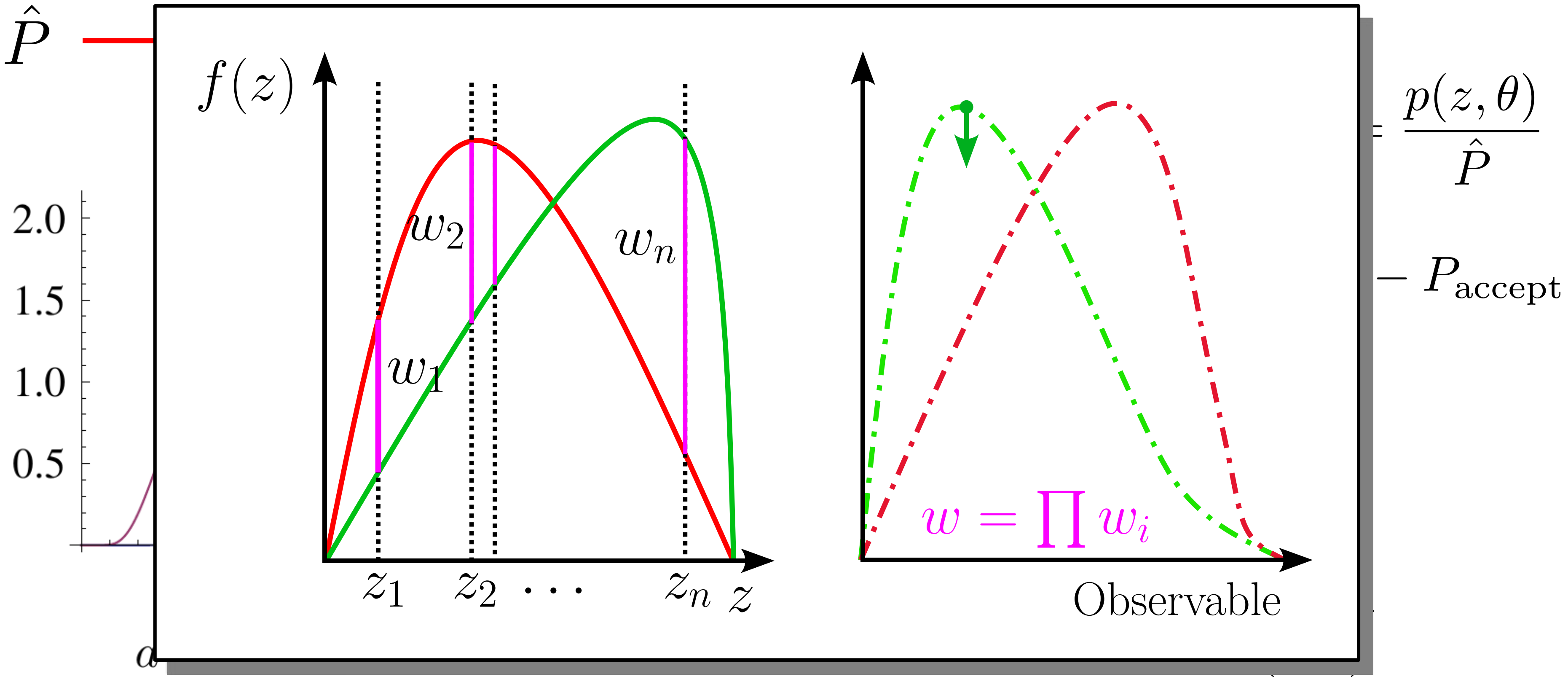


Kinematic reweighting (2308.13459)

$\hat{P}$

Rejection sampling:

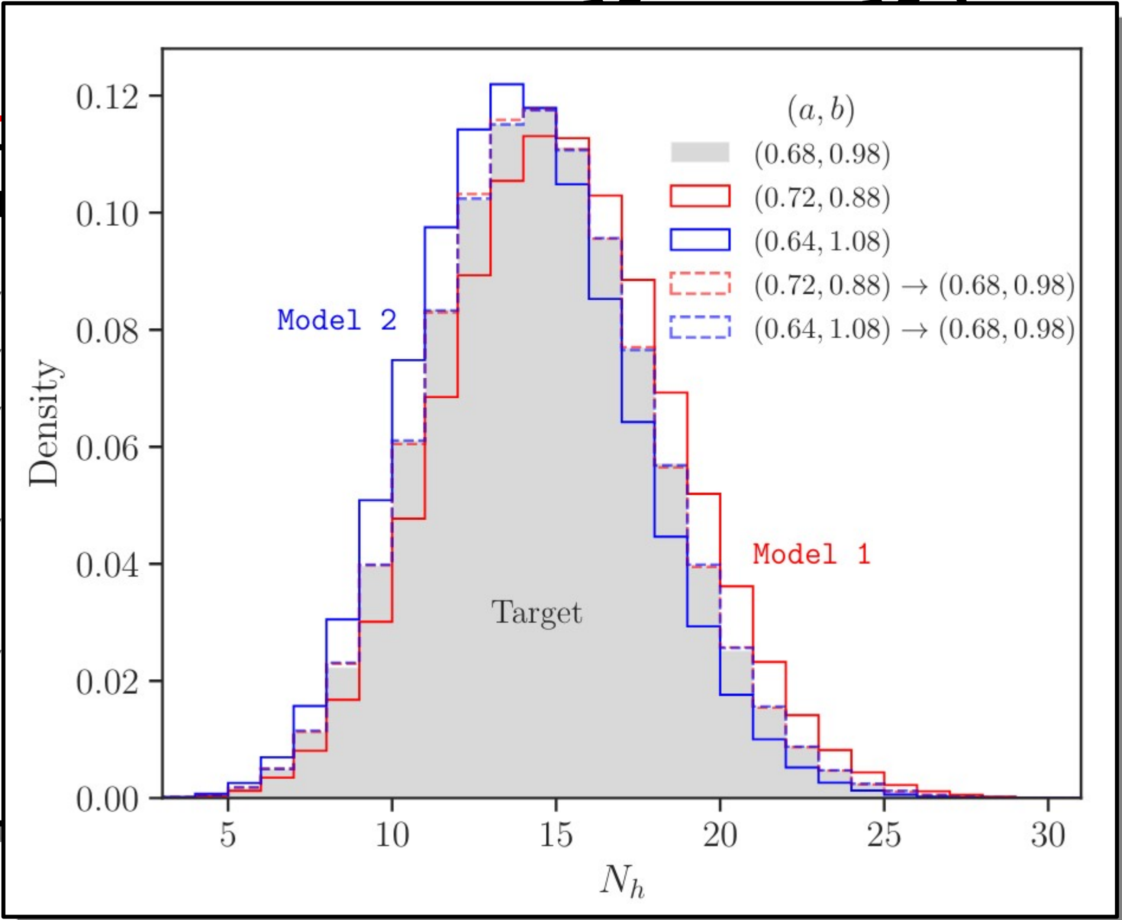
Data-structure:

$$\boldsymbol{z} = \begin{pmatrix} \boldsymbol{z}_1 = \begin{pmatrix} \{m_T^{h_1}, z_{\text{accept}}^{h_1}, z_{\text{reject}}^{1,h_1}, \cdots, z_{\text{reject}}^{n_{h_1},h_1}\} \\ \{m_T^{h_2}, z_{\text{accept}}^{h_2}, z_{\text{reject}}^{1,h_2}, \cdots, z_{\text{reject}}^{n_{h_2},h_2}\} \\ \{m_T^{h_3}, z_{\text{accept}}^{h_3}, z_{\text{reject}}^{1,h_3}, \cdots, z_{\text{reject}}^{n_{h_3},h_3}\} \end{pmatrix}_1 \\ \boldsymbol{z}_2 = \begin{pmatrix} \{m_T^{h_1}, z_{\text{accept}}^{h_1}, z_{\text{reject}}^{1,h_1}, \cdots, z_{\text{reject}}^{n_{h_1},h_1}\} \\ \vdots \\ \{m_T^{h_4}, z_{\text{accept}}^{h_4}, z_{\text{reject}}^{1,h_4}, \cdots, z_{\text{reject}}^{n_{h_4},h_4}\} \end{pmatrix}_2 \\ \vdots \\ \boldsymbol{z}_N = \cdots \end{pmatrix}$$

$$w_n = \prod_{i=1}^{\tilde{N}_{h,n}} \left(\frac{f(z_{\text{accept}}^{h_i}; \{a,b\}_P)}{f(z_{\text{accept}}^{h_i}; \{a,b\}_B)} \right) \times \prod_{j=1}^{n_{h_i}} \left(\frac{\hat{f} - f(z_{\text{reject}}^{j,h_i}; \{a,b\}_P)}{\hat{f} - f(z_{\text{reject}}^{j,h_i}; \{a,b\}_B)} \right)$$

Kinematic reweighting (2308.13459)

Data-structu



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\end{pmatrix}_1 \\
\\
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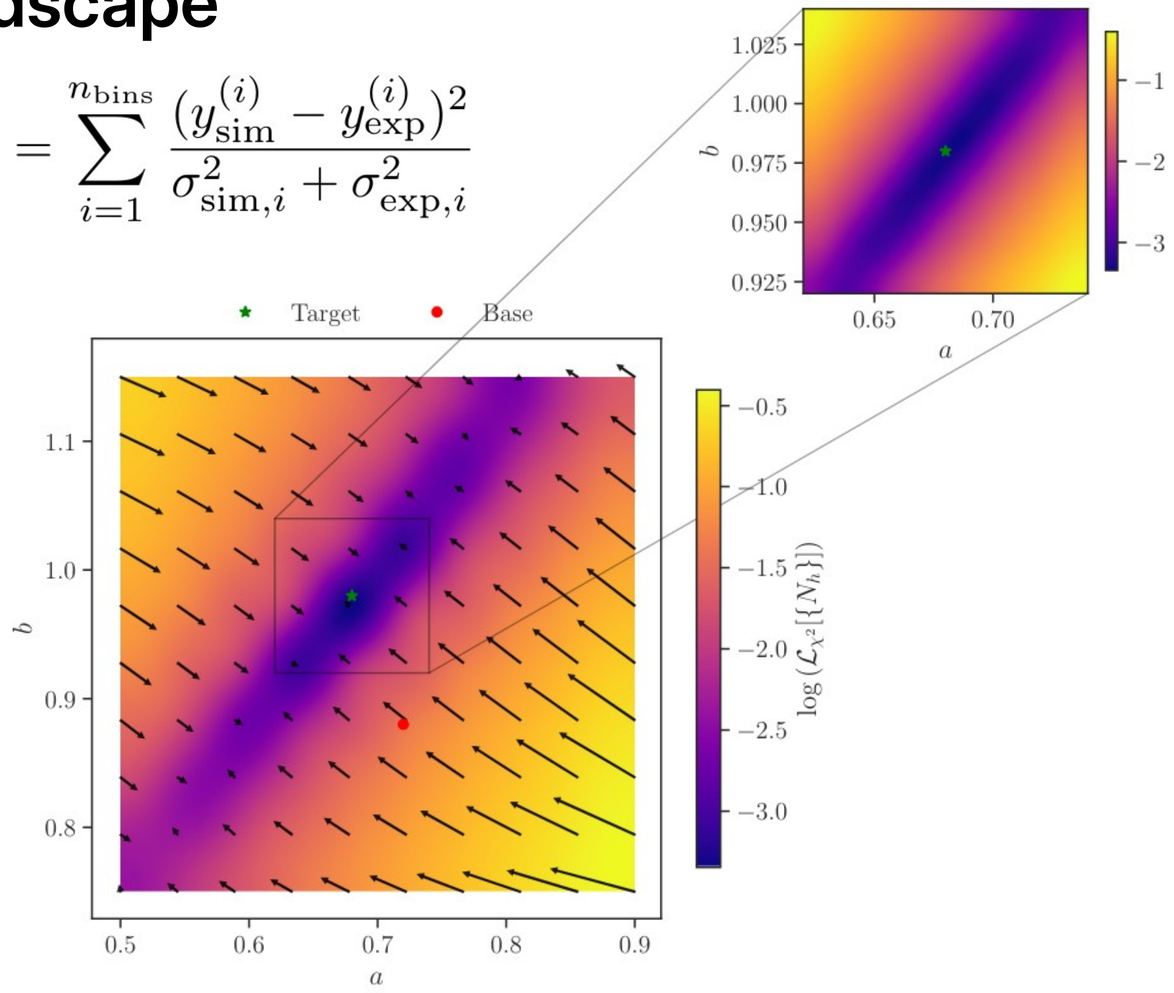
\vdots \\
\{m_T^{h_4}, z_{\text{accept}}^{h_4}, z_{\text{reject}}^{1,h_4}, \cdots, z_{\text{reject}}^{n_{h_4},h_4}\}
\end{pmatrix}_2 \\
\vdots \\
\boldsymbol{z}_N = \cdots
\end{pmatrix}
$$

Differentiable! → RSA

$$
w_n = \prod_{i=1}^{\tilde{N}_{h,n}} \left(\frac{f(z_{\text{accept}}^{h_i}; \{a,b\}_P)}{f(z_{\text{accept}}^{h_i}; \{a,b\}_B)} \right. \times \left. \prod_{j=1}^{n_{h_i}} \left(\frac{\hat{f} - f(z_{\text{reject}}^{j,h_i}; \{a,b\}_P)}{\hat{f} - f(z_{\text{reject}}^{j,h_i}; \{a,b\}_B)} \right) \right)
$$

χ^2-loss landscape

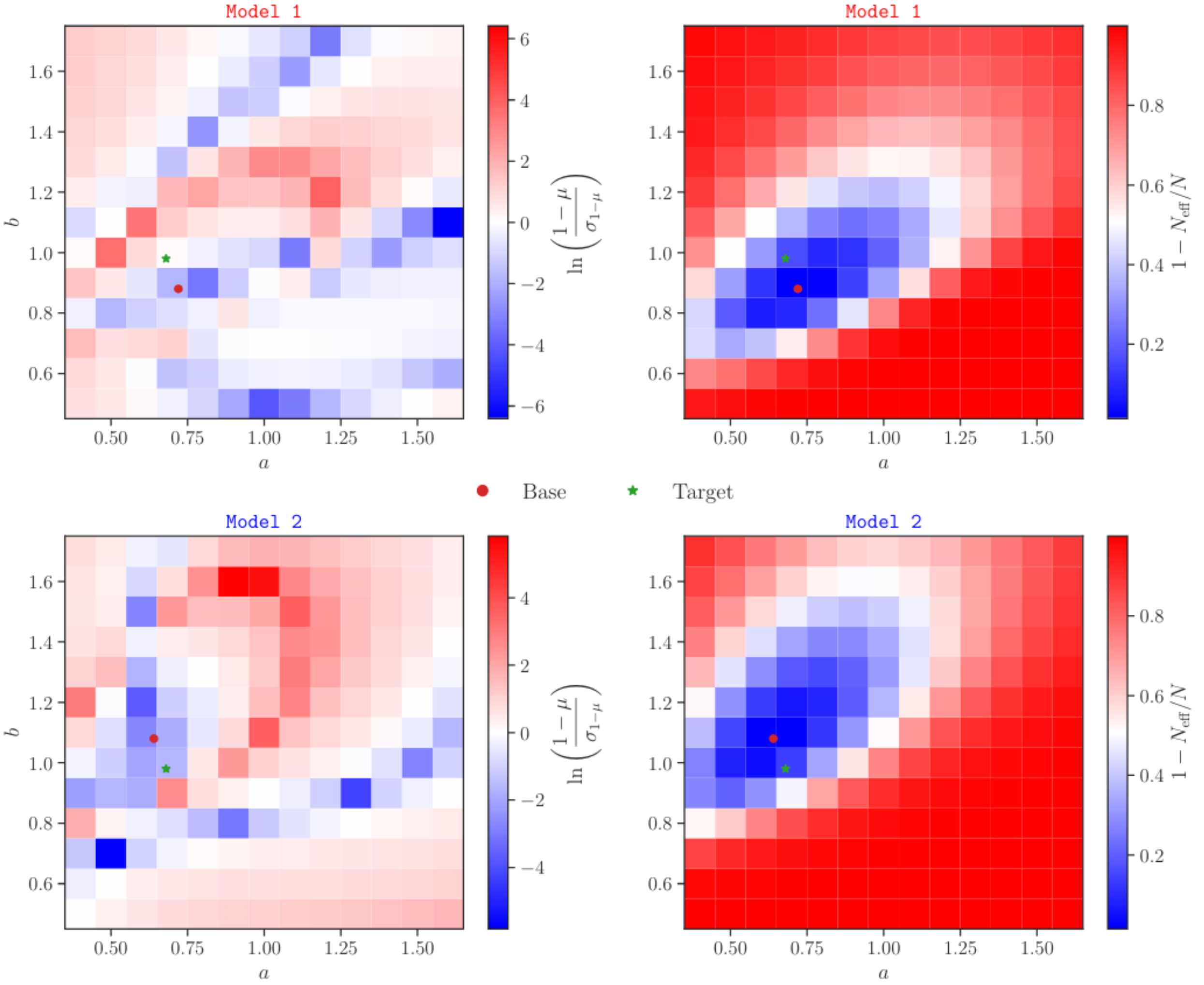
$$\mathcal{L}_{\chi^2}(\boldsymbol{y}_{\rm sim}, \boldsymbol{y}_{\rm exp}; \boldsymbol{w}) = \sum_{i=1}^{n_{\rm bins}} \frac{(y_{\rm sim}^{(i)} - y_{\rm exp}^{(i)})^2}{\sigma_{{\rm sim},i}^2 + \sigma_{{\rm exp},i}^2}$$



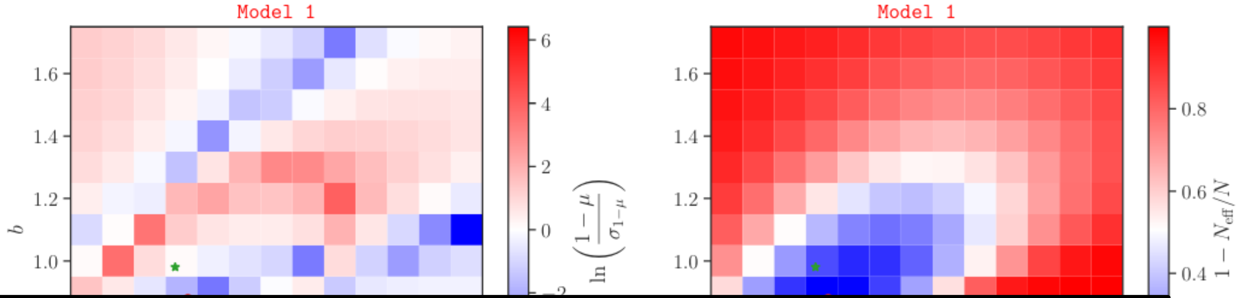
No free lunch

Statistical power drops off quickly as you move away from the base parameterization

$$\mu \equiv \sum_{i=1}^{N} \frac{w_i}{N}, \qquad N_{\text{eff}} = \frac{\left(\sum_{i=1}^{N} w_i\right)^2}{\sum_{i=1}^{N} w_i^2}$$



No free lunch



Stati
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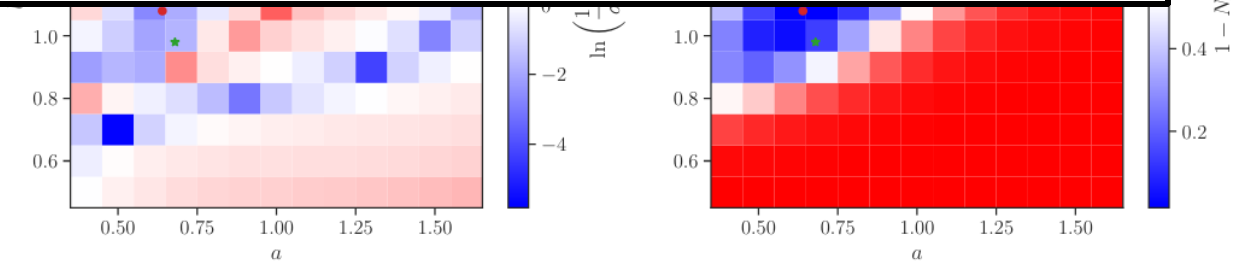
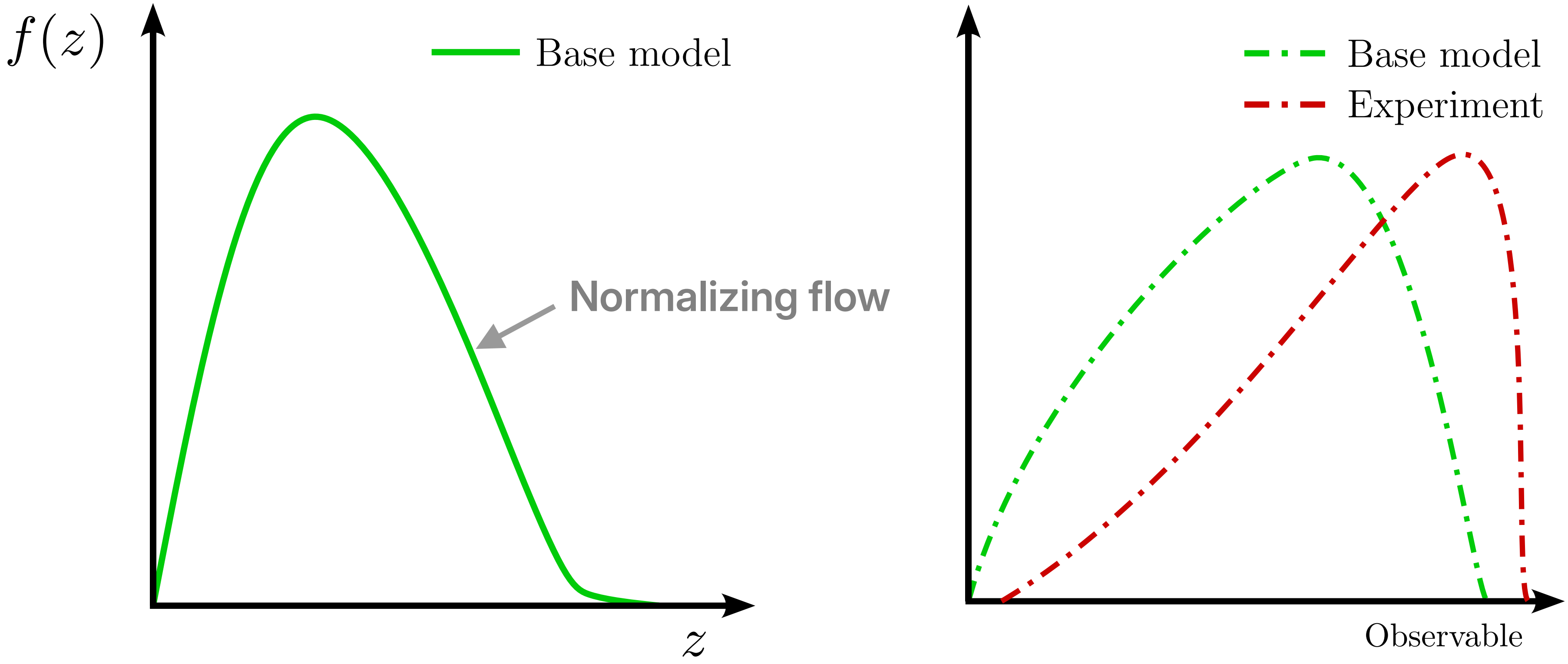
$\mu \equiv \sum_{i=}$

Road to differentiable Pythia?



MLHAD efforts: MAGIC



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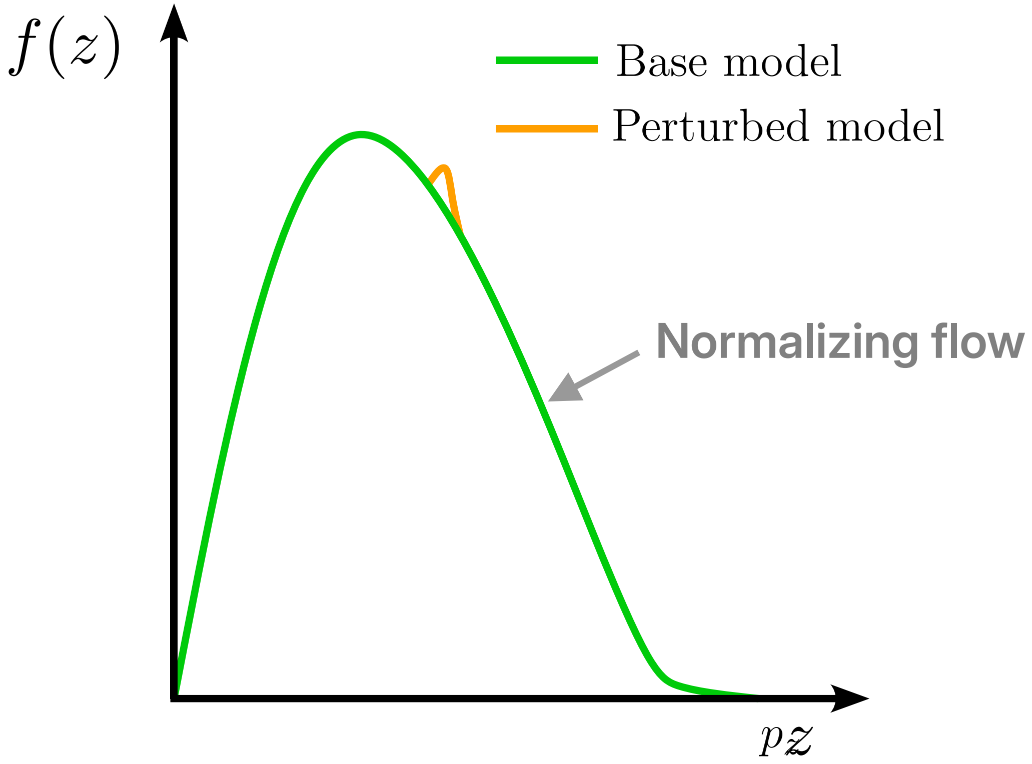

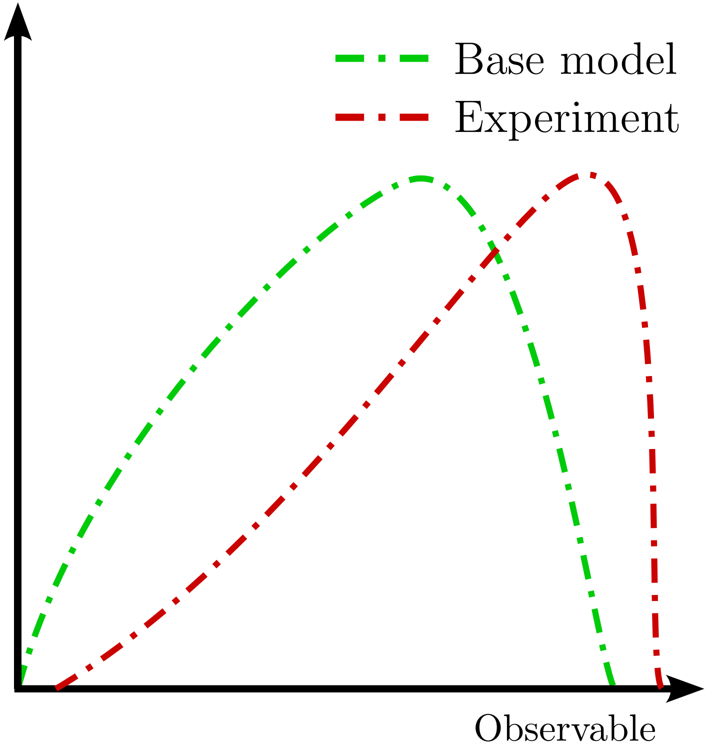

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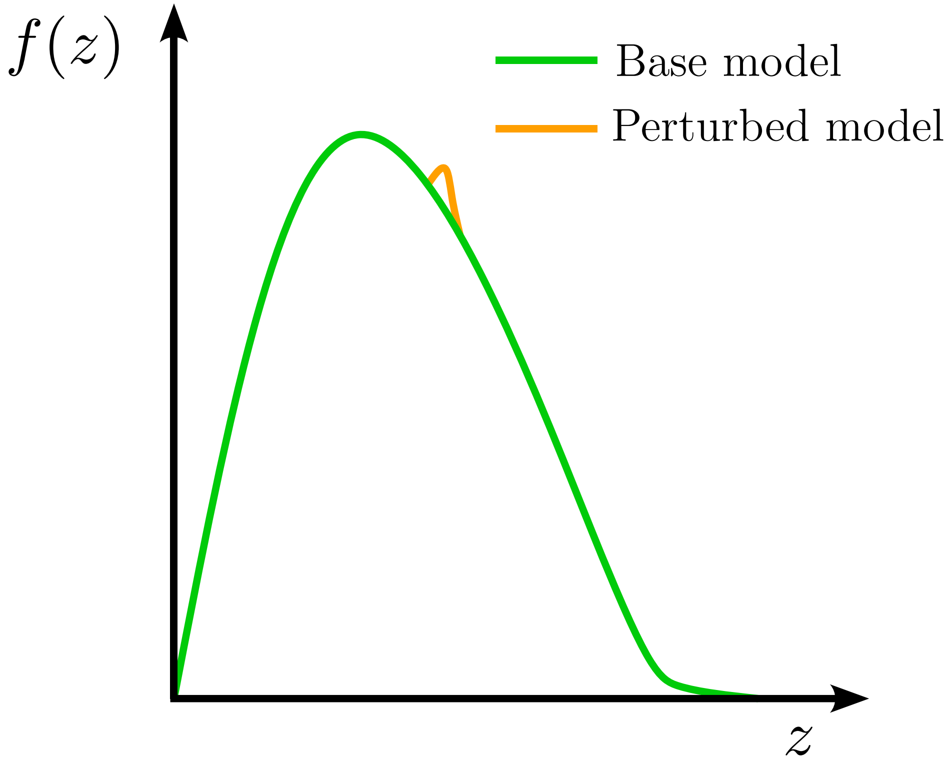

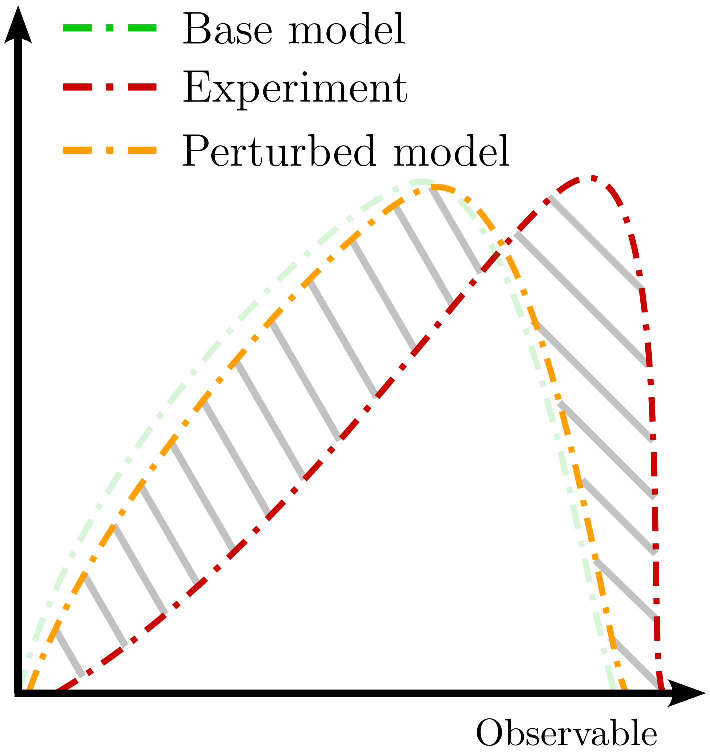

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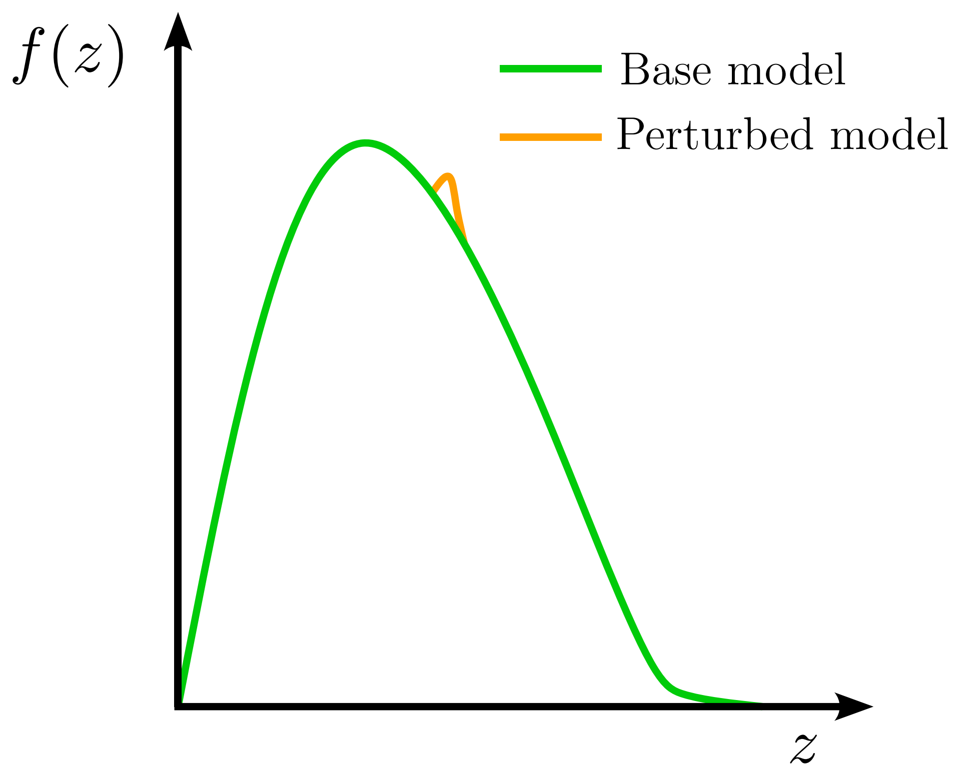

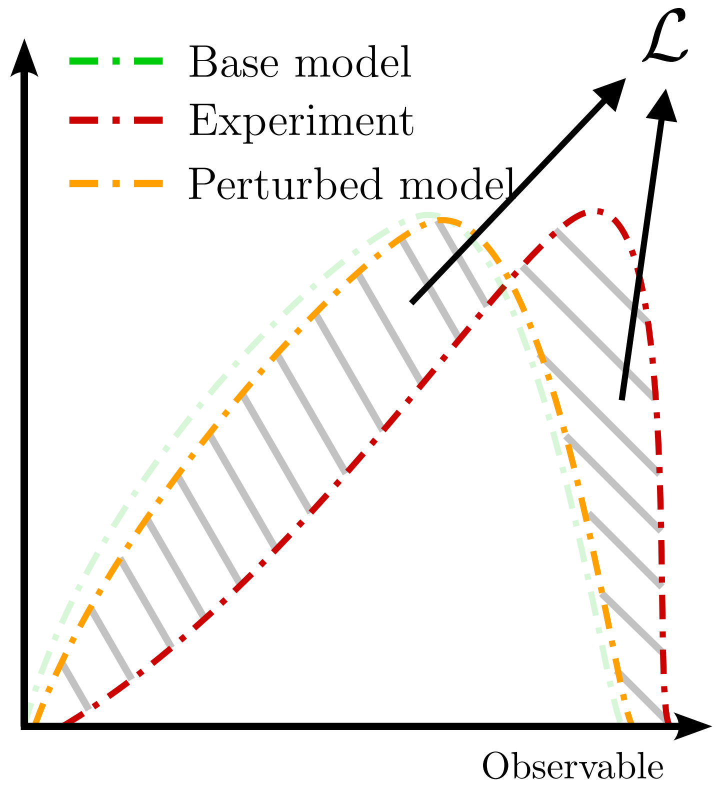

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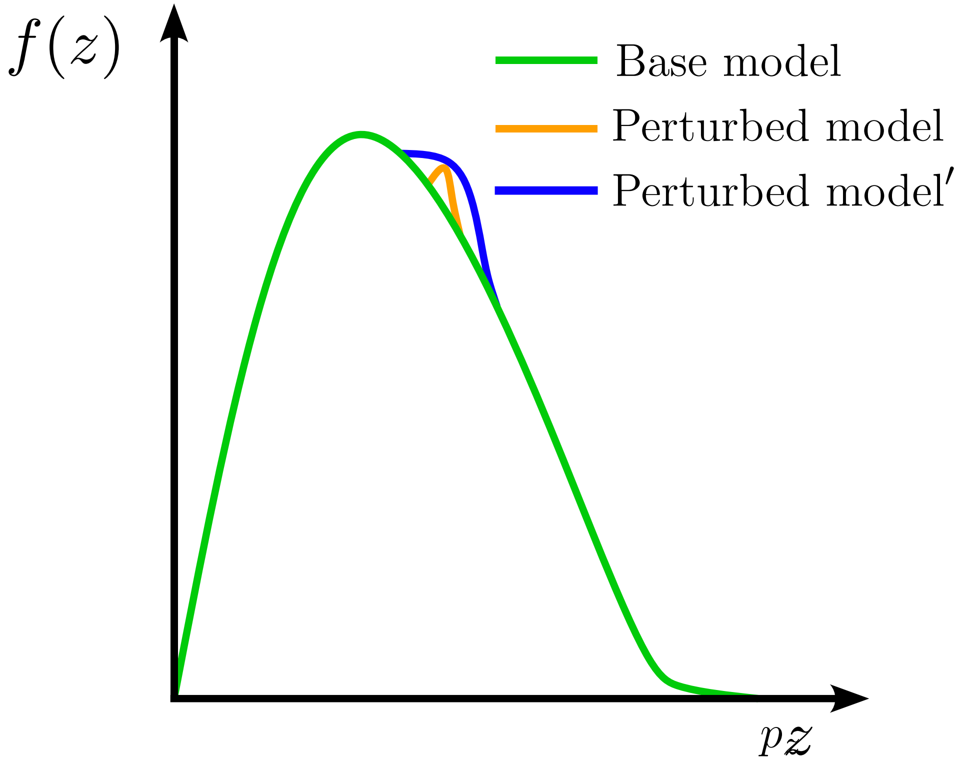
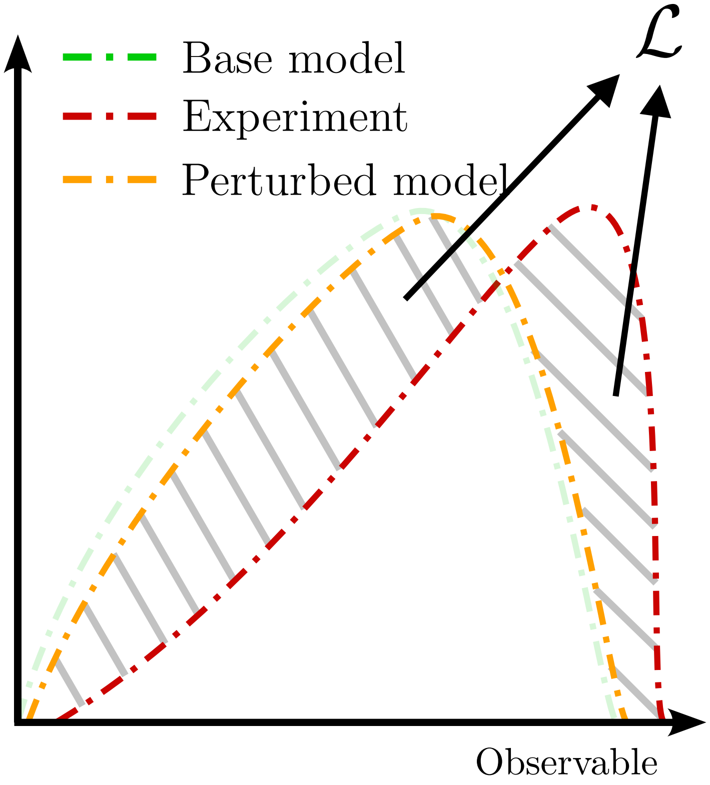
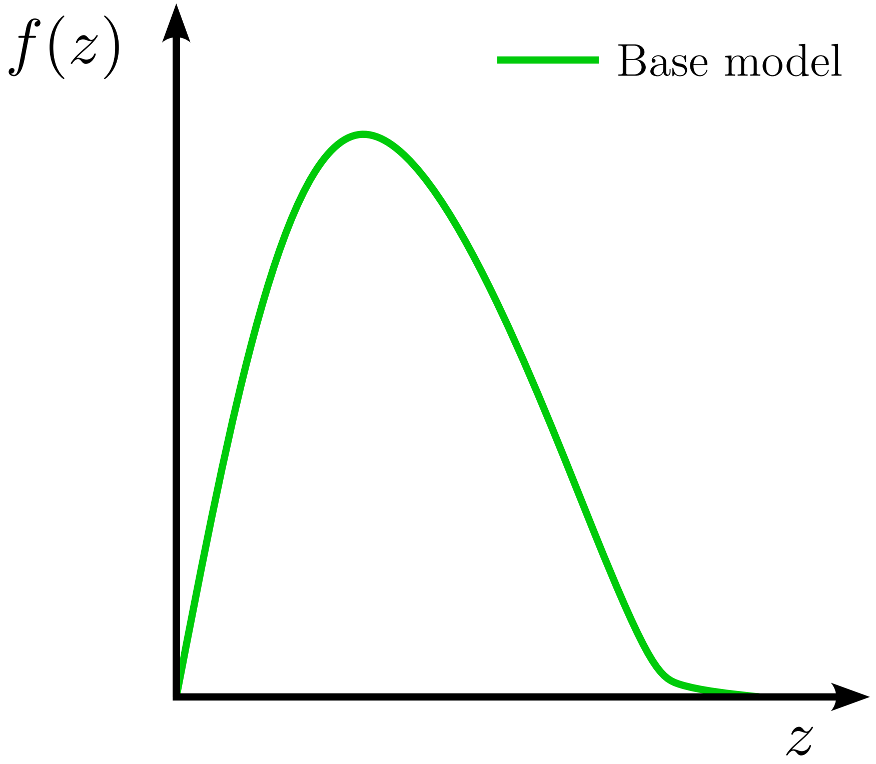
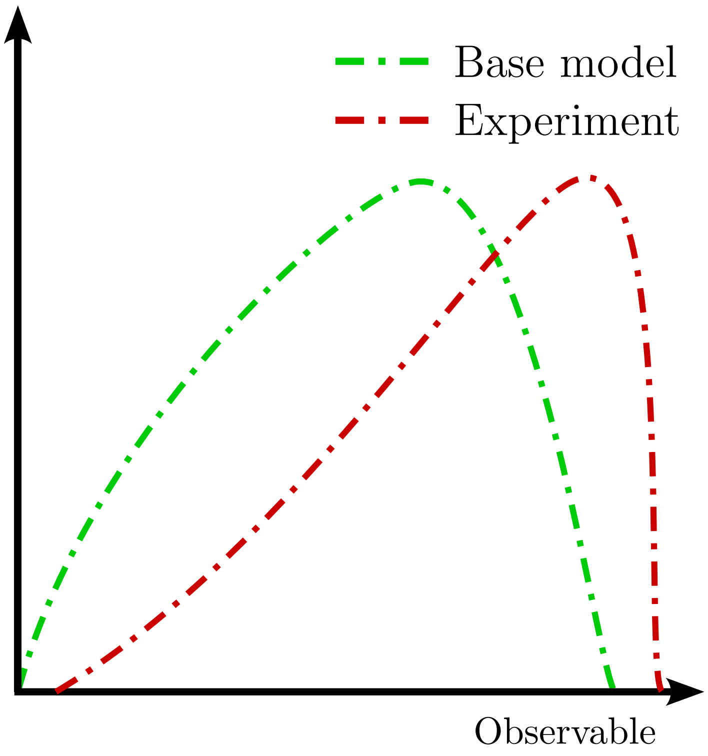




MLHAD efforts: HOMER





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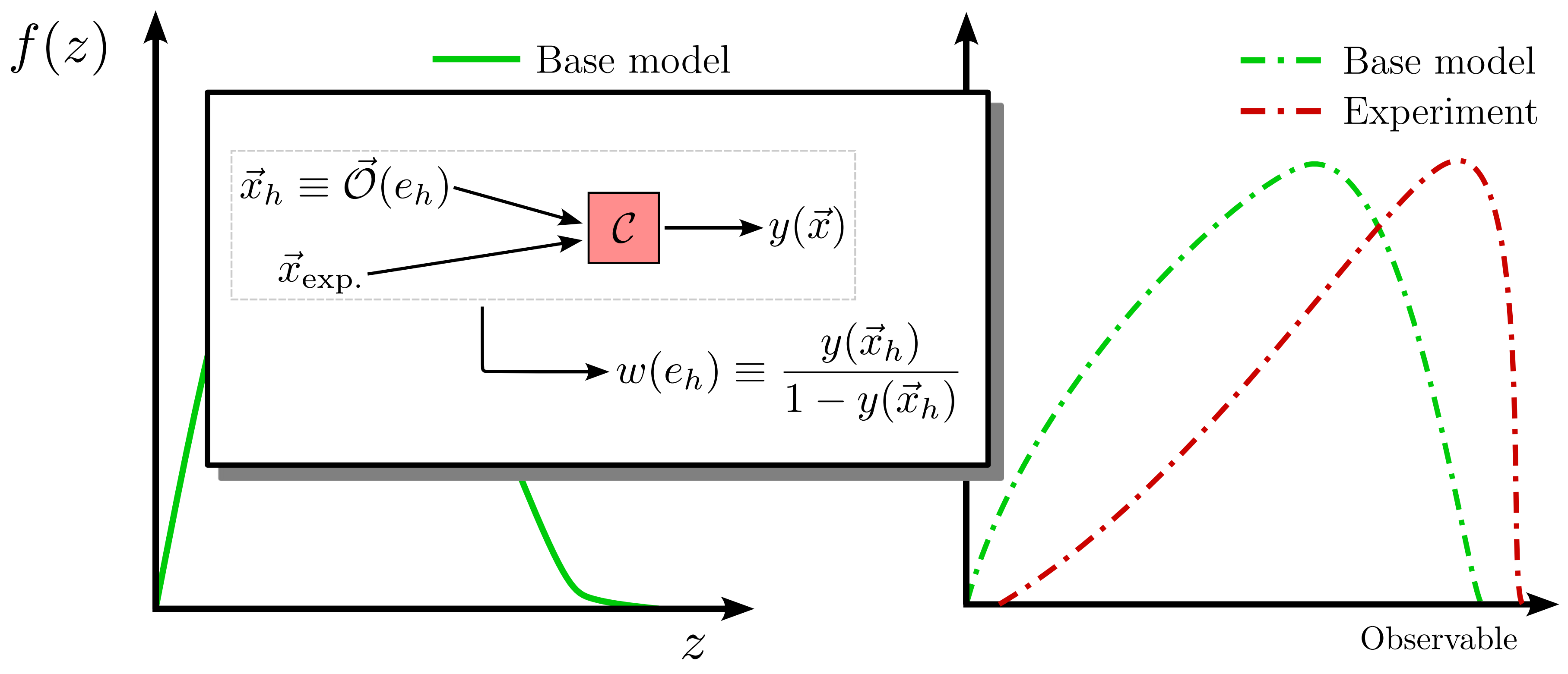

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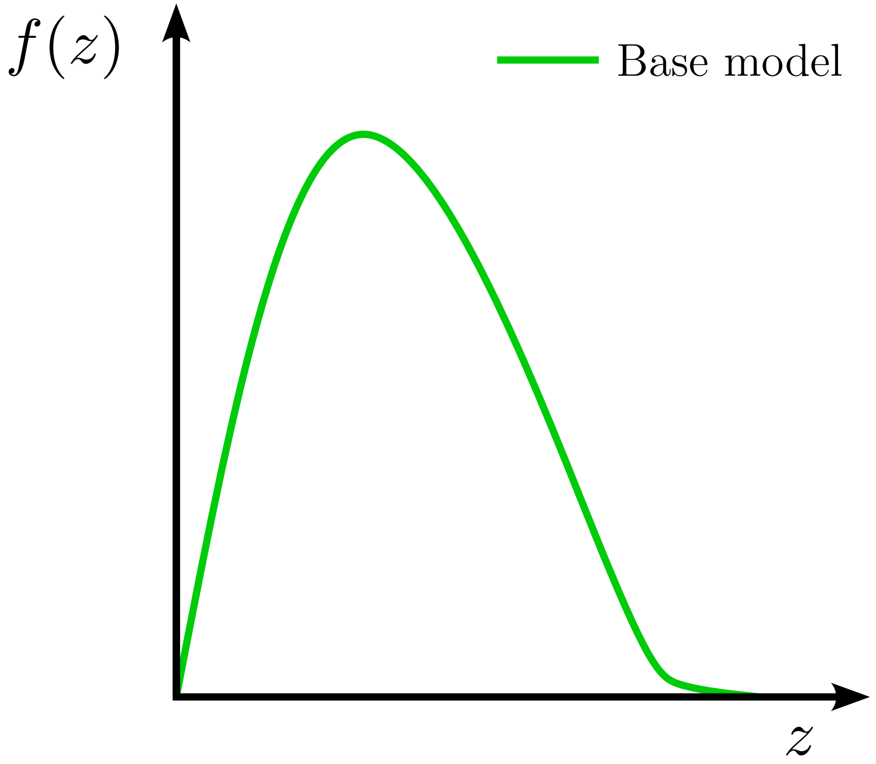
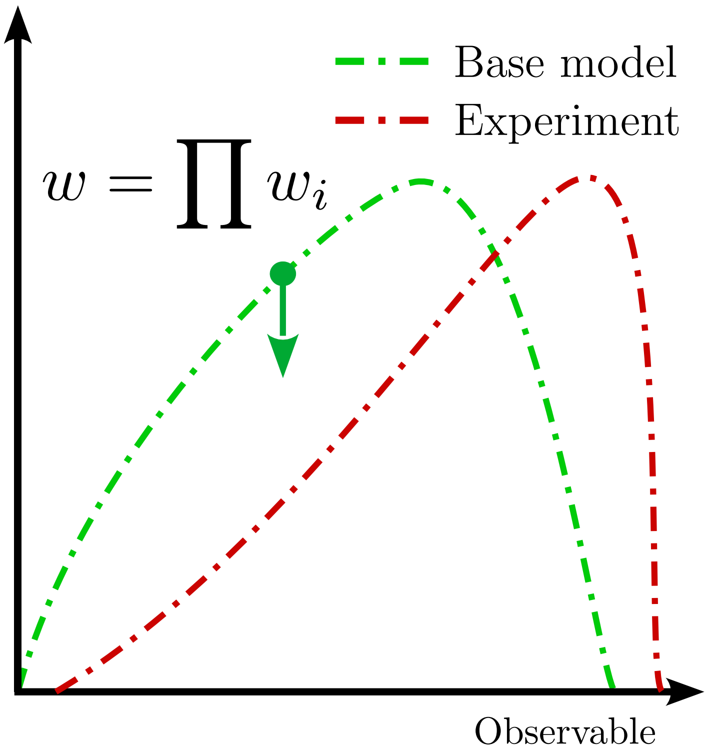
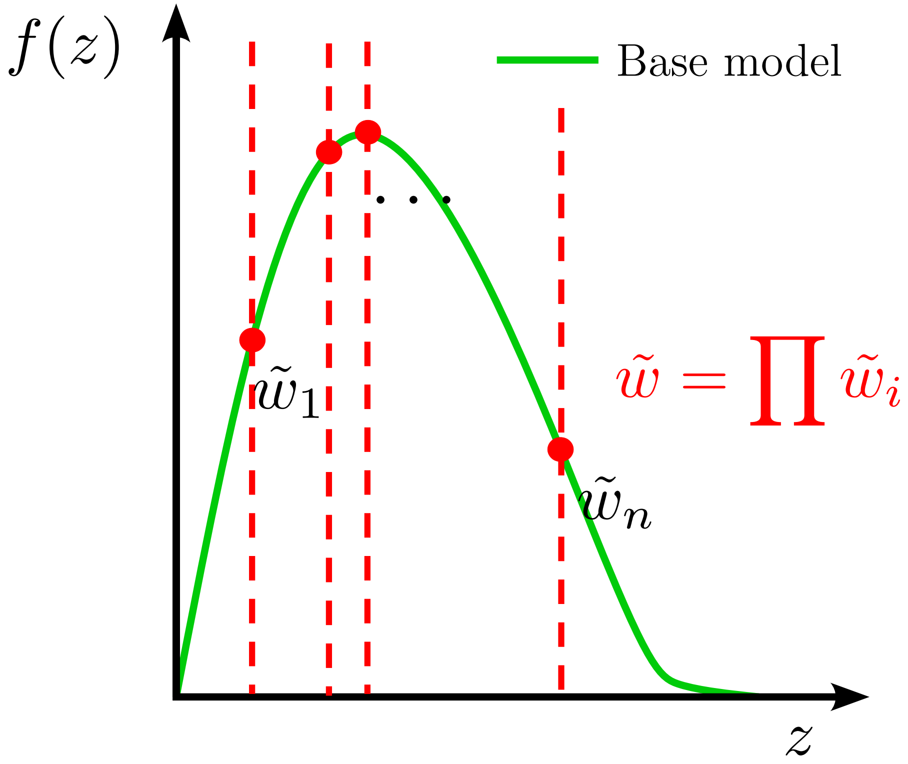
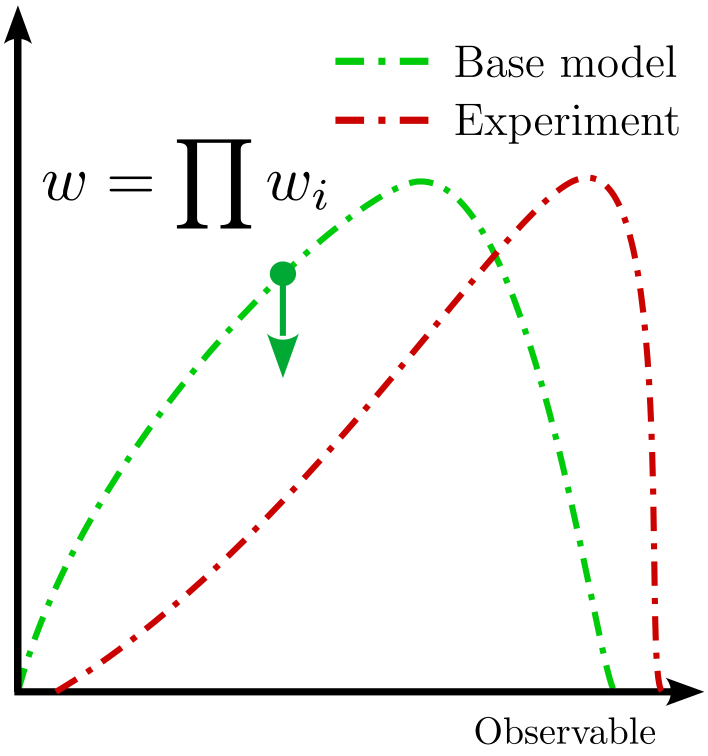




MLHAD efforts: HOMER





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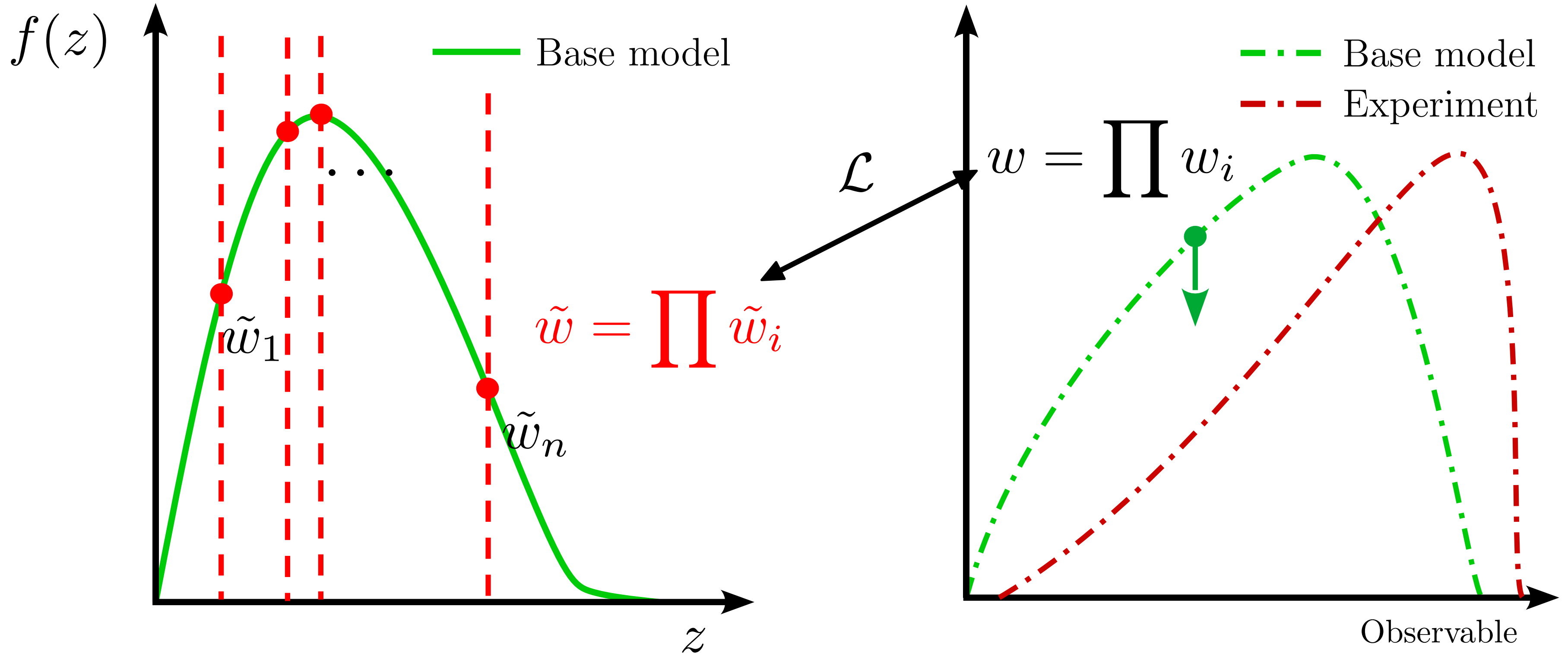

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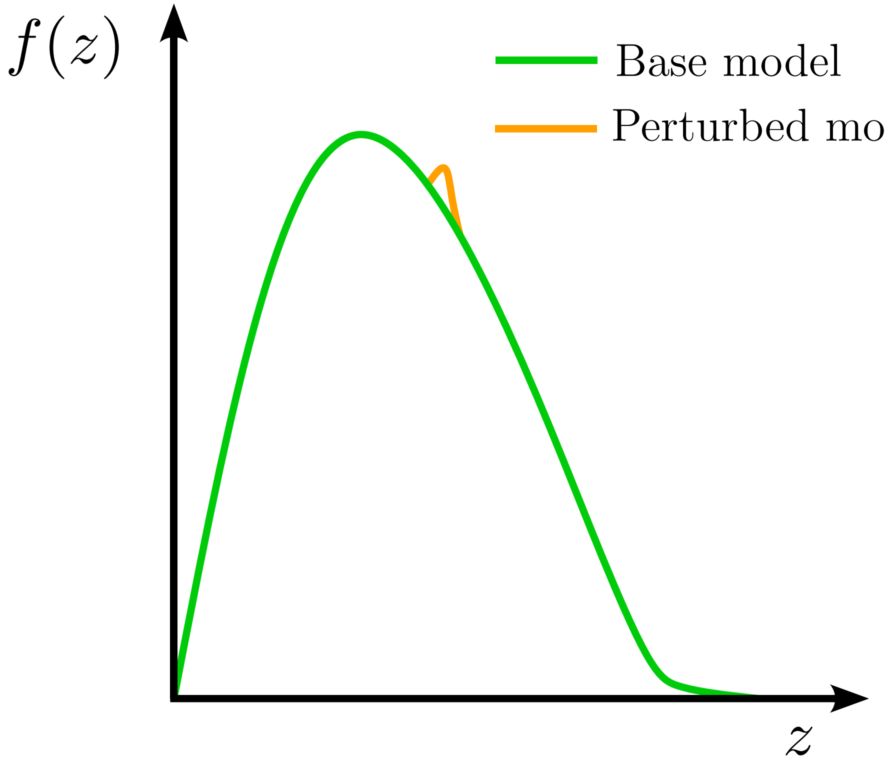

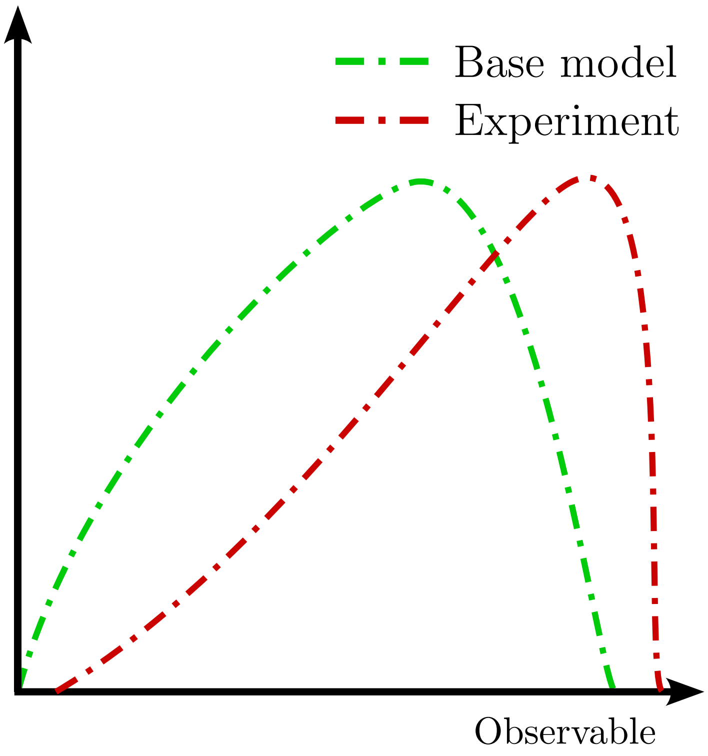

Conclusions

Part I:

- Interesting invasive and non-invasive avenues for making Pythia "fully" differentiable

Part II:

- Fun CLFVing signals and model building at the intensity frontier.

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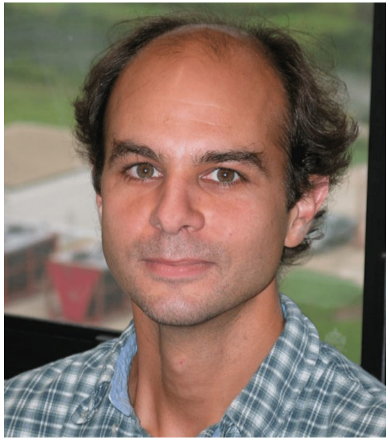
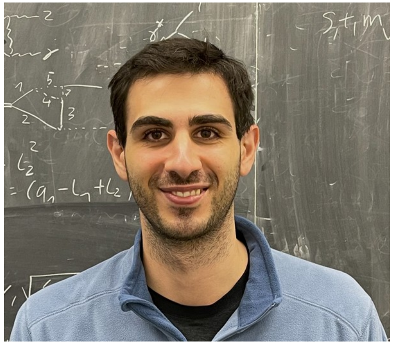
Thanks for your attention :)

Ackowledgements





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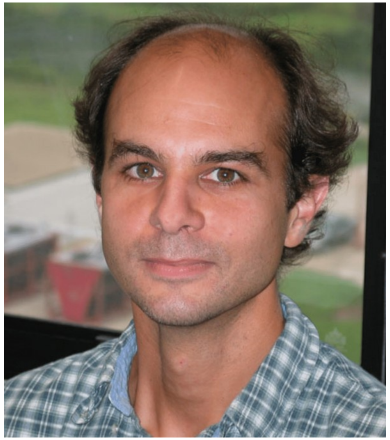

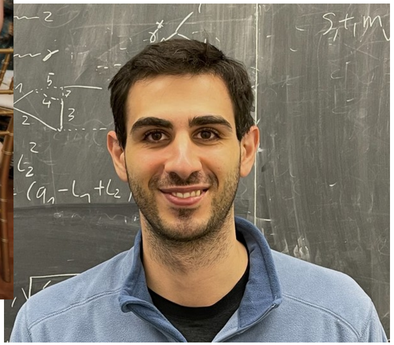

Acknowledgements

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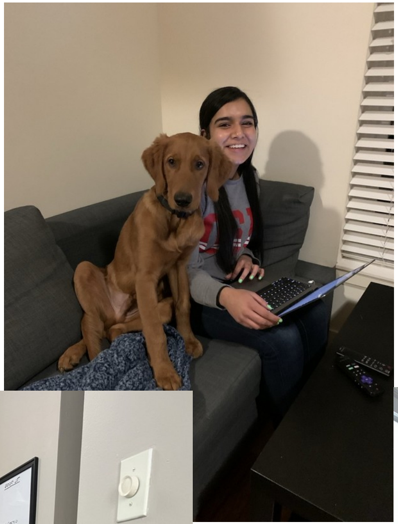
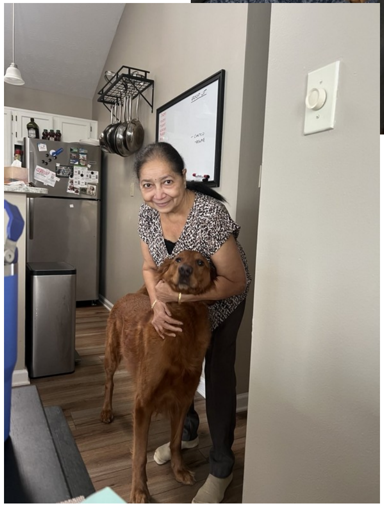
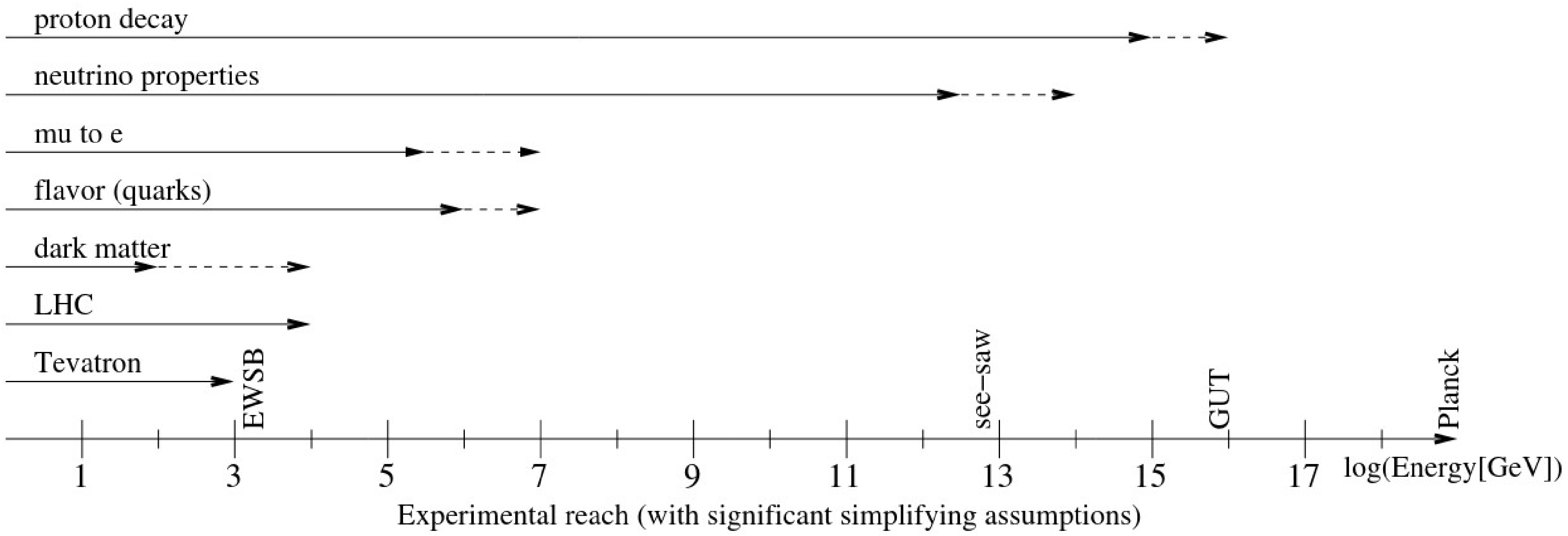




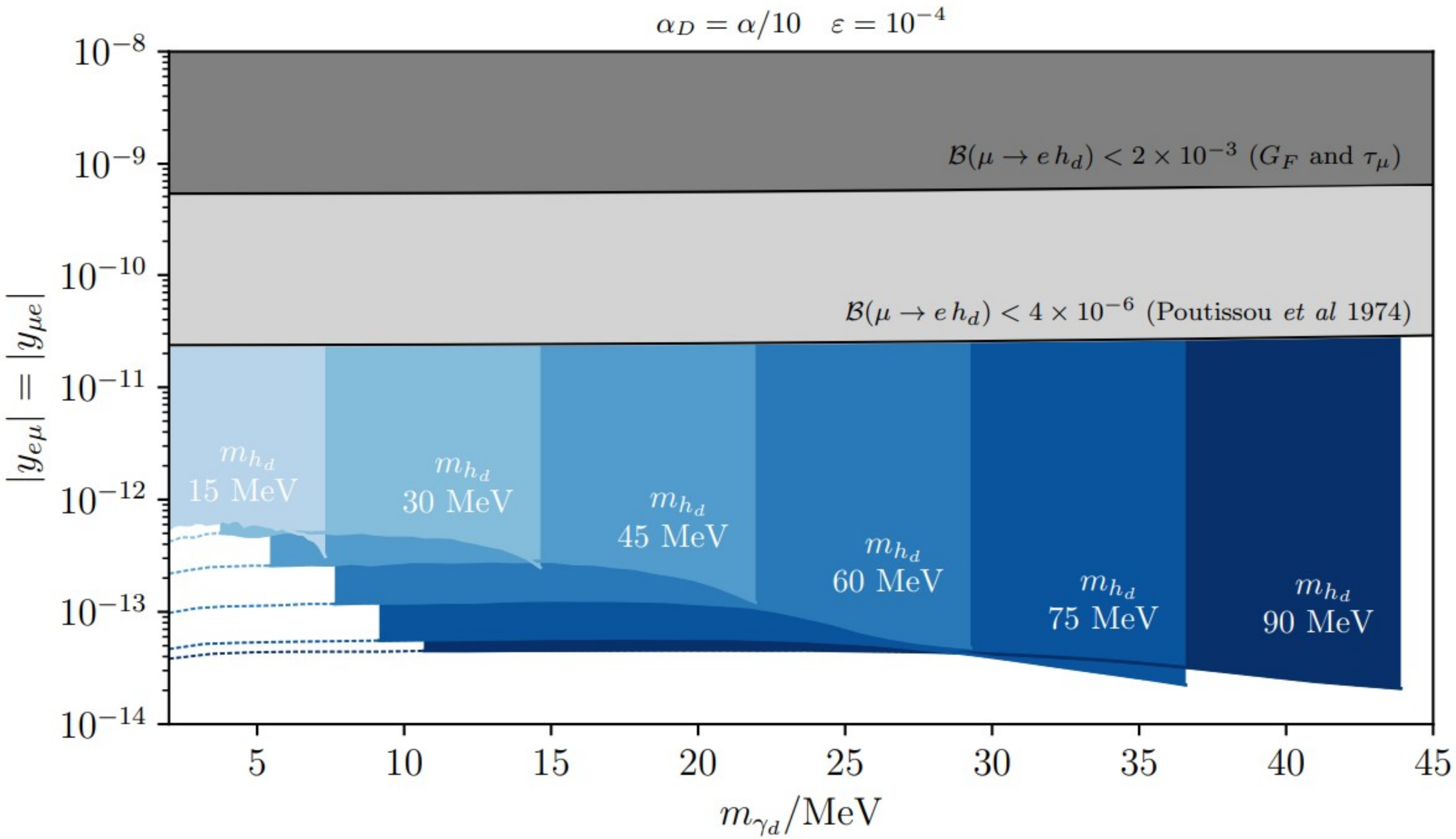
Ackowledgements

Back-up

Reach @ intensity and energy frontier

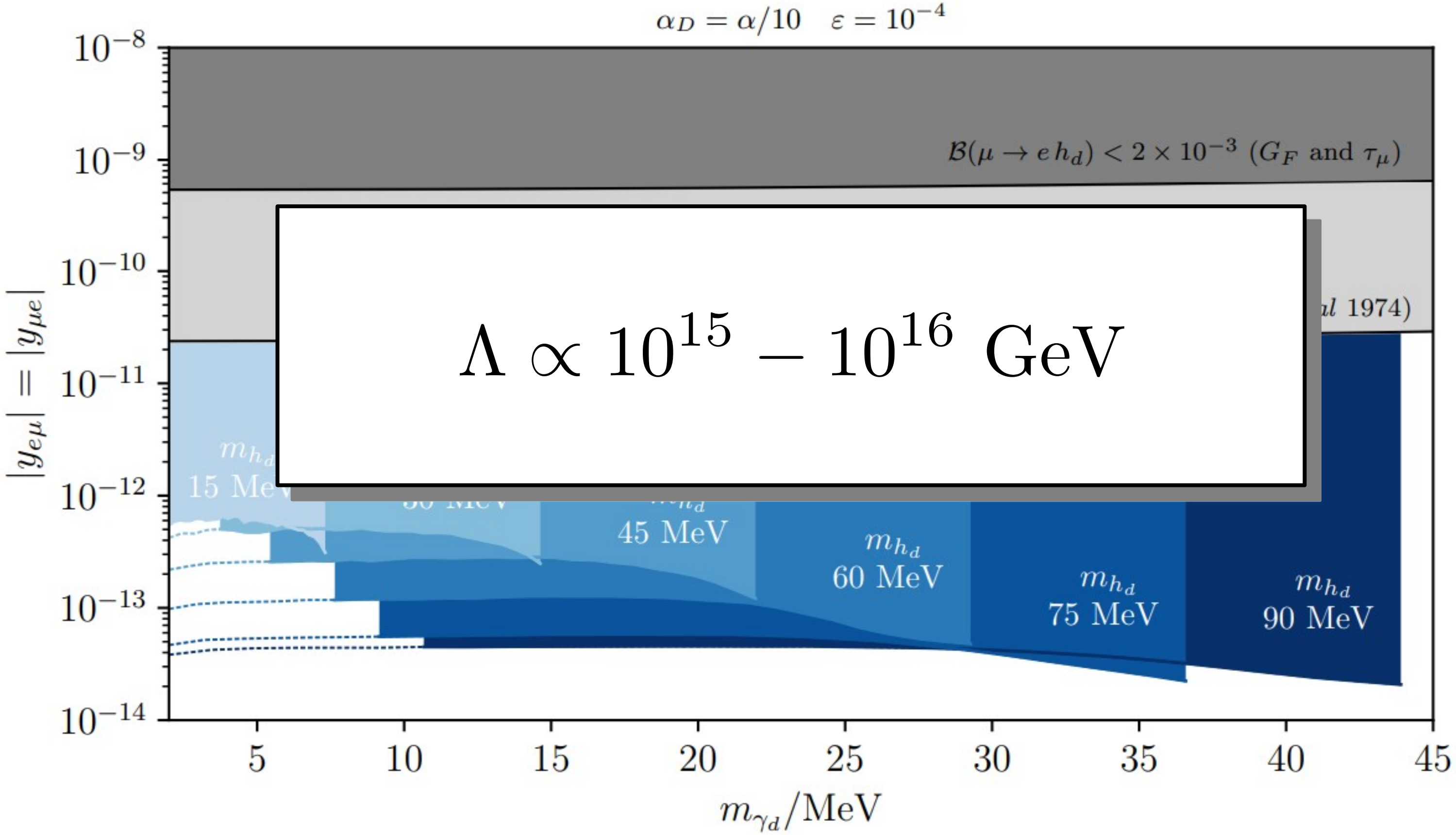


Reach - Mu5e



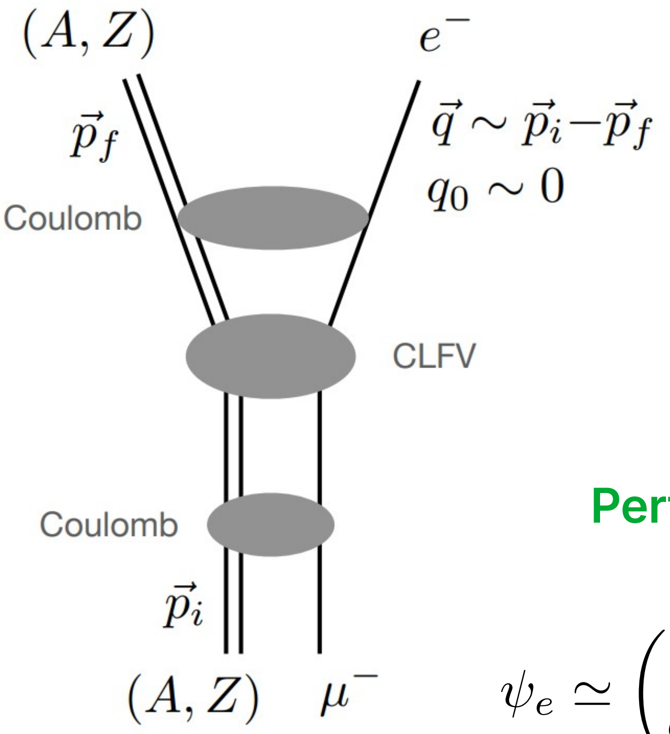
$\alpha_D = \alpha/10 \quad \varepsilon = 10^{-4}$
$\mathcal{B}(\mu \to e\, h_d) < 2 \times 10^{-3}$ (G_F and τ_μ)
$\mathcal{B}(\mu \to e\, h_d) < 4 \times 10^{-6}$ (Poutissou et al 1974)
m_{h_d} 15 MeV
m_{h_d} 30 MeV
m_{h_d} 45 MeV
m_{h_d} 60 MeV
m_{h_d} 75 MeV
m_{h_d} 90 MeV
$|y_{e\mu}| = |y_{\mu e}|$
m_{γ_d}/MeV

Reach - Mu5e



$$\Lambda \propto 10^{15} - 10^{16}\ \mathrm{GeV}$$

(A,Z) μ⁻ → (A,Z) e⁻



Schematically...

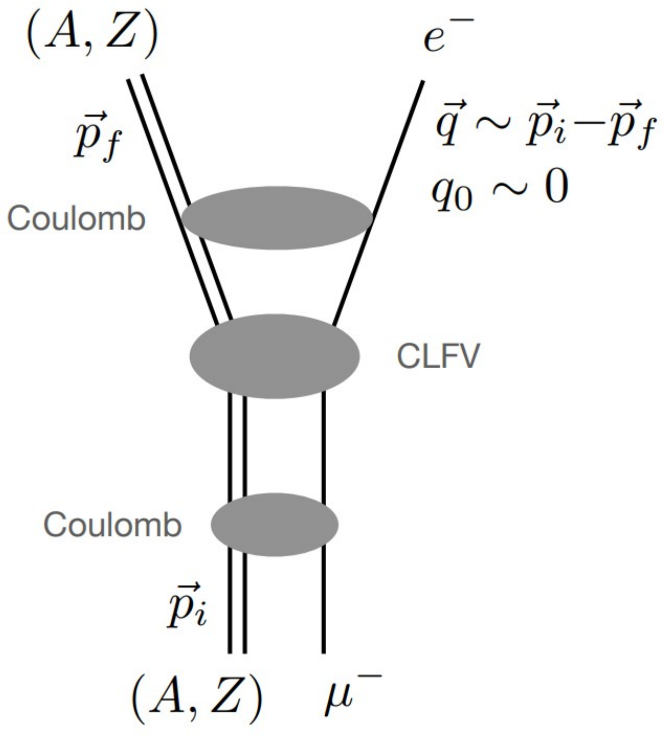
- Effective Hamiltonian:

$$\mathcal{H}_i = c_i \int d^3x \; \bar{\psi}_e(\vec{x}) O_L \psi_\mu(\vec{x}) \bar{\psi}_N(\vec{x}) O_N \psi_N(\vec{x})$$

Perform partial-wave and multipole expansion...

$$\psi_e \simeq \begin{pmatrix} \xi \\ \hat{q} \cdot \vec{\sigma}\xi \end{pmatrix} \quad \psi_\mu \simeq \begin{pmatrix} \xi \\ \frac{1}{2}\vec{v}_\mu \cdot \vec{\sigma}\xi \end{pmatrix} \quad \psi_N \simeq \begin{pmatrix} \xi \\ \frac{1}{2}\vec{v}_N \cdot \vec{\sigma}\xi \end{pmatrix}$$

(A,Z) μ⁻ → (A,Z) e⁻



At the end of the day:

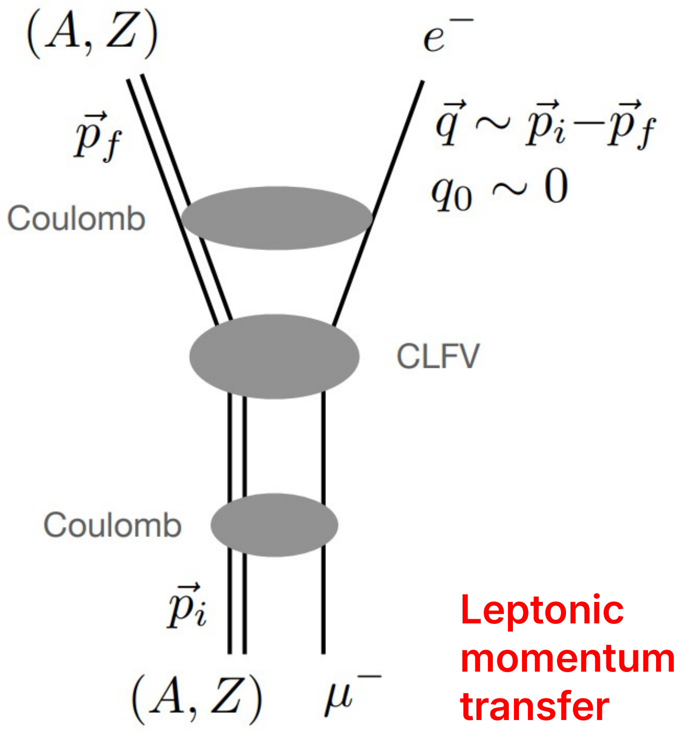
- Basis of CLFV'ing single-nucleon operators

$$i\hat{q} = \frac{i\vec{q}}{|\vec{q}|}, \qquad \vec{\sigma}_L, \qquad \vec{\sigma}_N, \qquad \vec{v}_N, \qquad \vec{v}_\mu.$$

- Natural hierarchy of dimensionless scales

$$y \equiv \left(\frac{qb}{2}\right)^2 > |\vec{v}_N| > |\vec{v}_\mu| > |\vec{v}_T|$$

(A,Z) μ⁻ → (A,Z) e⁻



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- Natural hierarchy of dimensionless scales

$^{27}\mathrm{Al}$

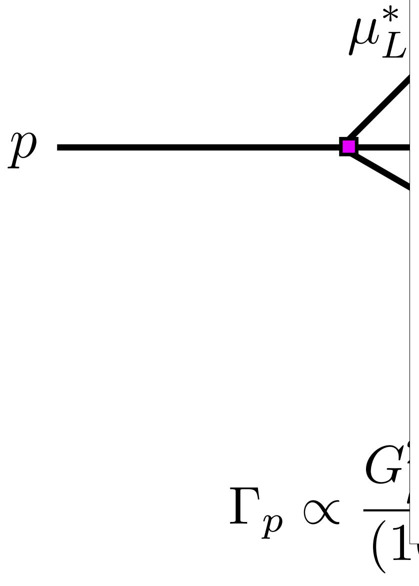
$$y \equiv \left(\frac{qb}{2}\right)^2 > |\vec{v}_N| > |\vec{v}_\mu| > |\vec{v}_T|$$

$\left(\frac{qb}{2}\right) \approx 0.25$, $|\vec{v}_N| \approx 0.2$

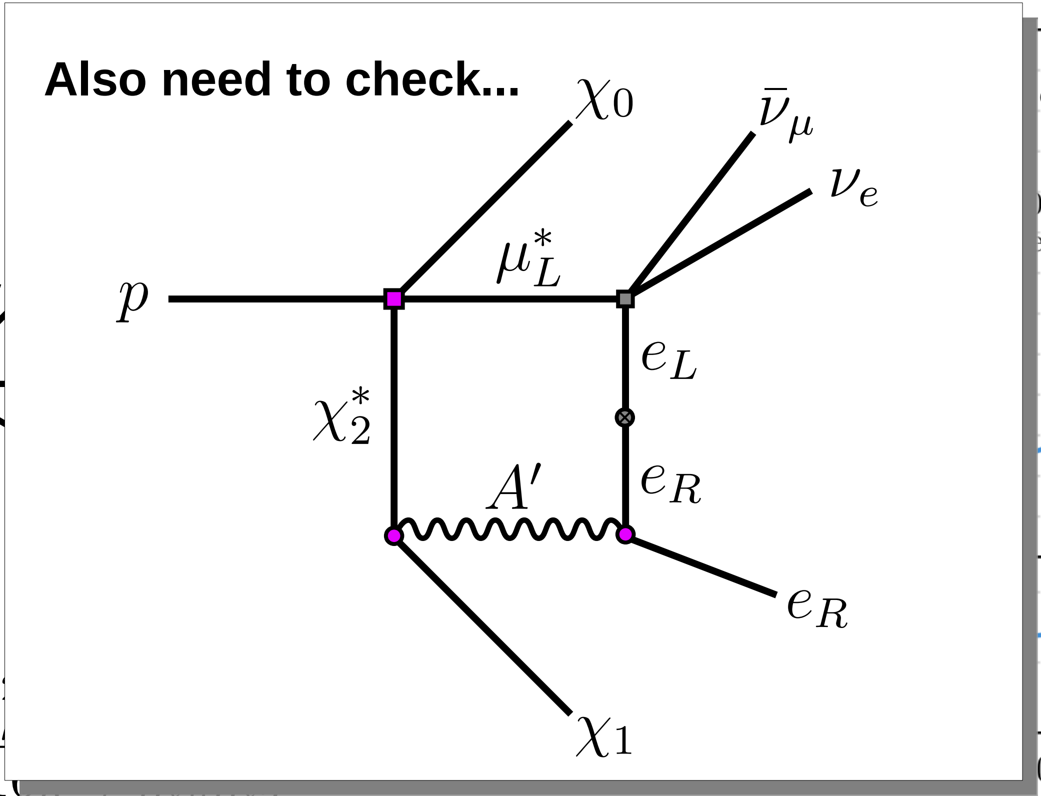
Leptonic momentum transfer: $q \approx m_\mu = 1/1.86\ \mathrm{fm}$

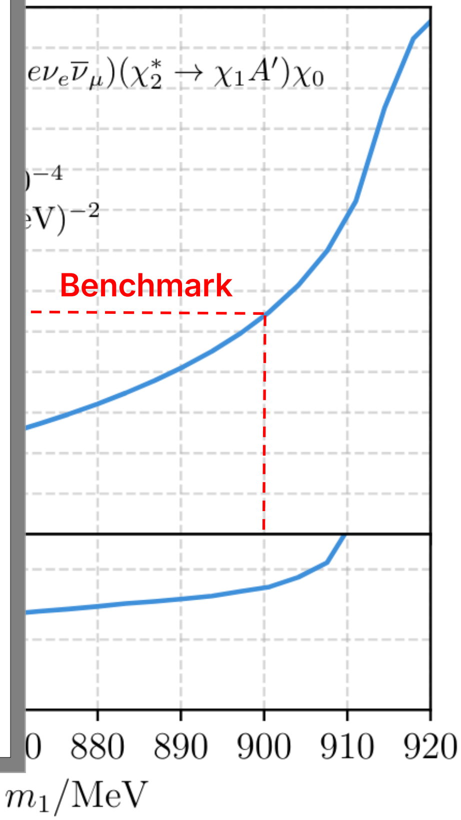
Harmonic oscillator parameter: $b = 1.85\ \mathrm{fm}$

Tug-of-war with nucleon decay



Also need to check...

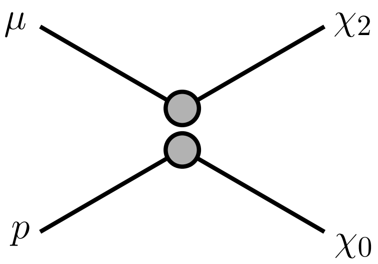




UV completion

$$\mathcal{L}_{\mu p} = G_{\mu p}(\bar{p}\chi_2)(\bar{\mu}\chi_0) + \text{h.c.}$$

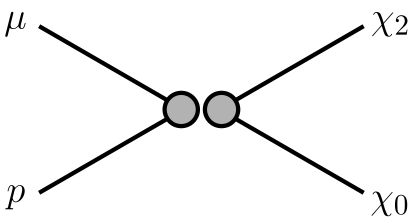
- **What's the new physics reach? Naively:**

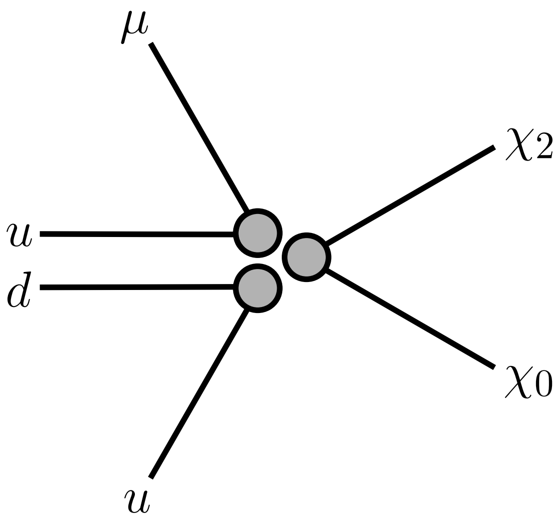


$$G_{\mu p} \simeq 10^{-8} G_F \to \Lambda \simeq 10^3 \text{ TeV}$$

but Λ isn't fundamental...

$$\frac{1}{\Lambda^2} \simeq \frac{\Lambda^3_{\text{QCD}}}{\Lambda^{d_{\text{col.}}}_{\text{col.}} \Lambda^{5-d_{\text{col.}}}_{\text{sing.}}}$$



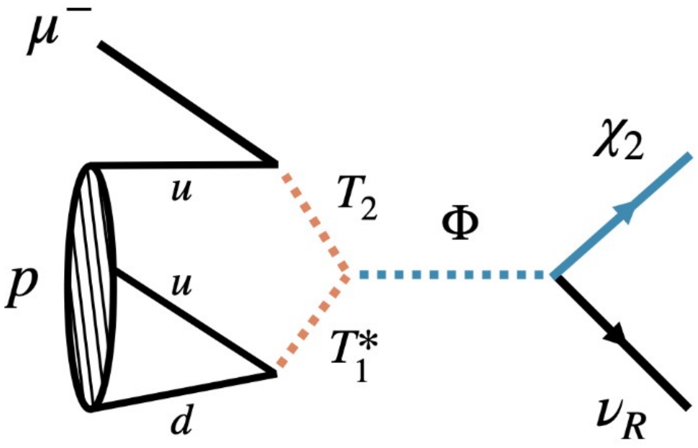


UV completion

Introduce two colored scalars - a diquark T_1 and a leptoquark T_2 as well as a complex scalar Φ.

$$\mathscr{L} \supset -y_{ud}(\overline{u_R^{iC}} d_R^j)\epsilon_{ijk}T_1^k - y_{\mu u}(\overline{u_R^{iC}}\mu_R)(T_2^*)_i + \text{h.c.}$$

$$\mathscr{L} \supset \rho T_1^{k*} T_{2k} \Phi^* + y_\chi \Phi(\overline{\nu_R}\chi_{2R}^C) + \text{h.c.}$$



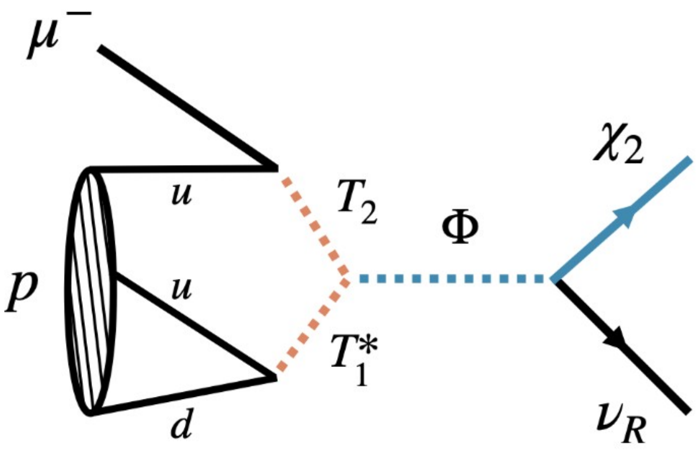
UV completion

Introduce two colored scalars - a diquark T_1 and a leptoquark T_2 as well as a complex scalar Φ.

Integrate out T_1 and T_2, for off-shell Φ:

$$\mathscr{L}_{\text{eff}} = \frac{1}{\Lambda_{\text{col}}^3}\frac{1}{\Lambda_{\text{sin}}^2}\left(\overline{u_R^{iC}}\mu_R\right)\epsilon_{ijk}\left(\overline{u_R^{jC}}d_R^k\right)\left(\overline{\nu_R}\chi_{2R}^C\right) + \text{h.c.}$$

$$\frac{1}{\Lambda_{\text{col}}^3} = y_{ud}y_{\mu u}\frac{\rho}{m_{T_1}^2 m_{T_2}^2} \qquad \frac{1}{\Lambda_{\text{sin}}^2} = \frac{y_\chi}{m_\Phi^2}$$



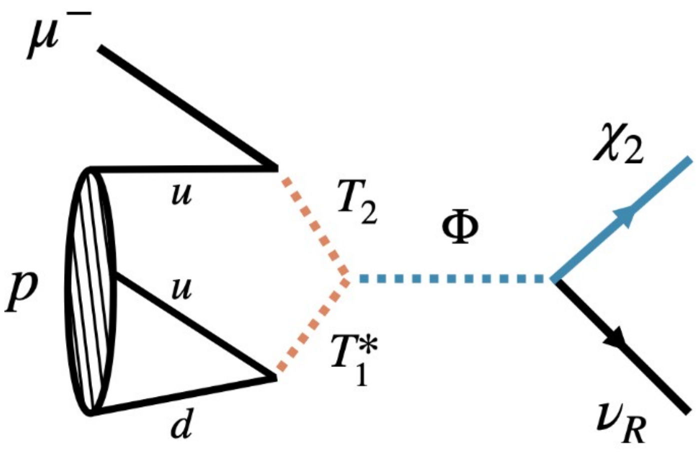
UV completion

Introduce two colored scalars - a diquark T_1 and a leptoquark T_2 as well as a complex scalar Φ.

Integrate out T_1 and T_2, for off-shell Φ:

$$\widetilde{G}_{\mu p} \simeq 10^{-6} G_F$$

$$\sim 10^{-6} G_F \, y_{ud} y_{\mu u} y_\chi \left(\frac{1 \text{ TeV}}{\sqrt{m_{T_1} m_{T_2}}} \right)^4 \left(\frac{\rho}{4 \text{ TeV}} \right) \left(\frac{2 \text{ GeV}}{m_\Phi} \right)^2$$



UV completion

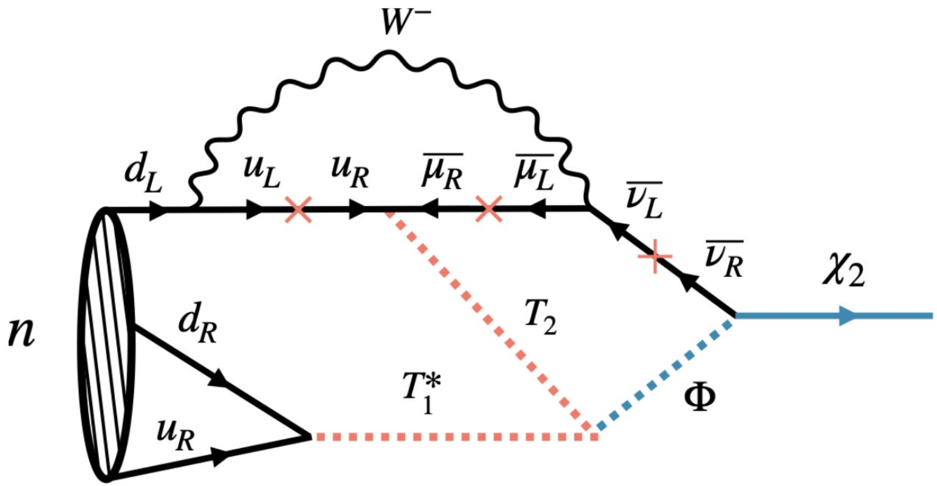
Introduce tv ... uark T_2
as well as a

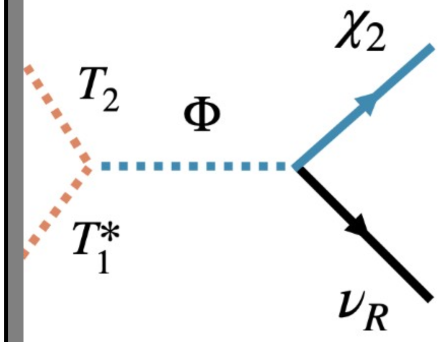
Integrate out T_1 and

$\widetilde{G}_{\mu p} \simeq$

$\sim 10^{-6} G_F\, y_{ud} y_{\mu u} y_\chi \Big($

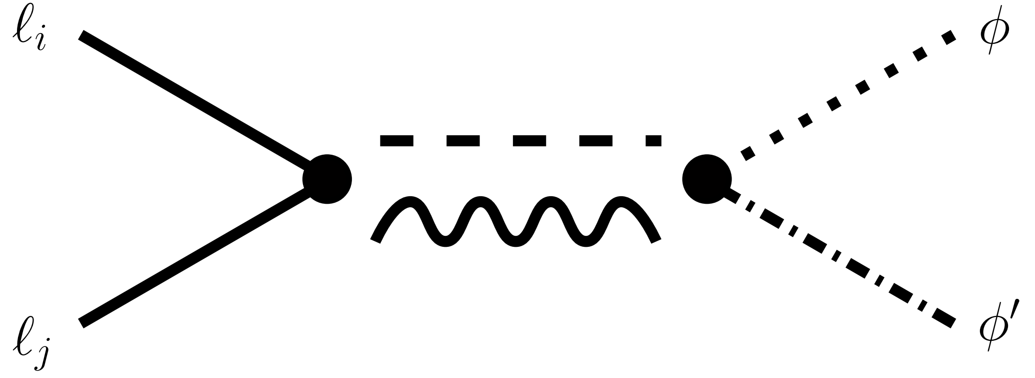
Also need to check neutron decay, neutron mixing...





Explicit realization

Consider: $$\frac{C_{ij}}{f^2}\phi\partial_\mu\phi'(\bar{\ell}_i\gamma^\mu\ell_j)$$



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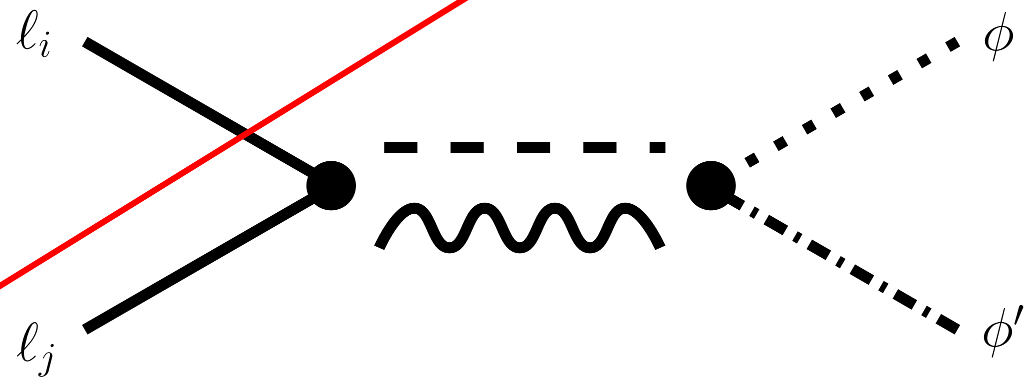


→ Non-abelian pseudo-NGB + portal

Explicit realization

- **Non-abelian pseudo-NGB + U(1)$_D$**

$$\mathcal{L}_D \supset \frac{f_{\rm UV}^2}{8}\left(D_\mu U D^\mu U^\dagger\right) + \frac{f_{\rm UV}^2}{8}\left(\chi^\dagger U + U^\dagger \chi\right) + \cdots$$

Explicit realization

- Non-abelian pseudo-NGB + $U(1)_D$

$$U \to e^{i\alpha} U$$

$$\mathcal{L}_D \supset \frac{f_{\rm UV}^2}{8}\left(D_\mu U D^\mu U^\dagger\right) + \frac{f_{\rm UV}^2}{8}\left(\chi^\dagger U + U^\dagger \chi\right) + \cdots$$

Explicit realization

- **Non-abelian pseudo-NGB + U(1)$_D$**

$$SU(3)_L \times SU(3)_R \times U(1)_D \to SU(3)_V \times U(1)_D$$

$$D_\mu \equiv \partial_\mu + i g_D A'_\mu [Q, U] \qquad\qquad U = \exp\left(i\sqrt{2}\Pi_D / f_D\right)$$

$$\Pi_D = \begin{pmatrix} \frac{\pi_D^0}{\sqrt{2}} + \frac{\eta_{D8}}{\sqrt{6}} & \pi_D^+ & K_D^+ \\ \pi_D^- & -\frac{\pi_D^0}{\sqrt{2}} + \frac{\eta_{D8}}{\sqrt{6}} & K_D^0 \\ K_D^- & \bar{K}_D^0 & -\sqrt{\frac{2}{3}}\eta_{D8} \end{pmatrix}$$

Explicit realization

- **Non-abelian pseudo-NGB + U(1)$_D$**

$$\mathcal{L}_D \supset \frac{f_{\rm UV}^2}{8}\left(D_\mu U D^\mu U^\dagger\right) + \frac{f_{\rm UV}^2}{8}\left(\chi^\dagger U + U^\dagger \chi\right) + \cdots$$

$$\mathrm{Q} \to q_U \lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & q_U \\ 0 & q_U & 0 \end{pmatrix}$$

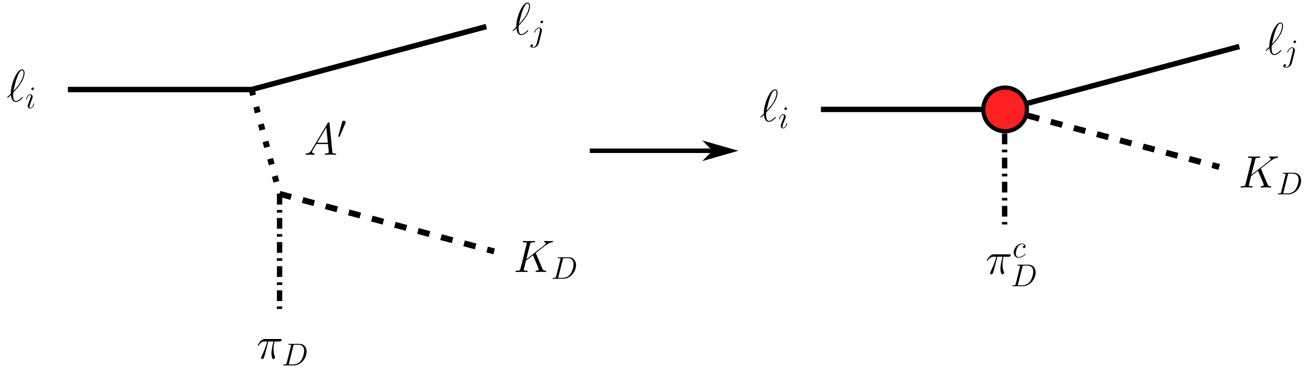
$$\supset -g' q_U A'_\mu \left(\pi_D^+ i\partial^\mu K_D^- - K_D^- i\partial^\mu \pi_D^+ + \text{h.c.}\right) + \cdots$$

Explicit realization

- Non-abelian pseudo-NGB + U(1)$_D$

$$\supset -g' q_U A'_\mu \left(\pi_D^+ i\partial^\mu K_D^- - K_D^- i\partial^\mu \pi_D^+ + \text{h.c.}\right) + \cdots$$

Charge the SM under U(1)$_D$: $$-\mathcal{L}_{\text{portal}} = ig' c_{ij} \ell_i \gamma^\mu A'_\mu \ell_j + \text{h.c.}$$



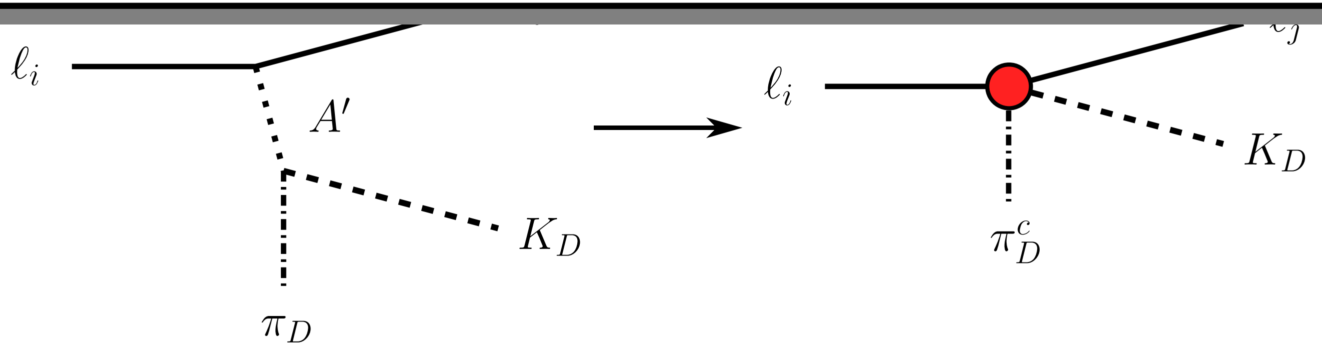
Explicit realization

- Non-abelian pseudo-NGB + $U(1)_D$

$$\supset -g' q_U A'_\mu \left(\pi_D^+ i\partial^\mu K_D^- - K_D^- i\partial^\mu \pi_D^+ + \mathrm{h.c.}\right) + \cdots$$

Charge t … + h.c.

$$\supset \frac{c_{ij} q_U g'^2}{m_{A'}^2} \left(\bar{\psi}_i \gamma^\mu \psi_j\right)\left(\pi_D^+ \partial_\mu K_D^-\right) + \cdots$$



Applications

- Efficient means of exploring parameter space
- Inherently differentiable
- Very useful for tuning!
  - Embed the computation of weights into numerical autodifferentiation engine
  - Picking new parameters in the update step facilitated by well-developed optimizers (SGD, Adam, etc.)

Rejection sampling with Autodifferentiation (RSA)

WHAT is a differentiable simulator?

- All parameters are differential parameters
  - Differential with respect to what?

WHY a differentiable simulator?

- Parameter exploration (tuning)
- Uncertainty quantification
- Reproducibility

HOW to build a differentiable simulator?

- Many roads to a fully differentiable event generator. Varying levels of “intrusivity” (from a developers perspective).

The "score" observable

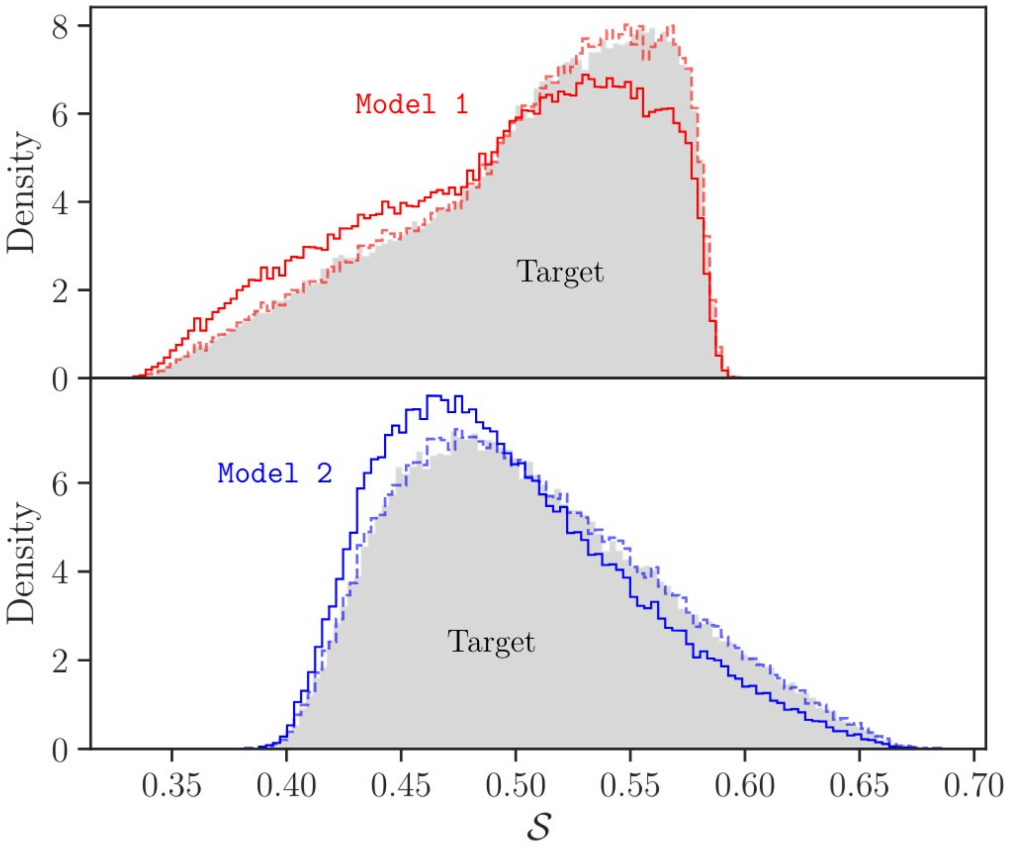
- Train a deepsets classifier to distinguish simulation from experiment
  - Takes full event information as input

$(E, p_x, p_y, p_z)_1$

$(E, p_x, p_y, p_z)_2$

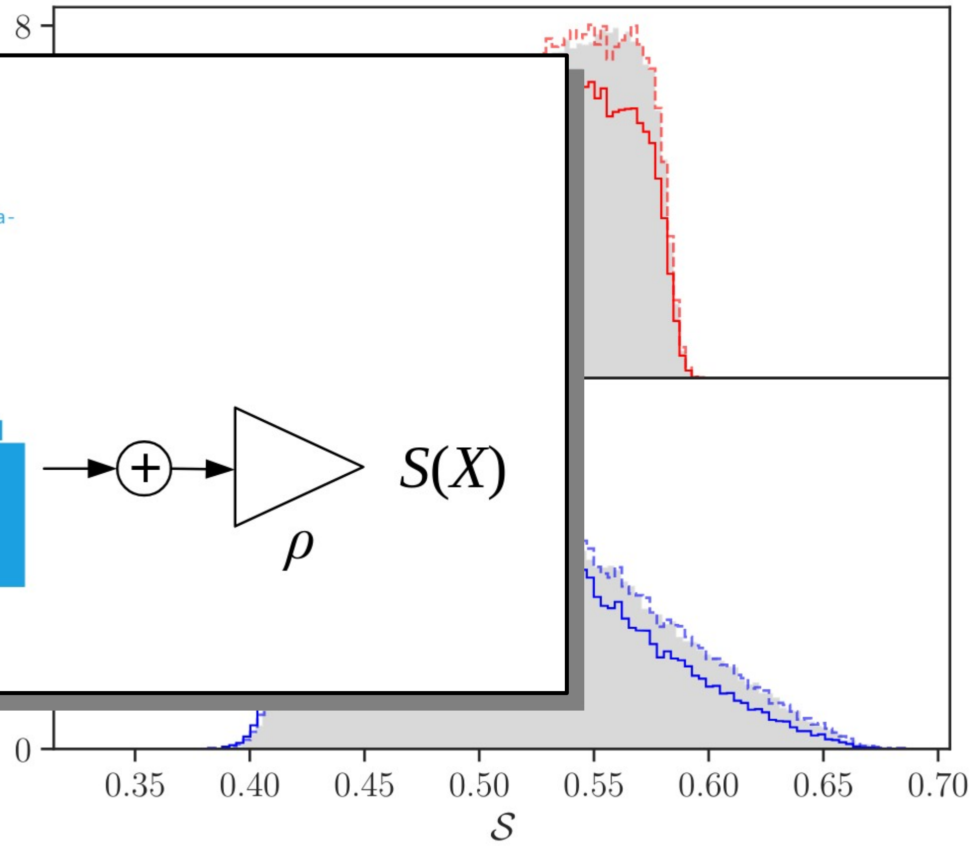
...

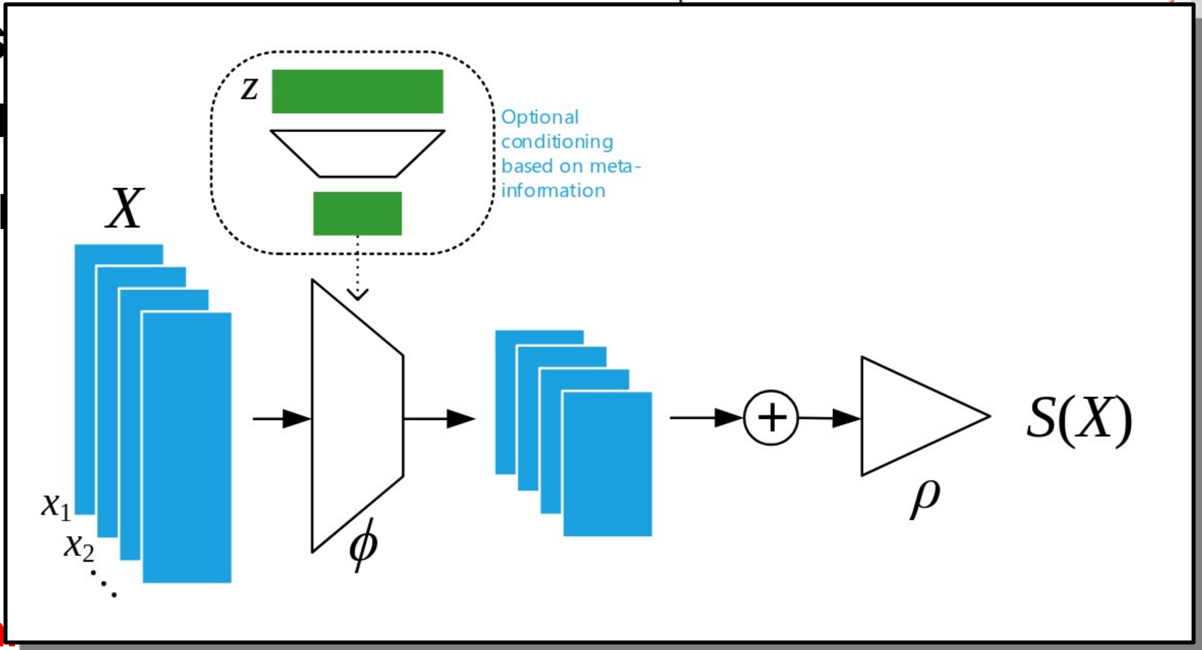
Use trained classifier output (score) as an observable



The "score" observable

- Train a deepsets classifier to distinguis experimer
  - Takes ful input





Use tra
(score) as an observable

Classifier score with full event info improves tuning convergence