

Differentiable hadronization models, heavy and light new physics in rare lepton decays, and more...

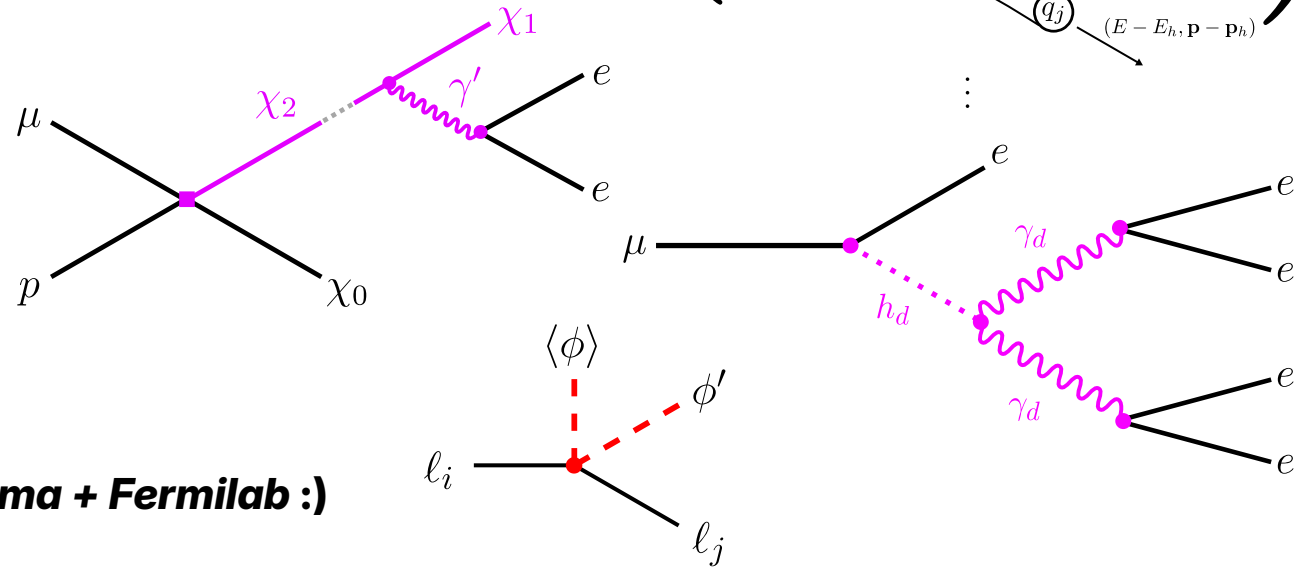
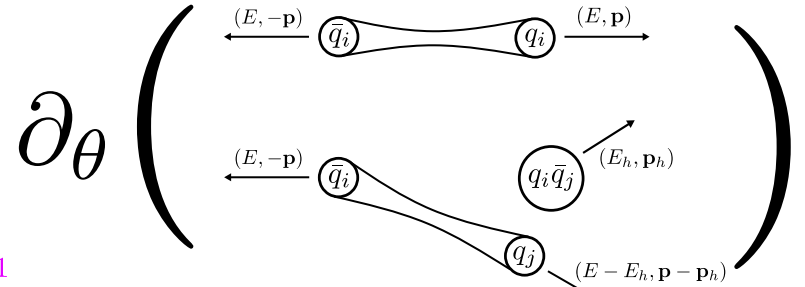
University of Alabama

July 31st, 2025

Tony Menzo

University of Cincinnati

Soon to be postdoc at *Alabama + Fermilab* :)



Based on work with MLHAD + MLhadML, Matheus, Maxim, Paddy, Evan, Wick, Ken, and Jure

Outline

1) Introduction to hadronization

2) The inverse problem of hadronization

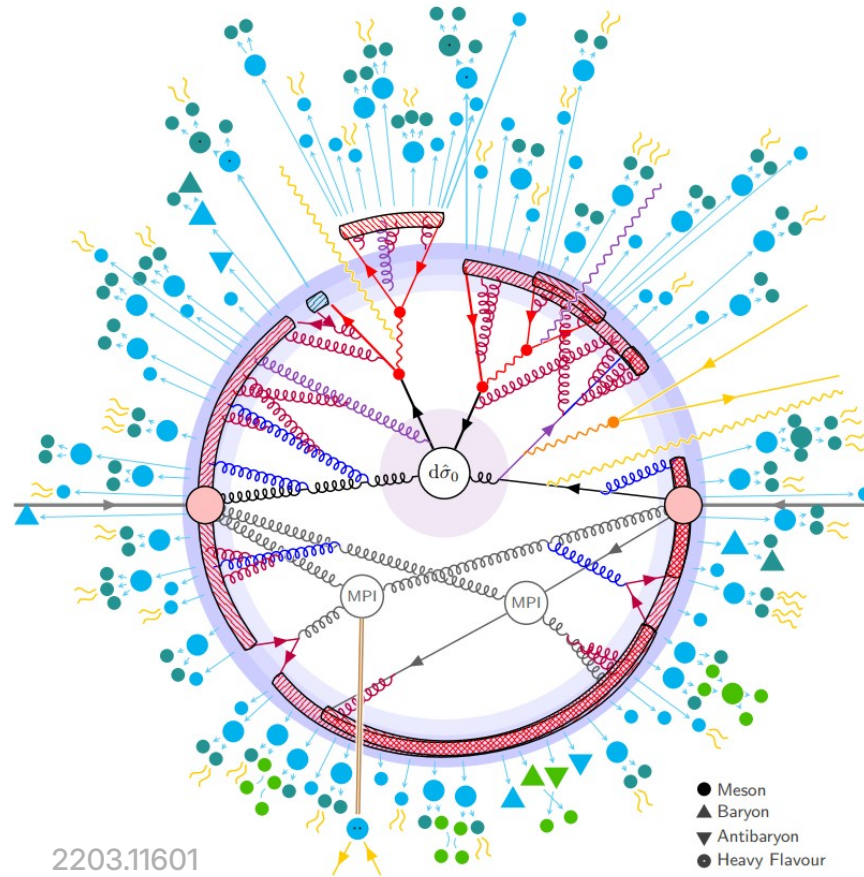
- Pure ML solutions, hybrid solutions

3) Introduction to charged lepton flavor violation

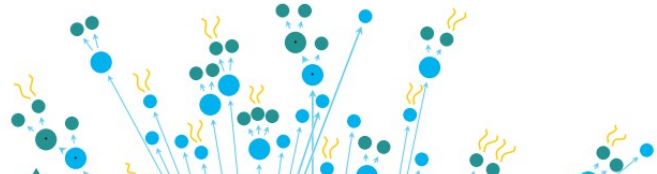
4) Fun signatures at rare muon decay experiments

- Multi-electron final states, exotic muon capture modes, time-dependent signals from ultralight dark matter

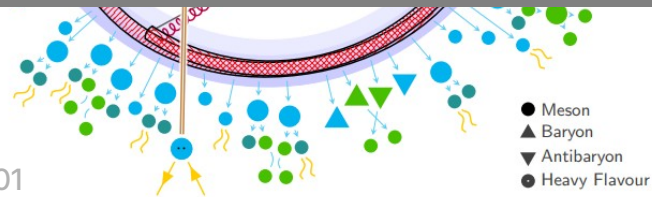
Monte Carlo Event generators



Monte Carlo Event generators



$$\mathcal{G} : \underbrace{\mathcal{S}(\mathcal{D}(\mathcal{H}(\mathcal{P}(\mathcal{M}))))}_{\text{Simulation}} = \mathcal{E} \simeq \underbrace{\begin{pmatrix} \{\text{id}, E, p_x, p_y, p_z, \dots\}_1 \\ \{\text{id}, E, p_x, p_y, p_z, \dots\}_2 \\ \vdots \\ \{\text{id}, E, p_x, p_y, p_z, \dots\}_N \end{pmatrix}}_{\text{Event record}} .$$

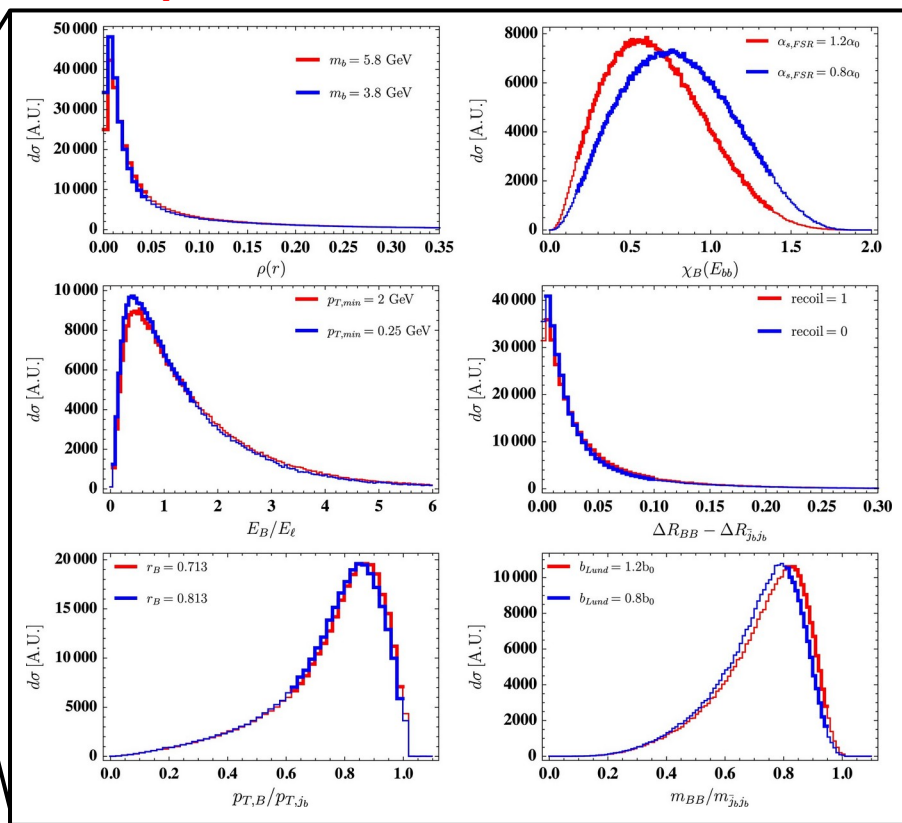


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Motivation

Precision (exclusive) measurements dependent on hadronization!

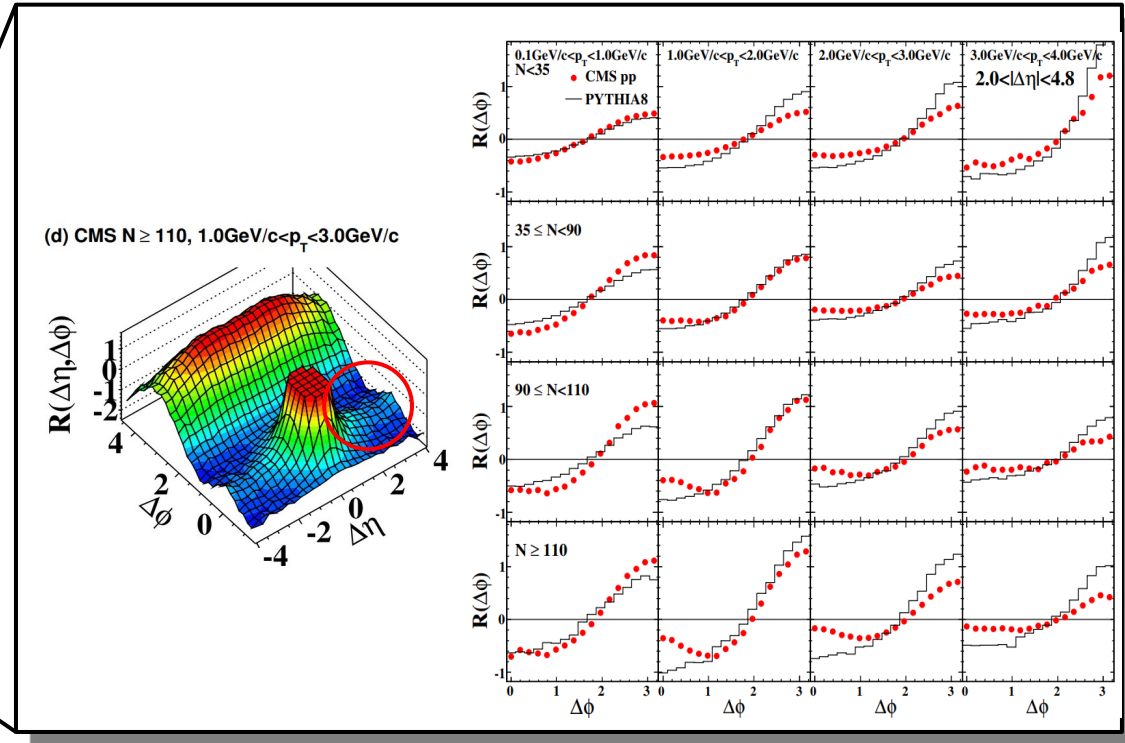
- Uncertainty reduction
 - Top quark mass measurement (r_b)
 - e^+e^- determination of α_s
- Mis-modeling
 - High-multiplicity events
 - Tuning discrepancies



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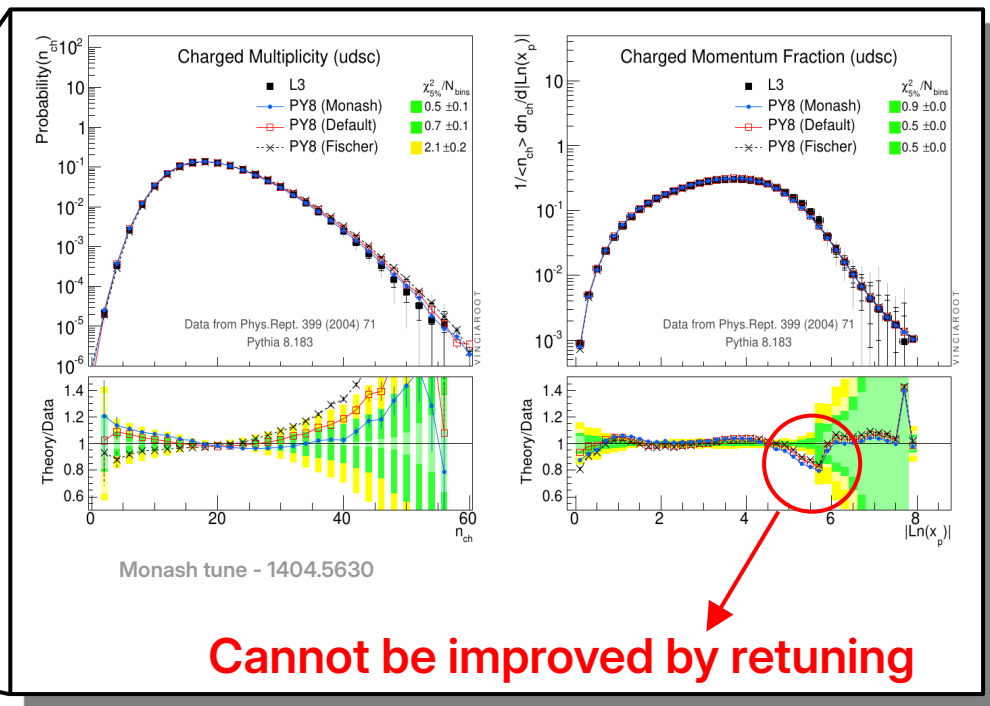
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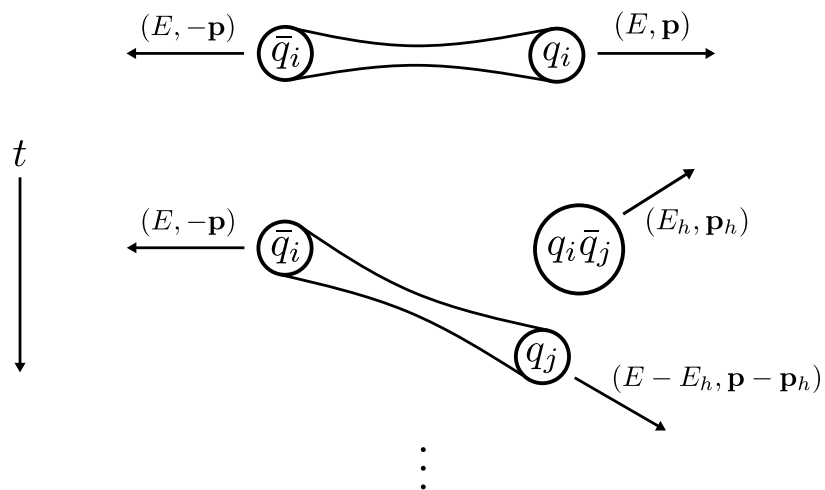
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Phenomenological models

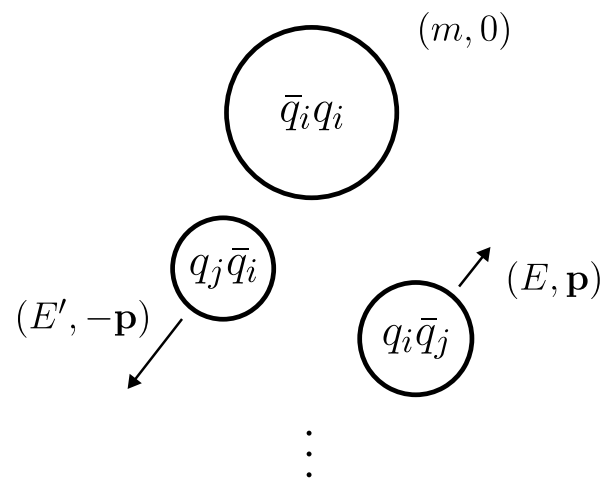
Lund string model

(used in Pythia)

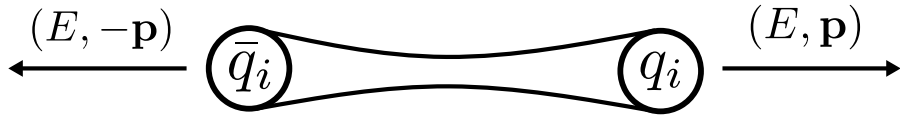


Cluster model

(used in Herwig, Sherpa)



The algorithm ($q\bar{q}$)



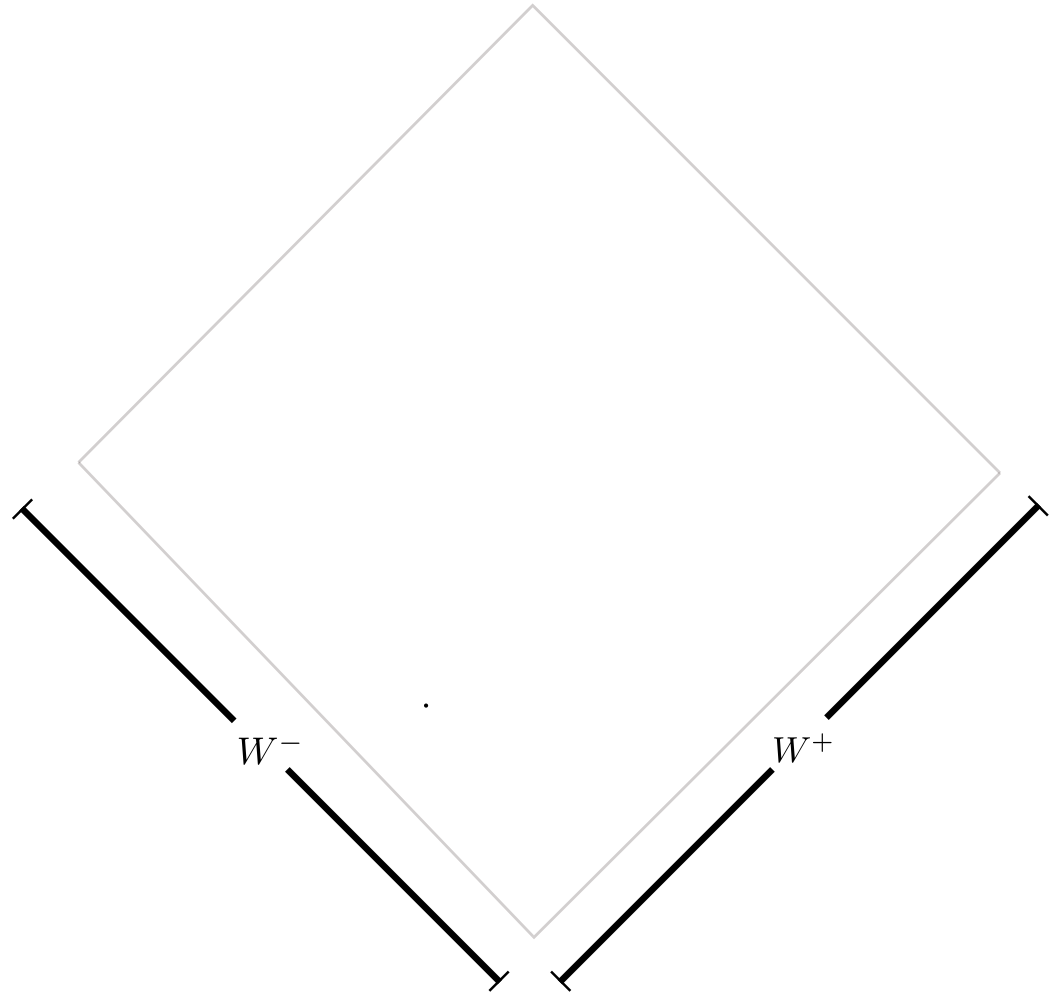
- 1) Randomly select one of the string ends
- 2) Sample new quark flavor
- 3) Sample transverse momentum of new quarks

$$\mathcal{P}(p_x, p_y; \sigma_{p_T}) = \frac{1}{\pi \sigma_{p_T}^2} \exp\left(-\frac{p_x^2 + p_y^2}{\sigma_{p_T}^2}\right)$$

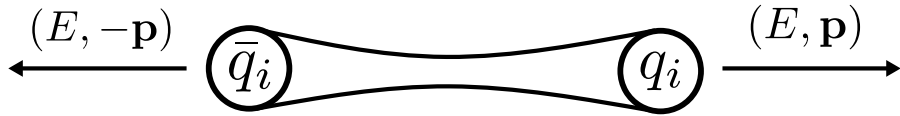
- 4) Sample longitudinal momentum fraction of new hadron

$$f(z) \propto \frac{(1-z)^a}{z} \exp\left(\frac{-bm_T^2}{z}\right), \quad z = \frac{p_z + E_h}{2E}$$

- 5) Repeat steps 1-4



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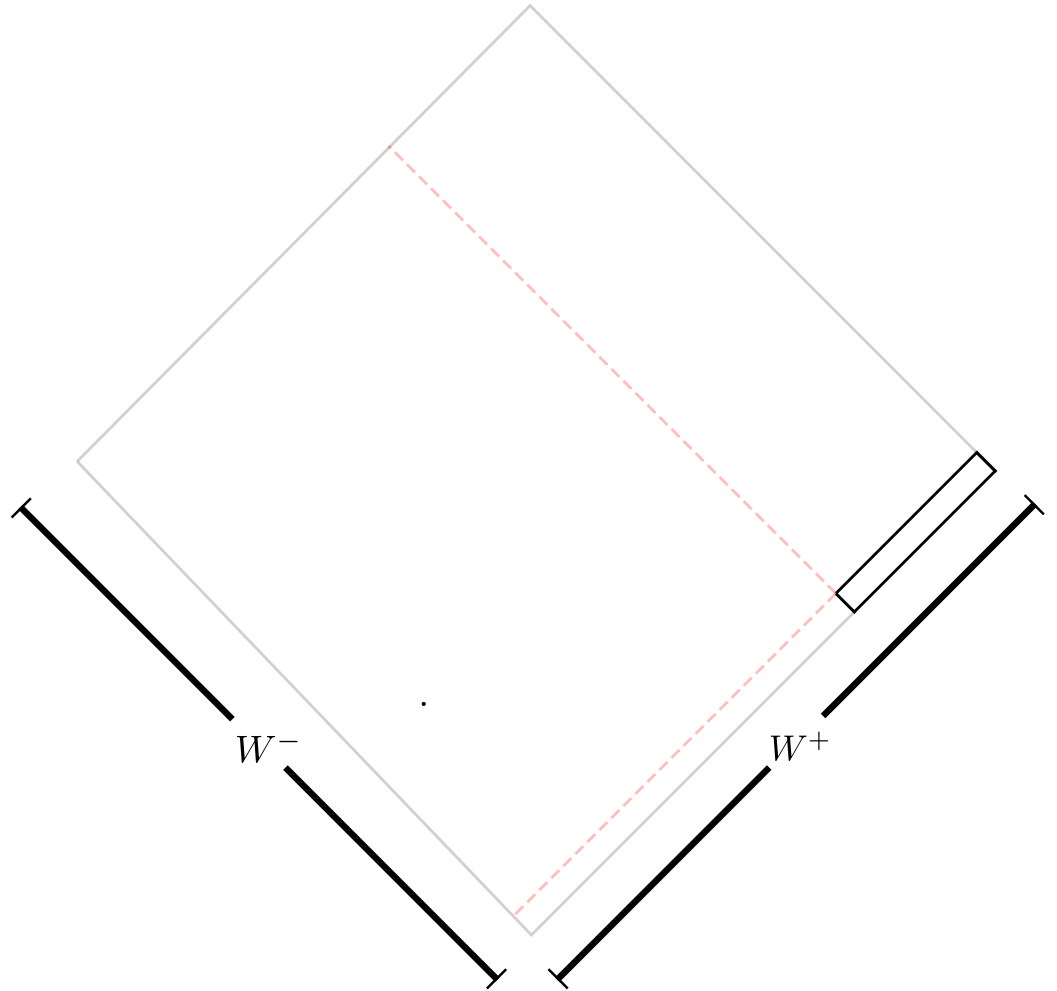
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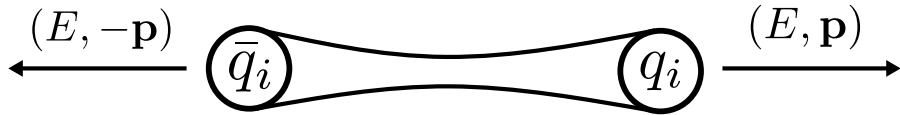
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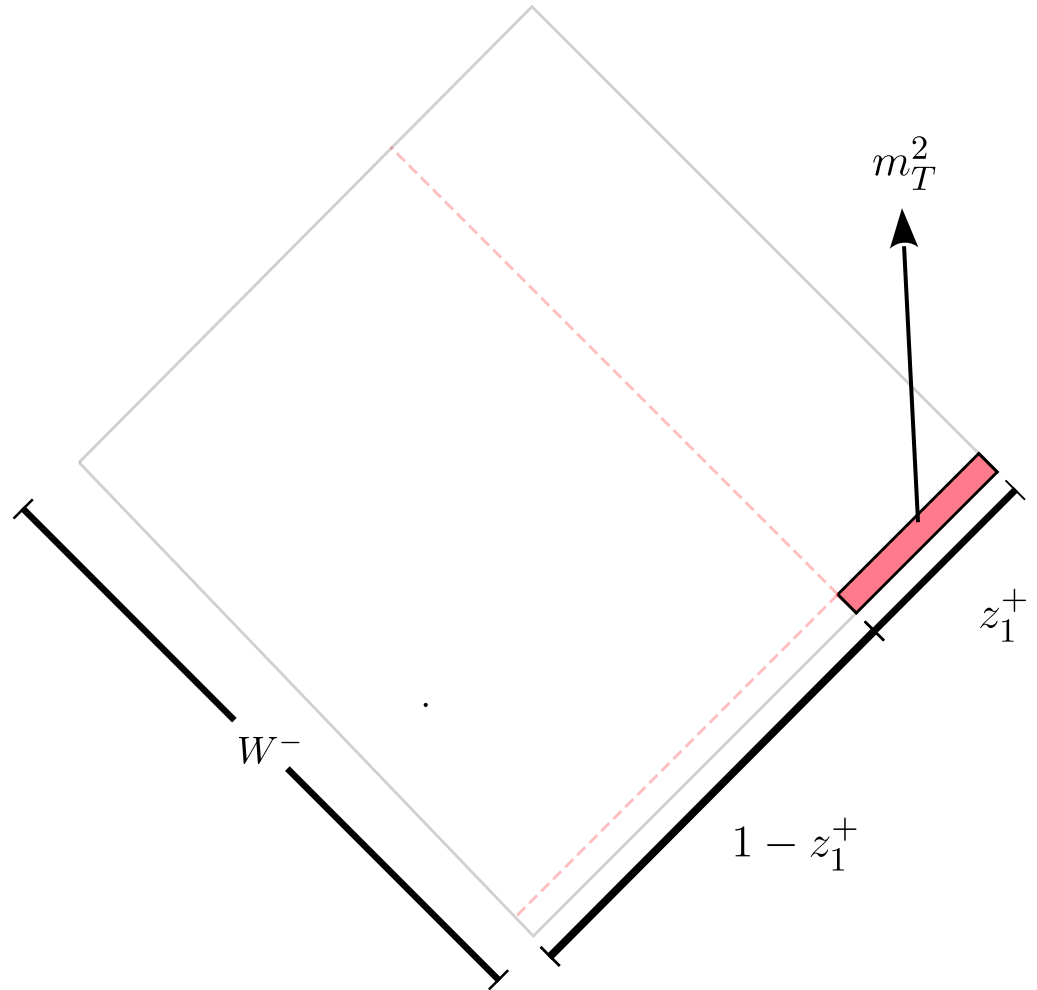
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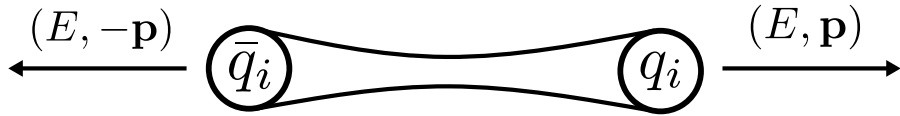
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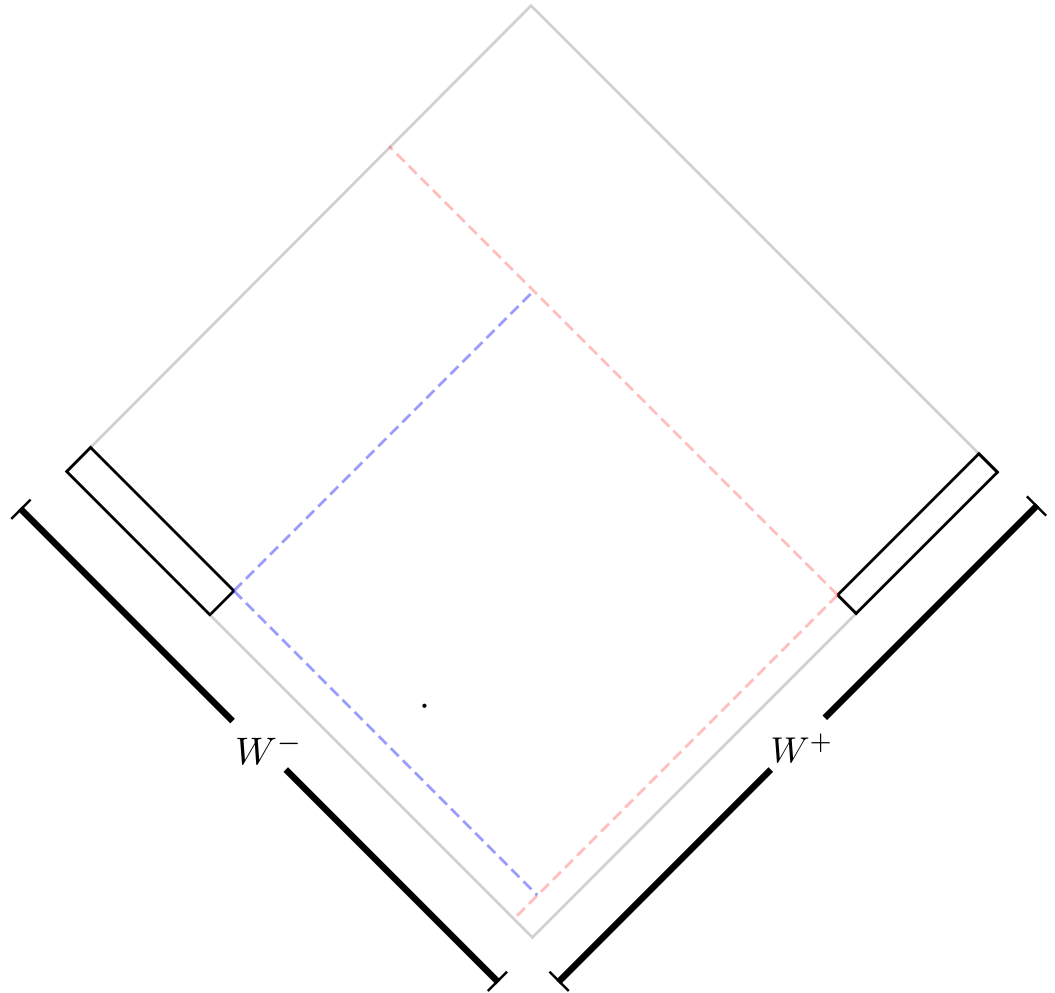
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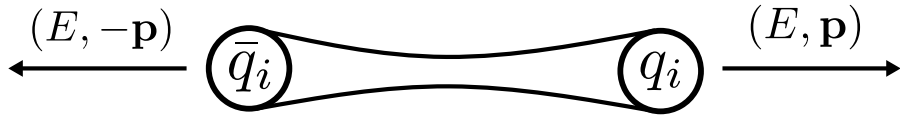
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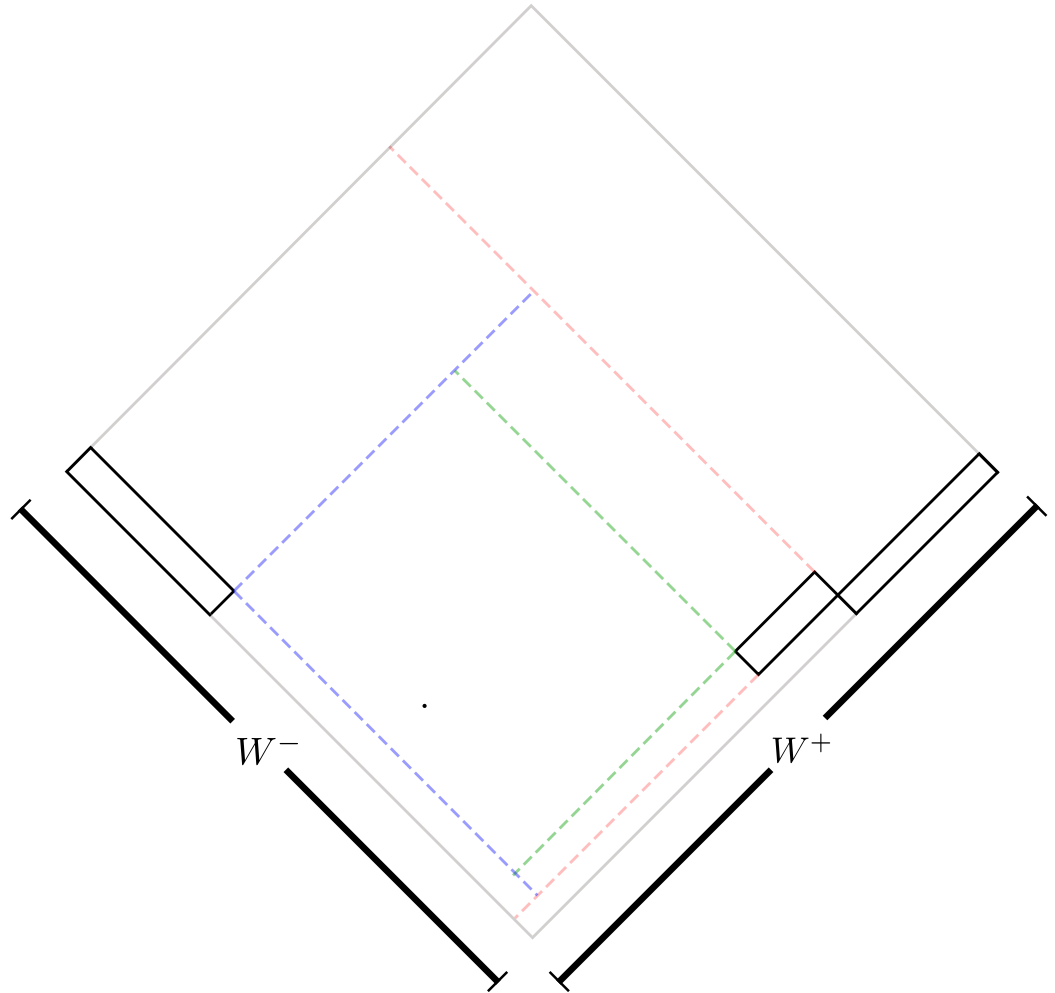
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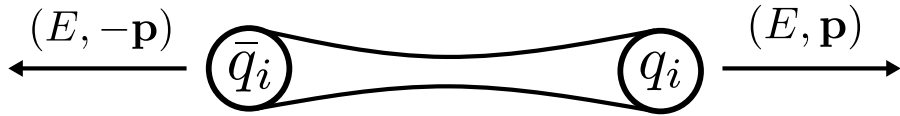
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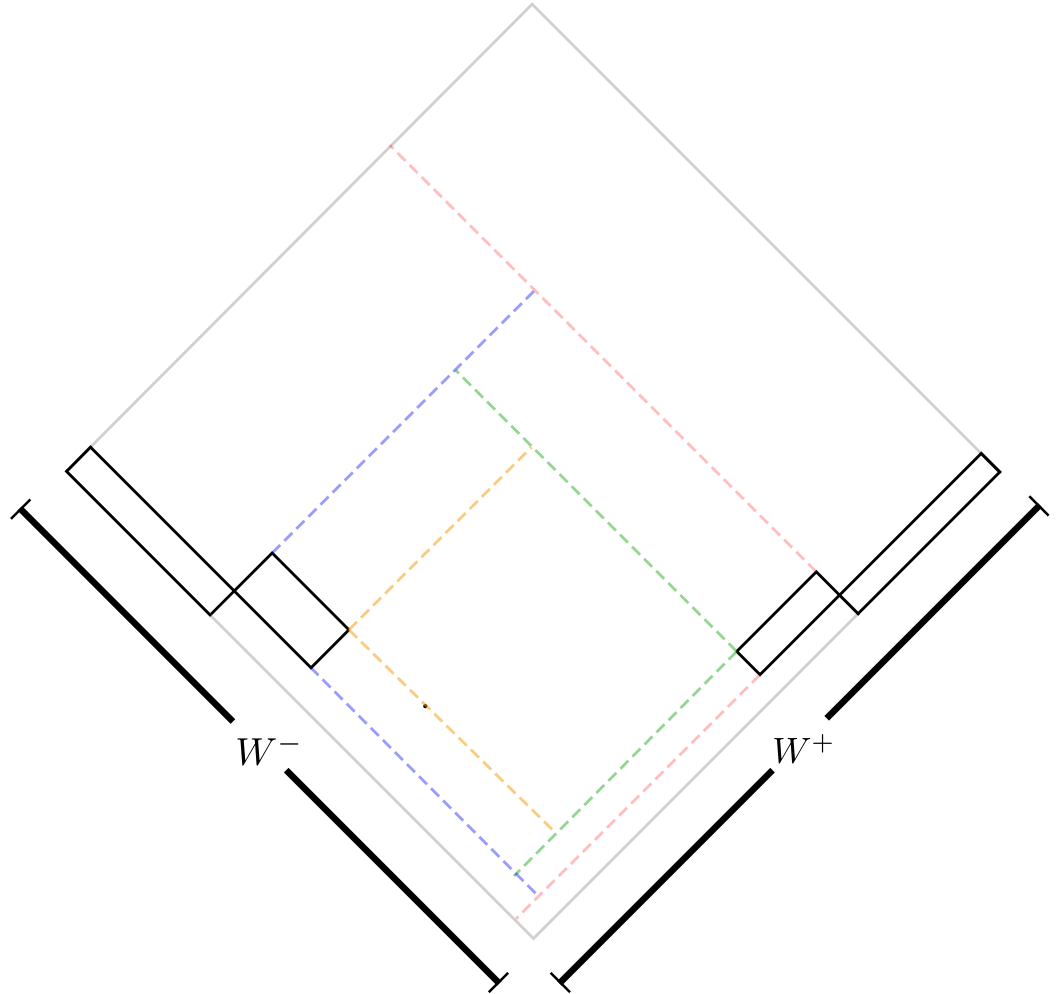
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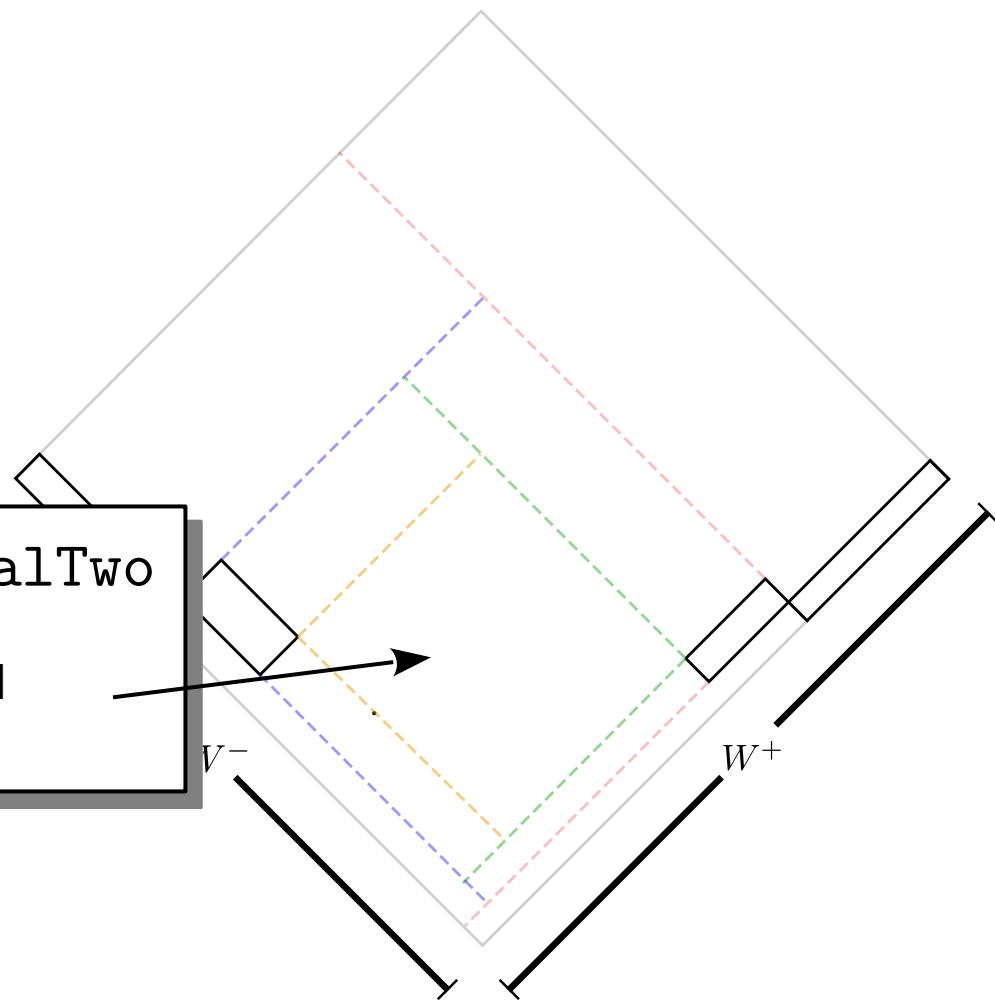
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$\mathcal{P}(z)$
4) Sample new hadron

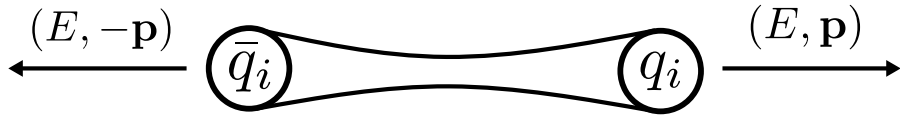
When E_{CM} goes below ~ 2 GeV, finalTwo is called. This can fail, when it does, the full string system is re-simulated from the beginning.

$$f(z) \propto \frac{1}{z} \exp\left(-\frac{1}{z}\right), \quad z = \frac{p_T}{2E}$$

- 5) Repeat steps 1-4



The algorithm ($q\bar{q}$)



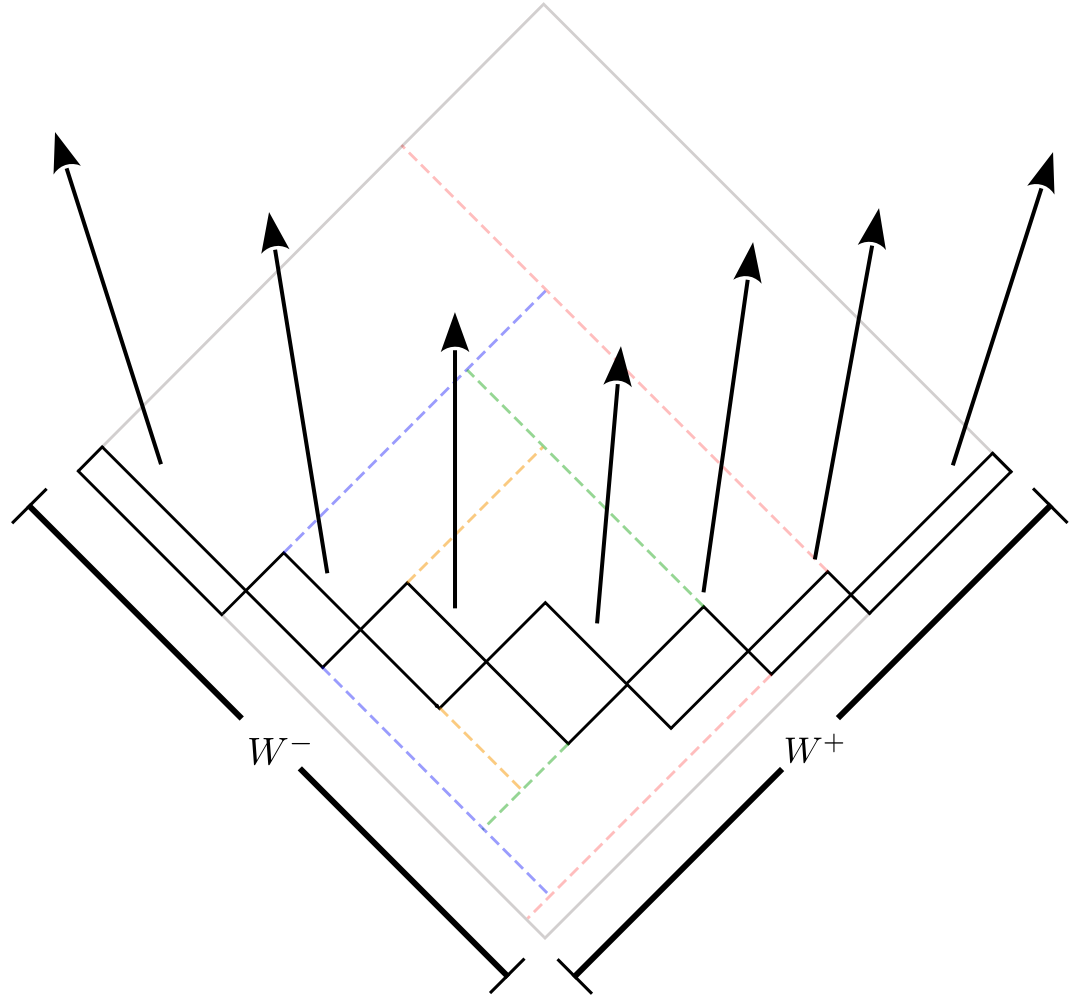
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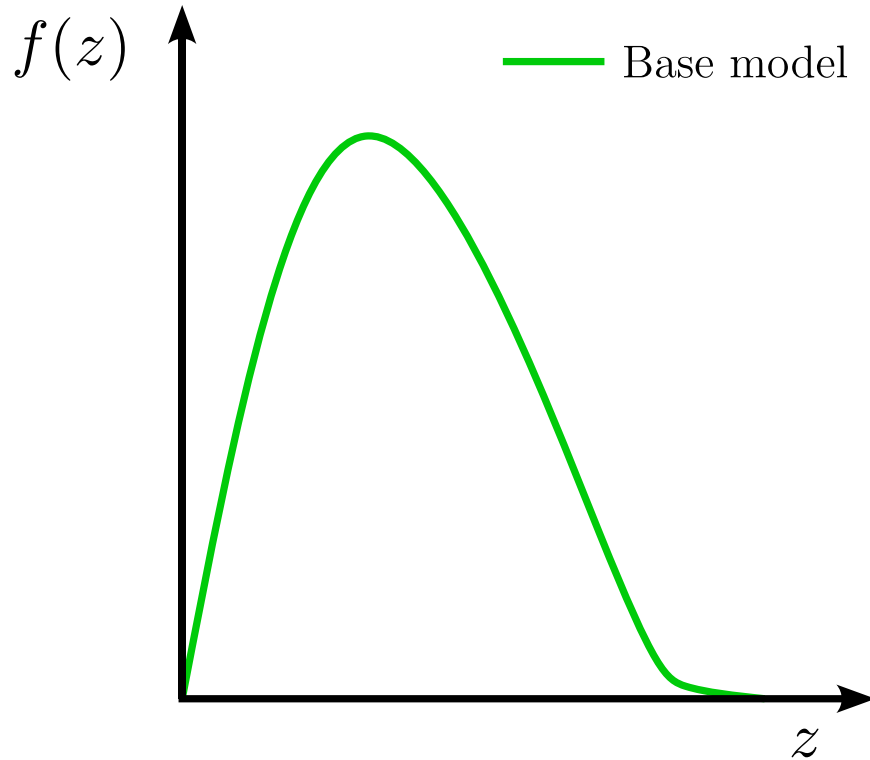
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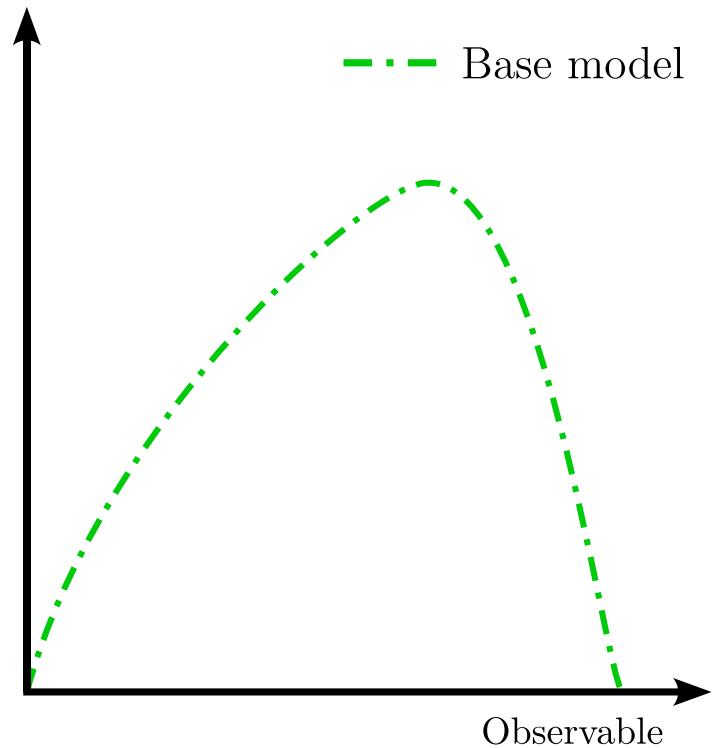
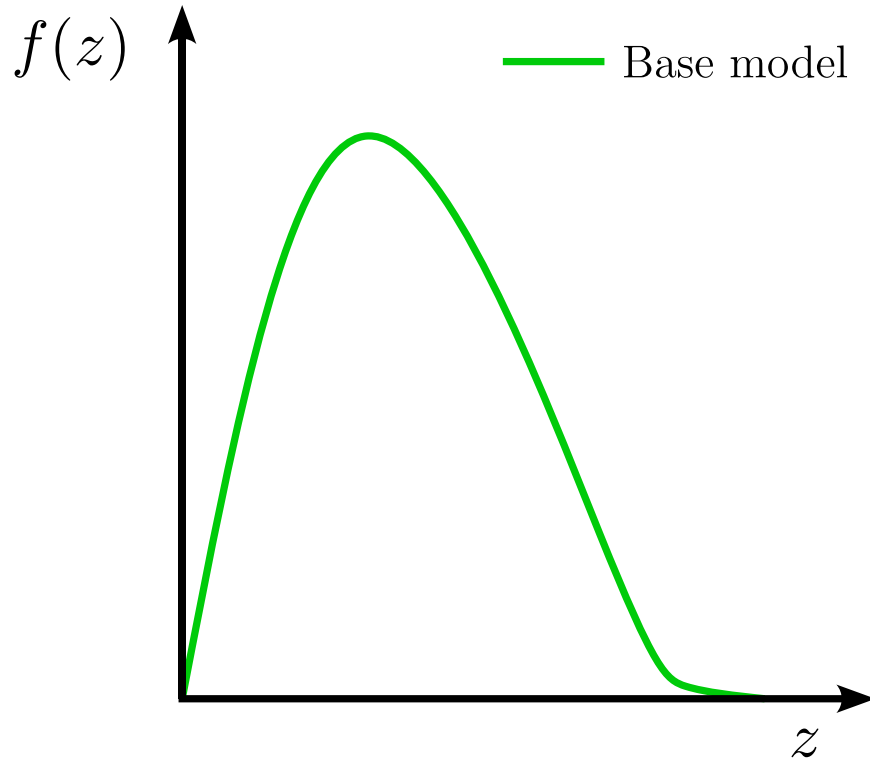
MLHAD efforts: big picture

Solving the “inverse problem of hadronization”



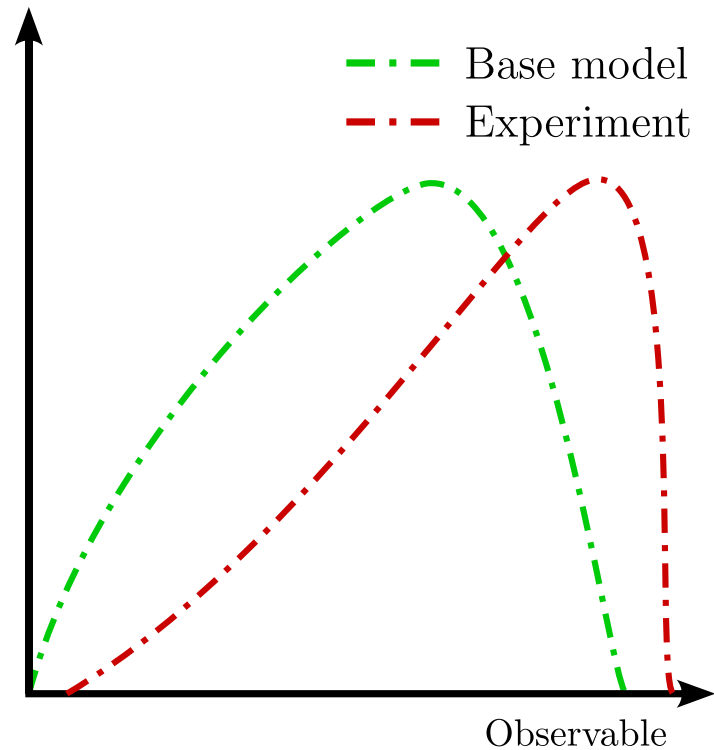
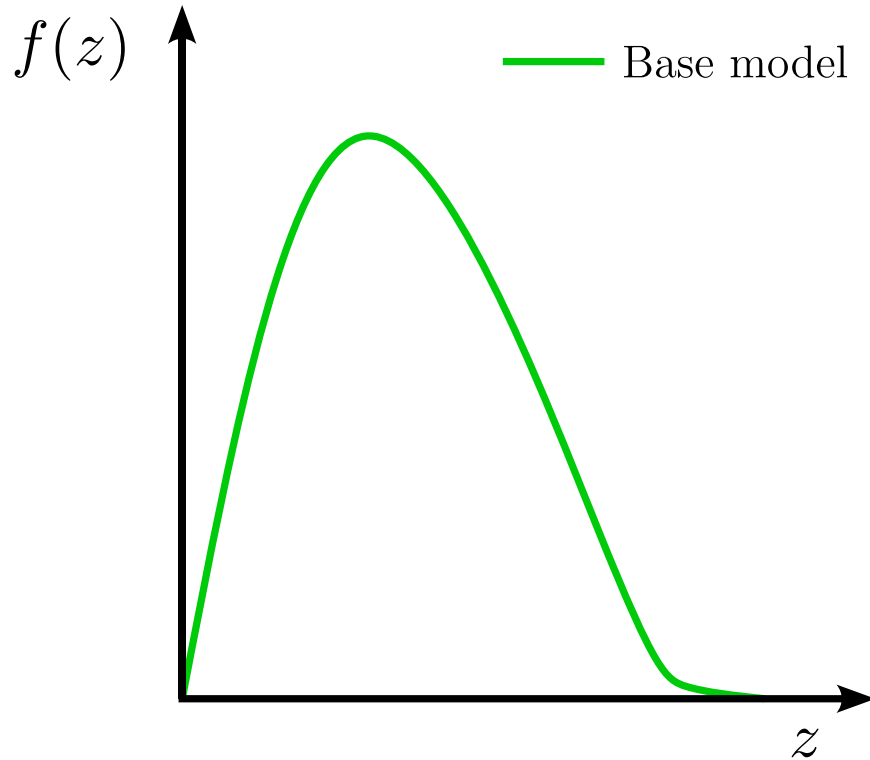
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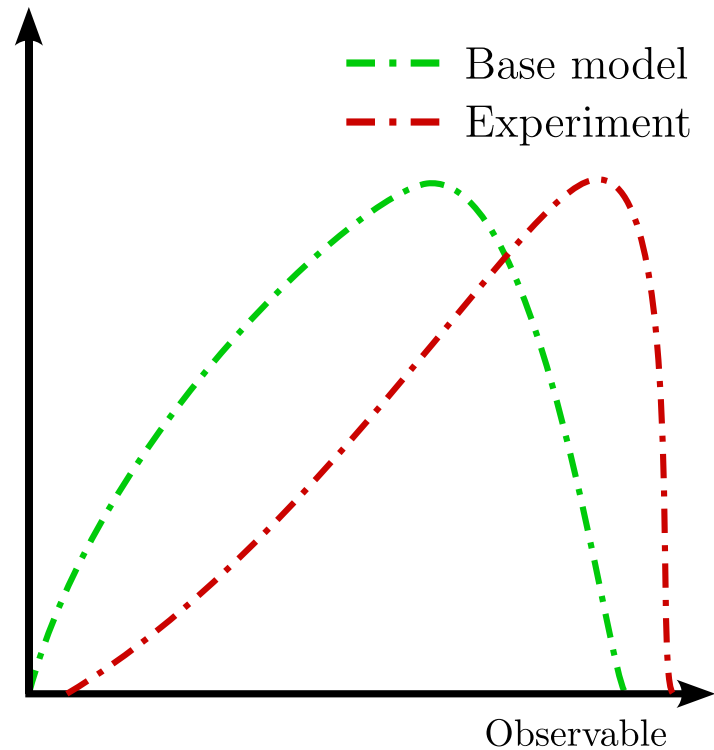
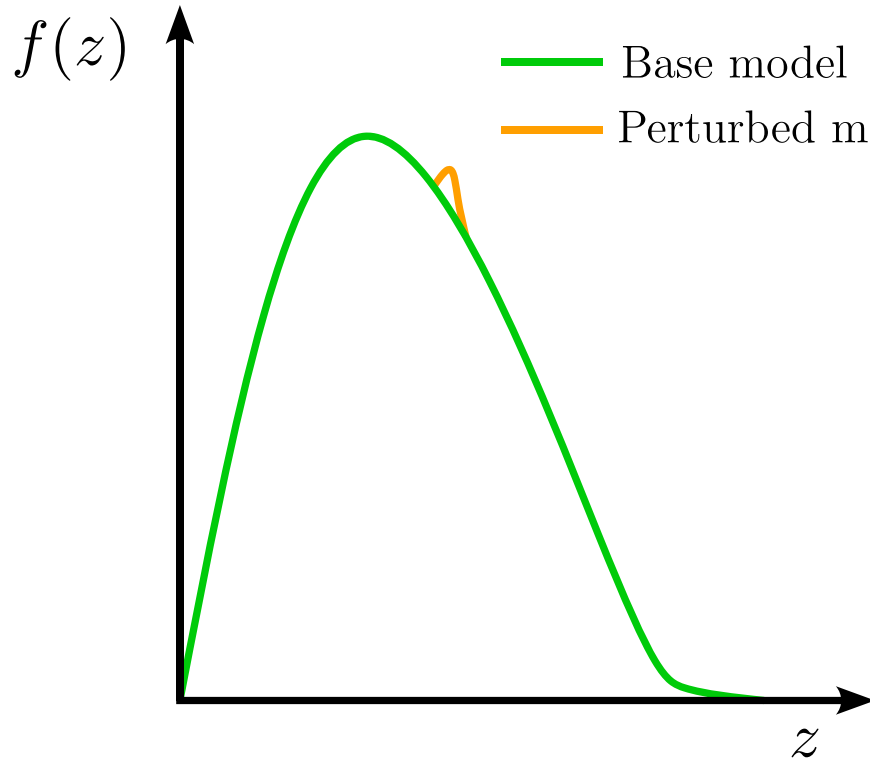
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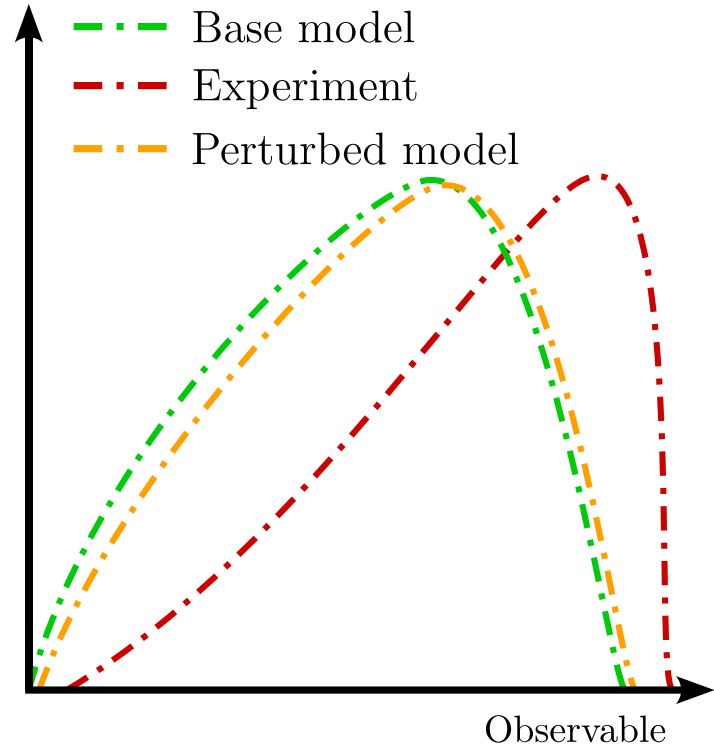
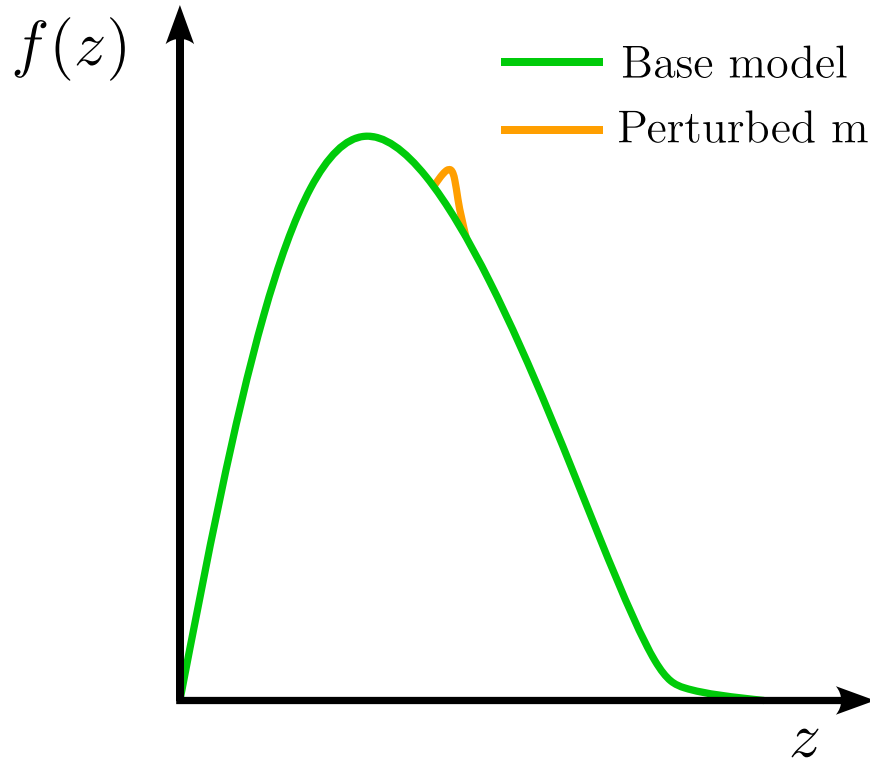
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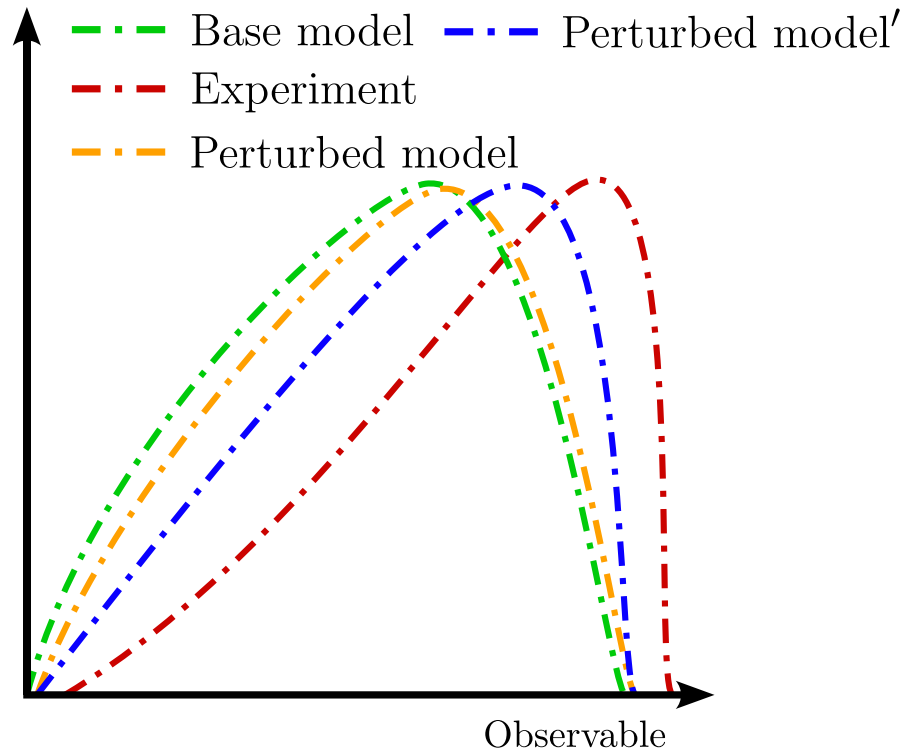
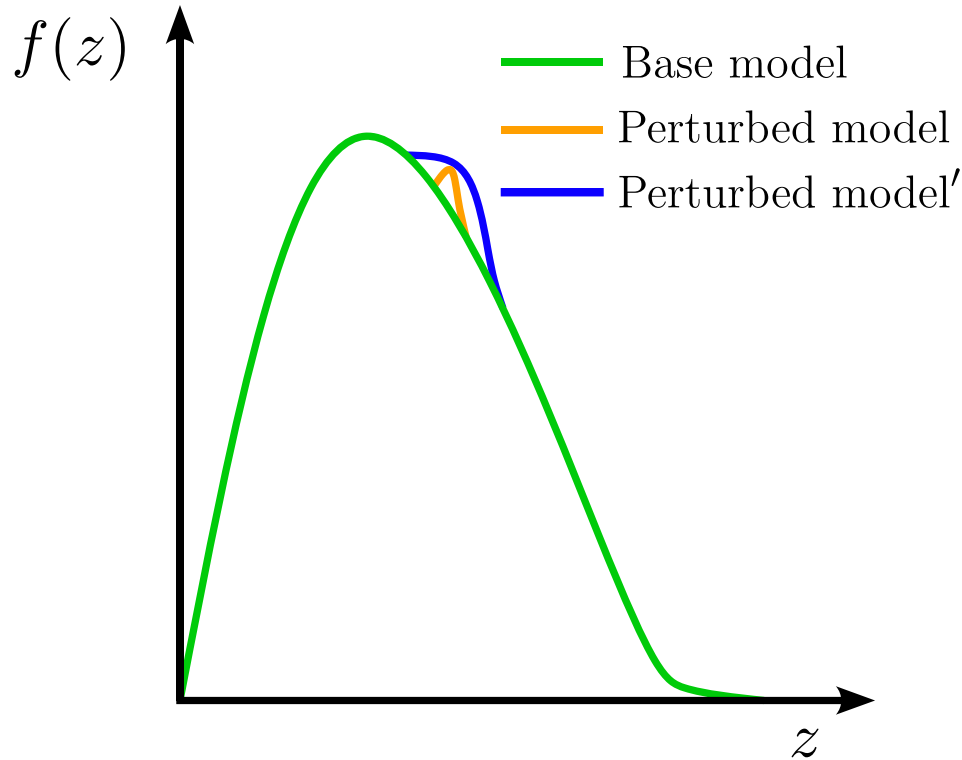
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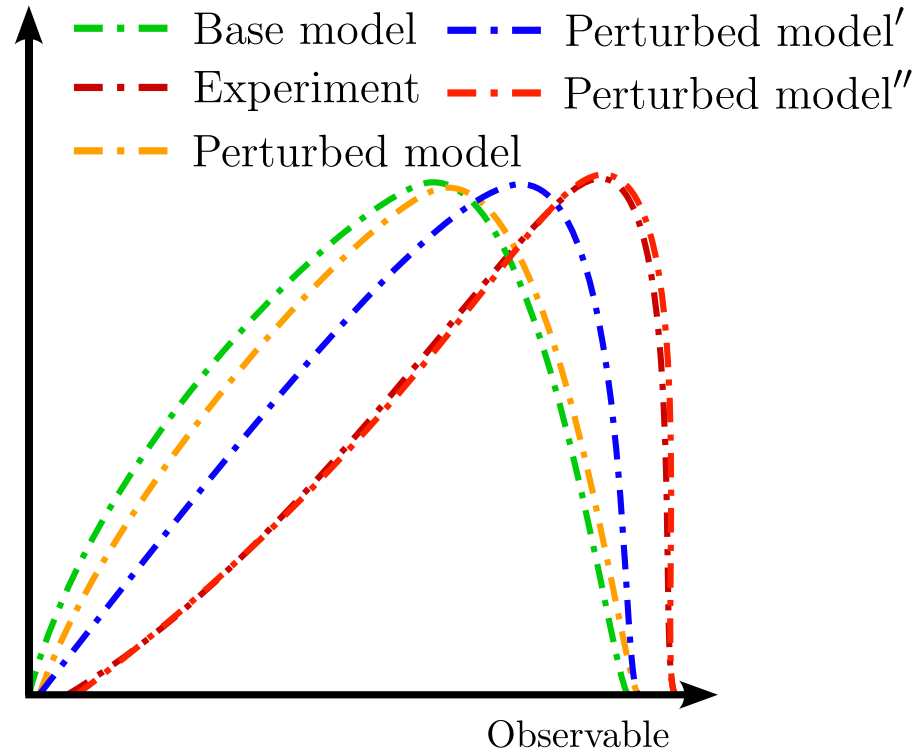
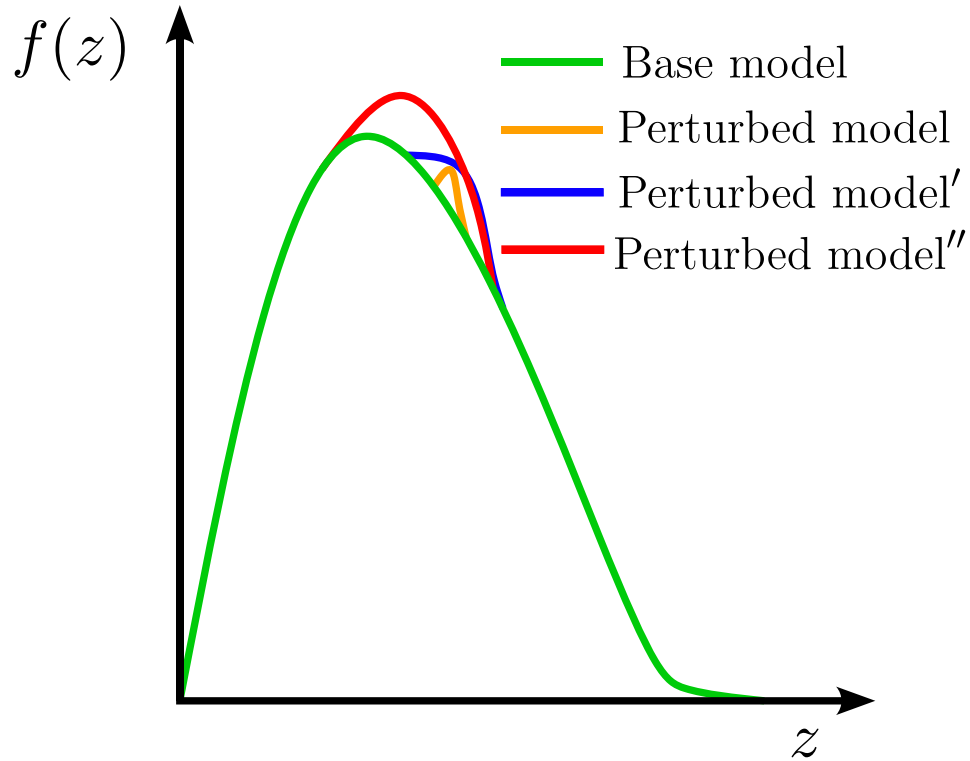
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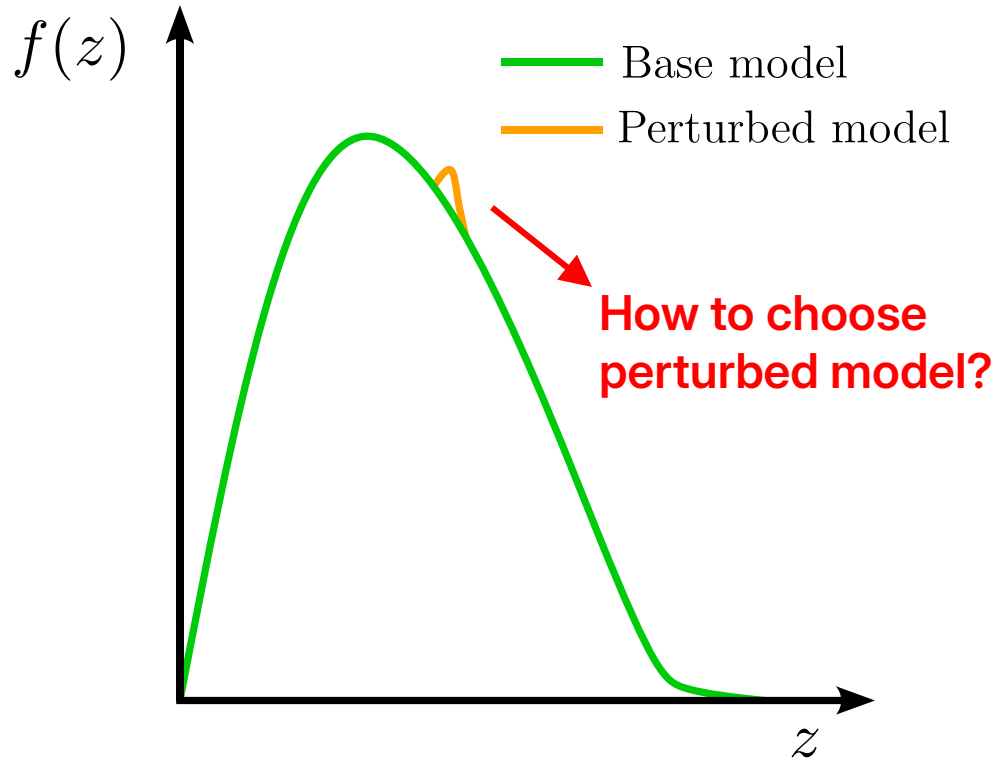
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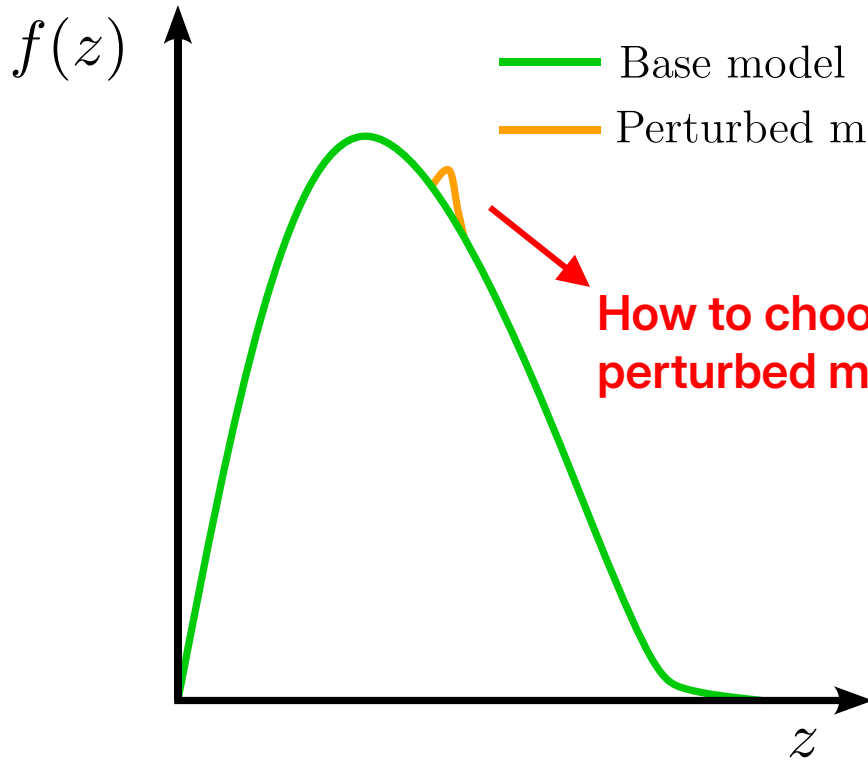
MLHAD efforts: big picture

Solving the “inverse problem of hadronization”



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Solving the "inverse problem of hadronization"



~~ML~~

vs

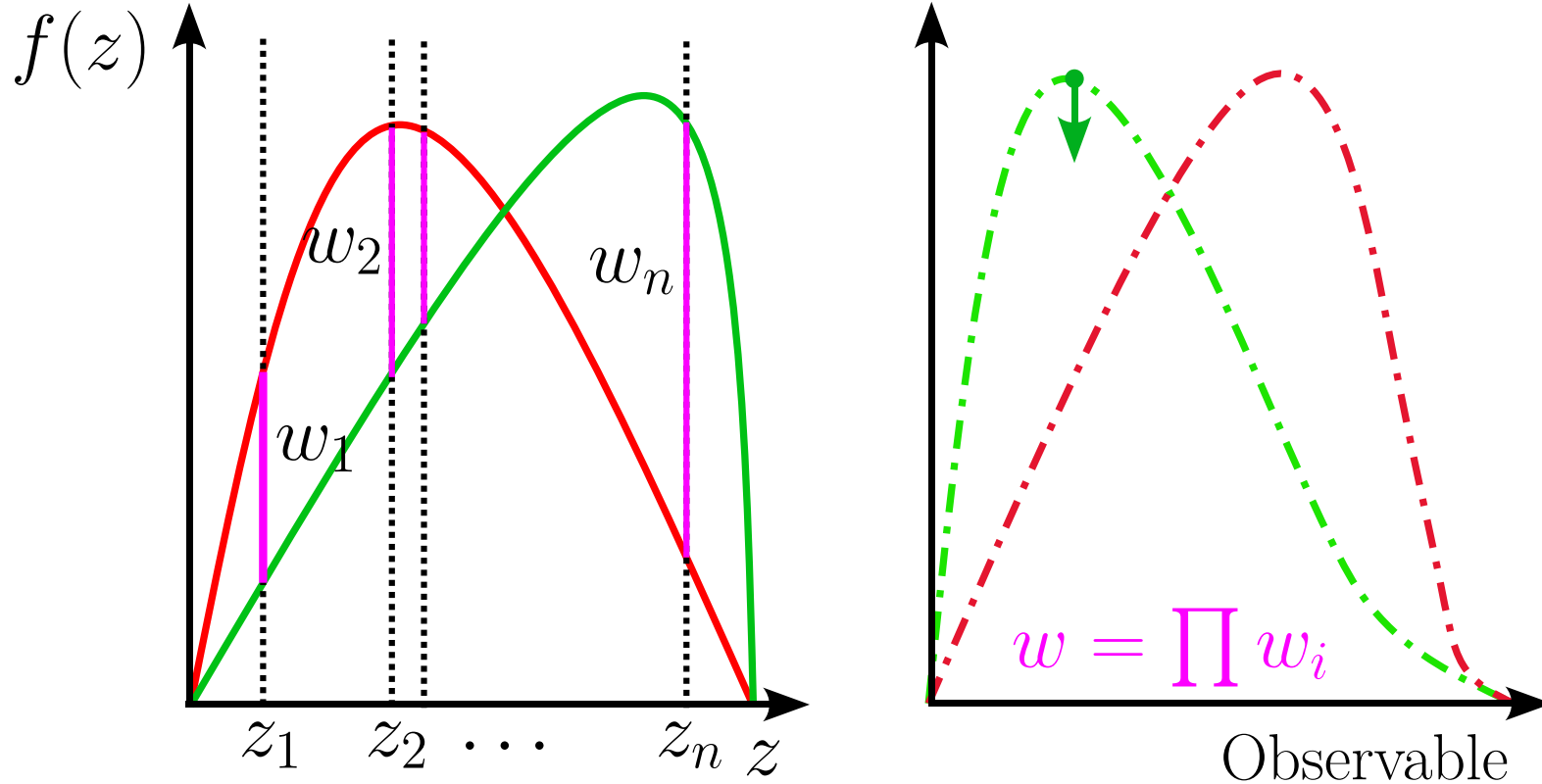
ML

- Vary Lund parameters (traditional "manual"/semi-"manual" tuning)

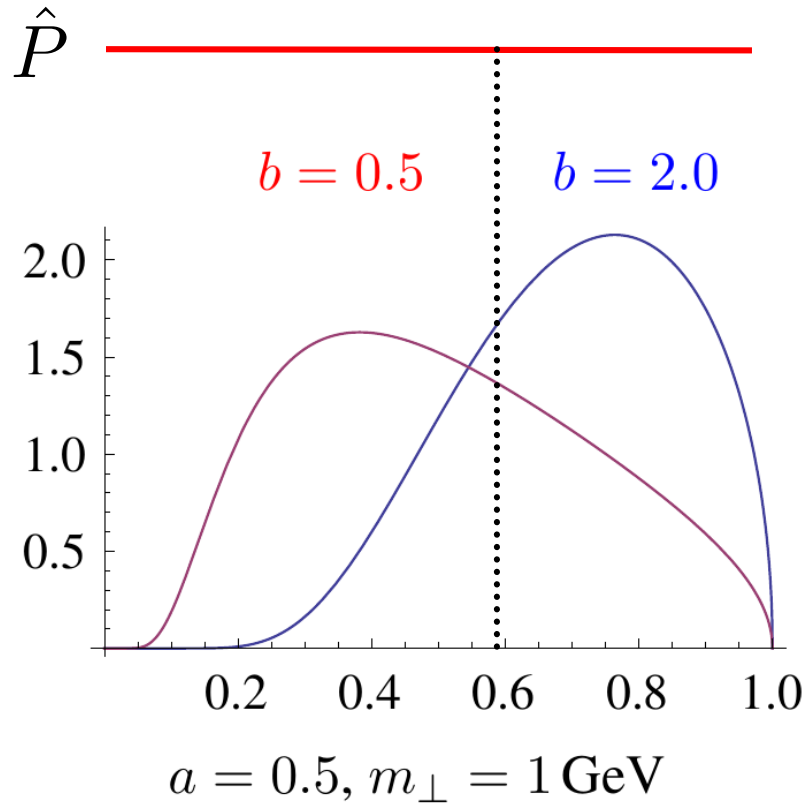
- ML-based (data-driven) fragmentation function
 - **MAGIC, HOMER**

- Hybrid – keep Lund fragmentation function, promote Lund parameters to differentiable objects
 - **Rejection sampling with autodifferentiation (RSA)**

Aside - Reweighting



Kinematic reweighting (2308.13459)



Rejection sampling:

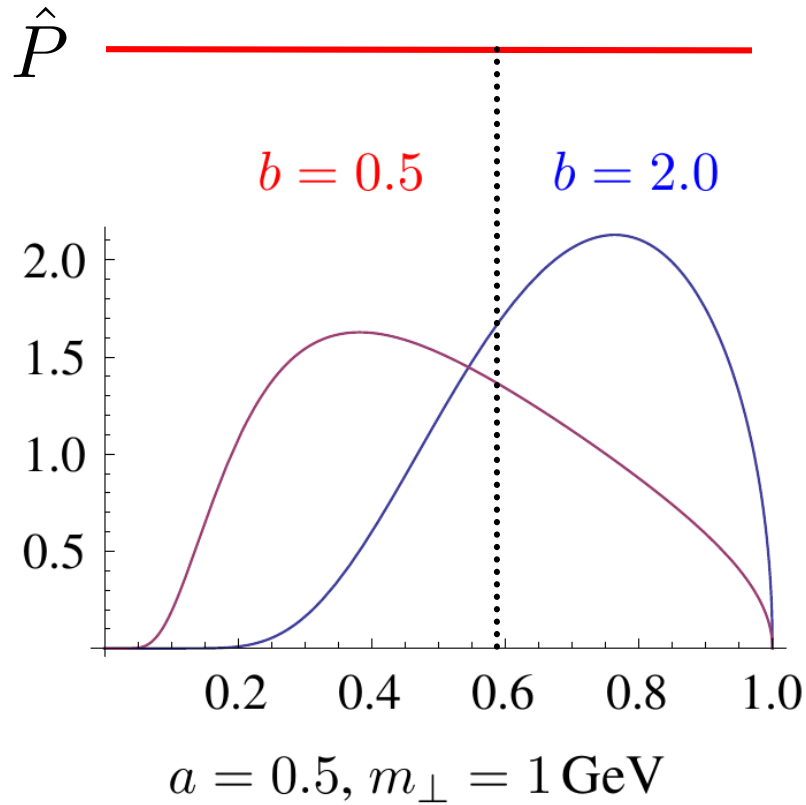
Acceptance probability: $P_{\text{accept}} = \frac{p(z, \theta)}{\hat{P}}$

Rejection probability: $P_{\text{reject}} = 1 - P_{\text{accept}}$

$$w_{\text{accept}} = \frac{p(z; \theta')}{p(z; \theta)}$$

$$w_{\text{reject}} = \frac{\hat{P} - p(z; \theta')}{\hat{P} - p(z; \theta)}$$

Kinematic reweighting (2308.13459)



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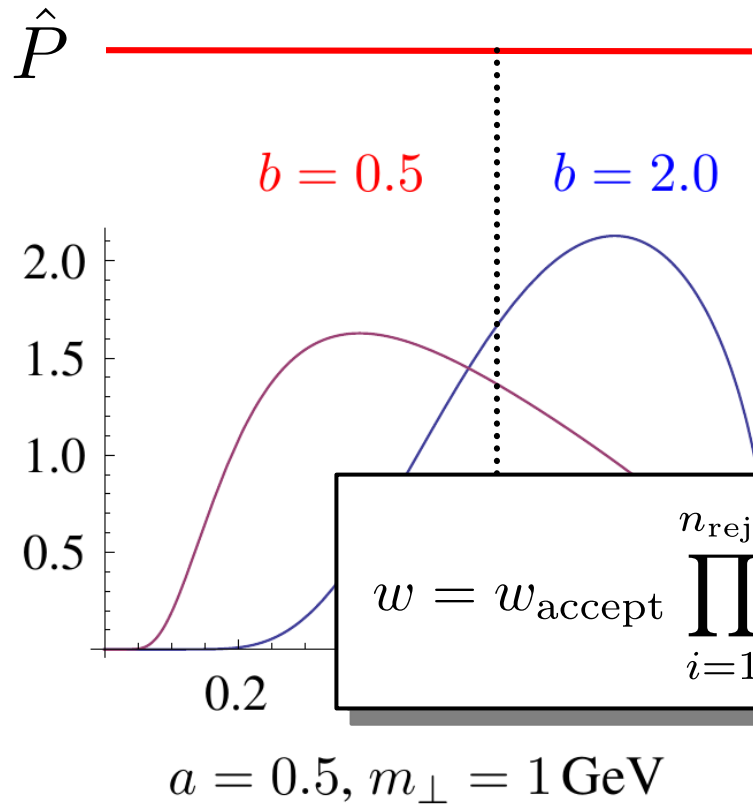
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Rejection sampling:

Acceptance probability: $P_{\text{accept}} = \frac{p(z, \theta)}{\hat{P}}$

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$$w = w_{\text{accept}} \prod_{i=1}^{n_{\text{rej.}}} w_{\text{reject}}^i$$

$$w_{\text{accept}} = \frac{p(z; \theta')}{p(z; \theta)}$$

$$w_{\text{reject}} = \frac{\hat{P} - p(z; \theta')}{\hat{P} - p(z; \theta)}$$

Kinematic reweighting (2308.13459)

\hat{P}

Rejection sampling:

Data-structure:

$$z = \begin{pmatrix} z_1 = \begin{pmatrix} \{m_T^{h_1}, z_{\text{accept}}^{h_1}, z_{\text{reject}}^{1,h_1}, \dots, z_{\text{reject}}^{n_{h_1},h_1}\} \\ \{m_T^{h_2}, z_{\text{accept}}^{h_2}, z_{\text{reject}}^{1,h_2}, \dots, z_{\text{reject}}^{n_{h_2},h_2}\} \\ \{m_T^{h_3}, z_{\text{accept}}^{h_3}, z_{\text{reject}}^{1,h_3}, \dots, z_{\text{reject}}^{n_{h_3},h_3}\} \\ \vdots \\ \{m_T^{h_4}, z_{\text{accept}}^{h_4}, z_{\text{reject}}^{1,h_4}, \dots, z_{\text{reject}}^{n_{h_4},h_4}\} \\ \vdots \\ z_N = \dots \end{pmatrix} \\ z_2 = \begin{pmatrix} \{m_T^{h_1}, z_{\text{accept}}^{h_1}, z_{\text{reject}}^{1,h_1}, \dots, z_{\text{reject}}^{n_{h_1},h_1}\} \\ \vdots \\ \{m_T^{h_4}, z_{\text{accept}}^{h_4}, z_{\text{reject}}^{1,h_4}, \dots, z_{\text{reject}}^{n_{h_4},h_4}\} \\ \vdots \end{pmatrix} \end{pmatrix}$$

$$w_n = \prod_{i=1}^{\tilde{N}_{h,n}} \left(\frac{f(z_{\text{accept}}^{h_i}; \{a, b\}_P)}{f(z_{\text{accept}}^{h_i}; \{a, b\}_B)} \right) \times \prod_{j=1}^{n_{h_i}} \left(\frac{\hat{f} - f(z_{\text{reject}}^{j,h_i}; \{a, b\}_P)}{\hat{f} - f(z_{\text{reject}}^{j,h_i}; \{a, b\}_B)} \right)$$

θ

accept

$a = 0.5, m_{\perp} = 1 \text{ GeV}$

$1 - P(z, \theta)$

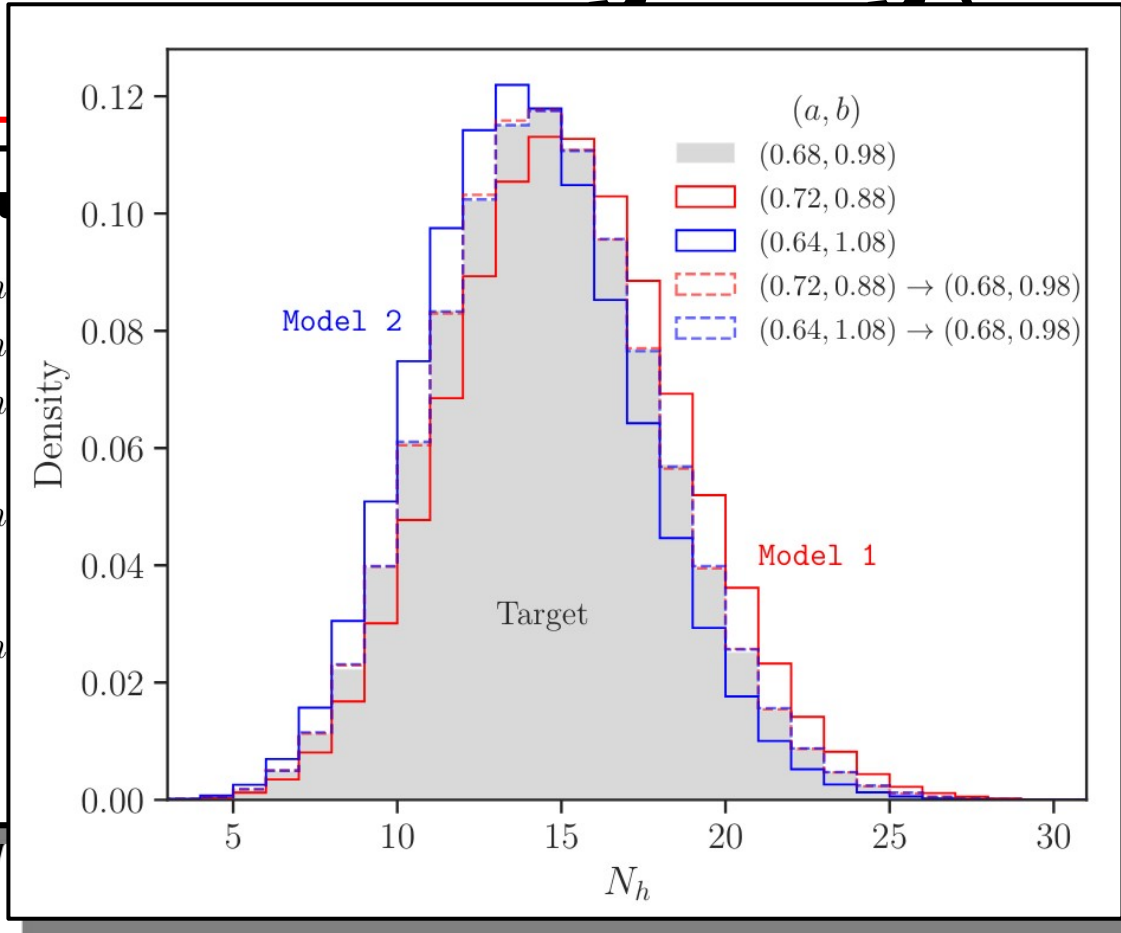
Kinematic reweighting (2308.13459)

\hat{P}

Data-structure

$$z = \begin{pmatrix} z_1 \\ z_2 \end{pmatrix} = \begin{pmatrix} \{m_1\} \\ \{m_2\} \\ \{m_3\} \\ \{m_4\} \\ \{m_5\} \\ \{m_6\} \\ \{m_7\} \\ \{m_8\} \\ \{m_9\} \\ \{m_{10}\} \\ \{m_{11}\} \\ \{m_{12}\} \\ \{m_{13}\} \\ \{m_{14}\} \\ \{m_{15}\} \\ \{m_{16}\} \\ \{m_{17}\} \\ \{m_{18}\} \\ \{m_{19}\} \\ \{m_{20}\} \end{pmatrix}$$

$a = 0.5, \tau$



$$\left. \begin{matrix} \theta \\ \text{cept} \\ \left. \begin{matrix} \text{ct; } \{a, b\}_P \\ \text{ct; } \{a, b\}_B \end{matrix} \right) \\ \left. \begin{matrix} z, \sigma \end{matrix} \right) \end{matrix} \right)$$

Kinematic reweighting (2308.13459)

\hat{P}

Rejection sampling:

Data-structure:

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Differentiable! → RSA

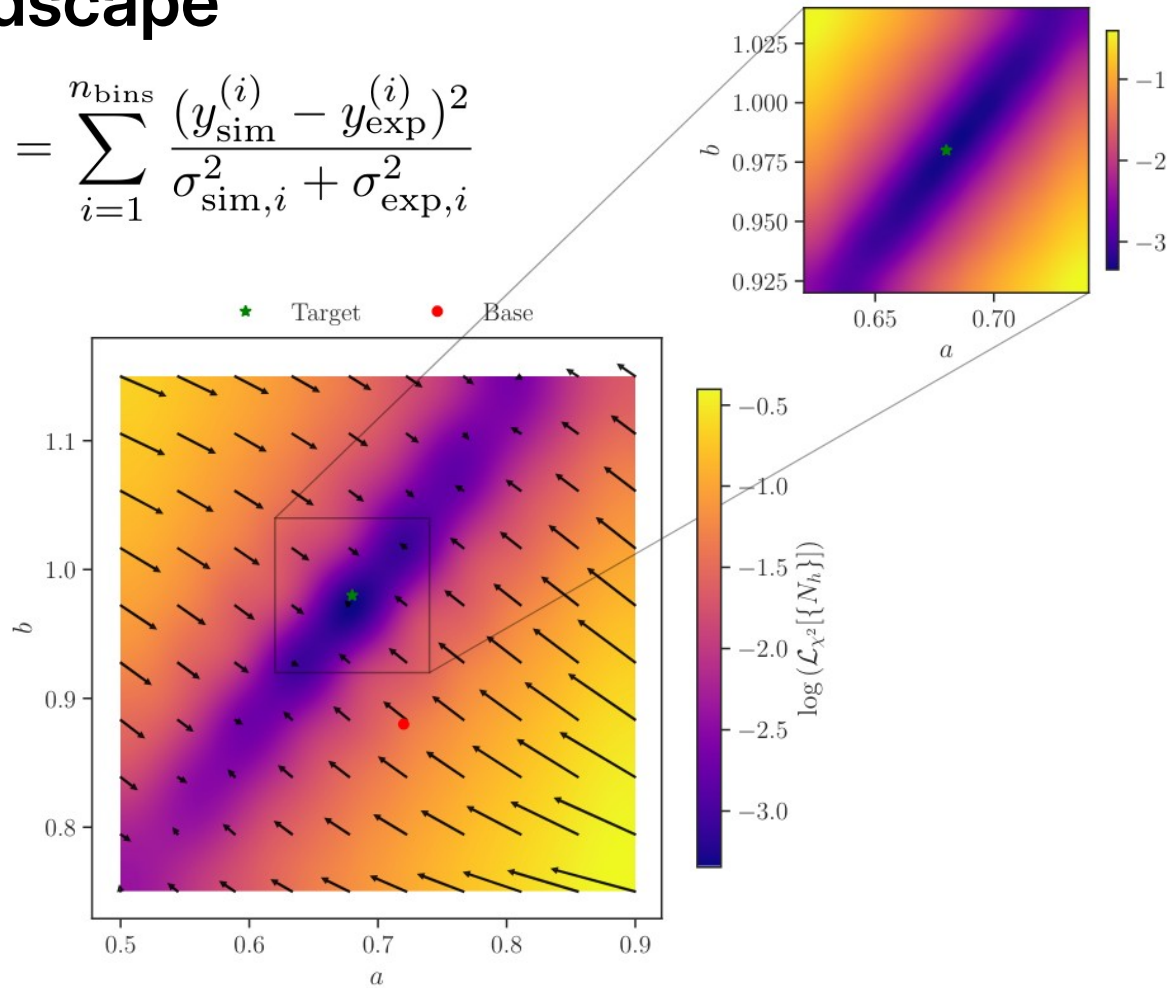
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$a = 0.5, m_{\perp} = 1 \text{ GeV}$

$1 - P(z, \theta)$

χ^2 -loss landscape

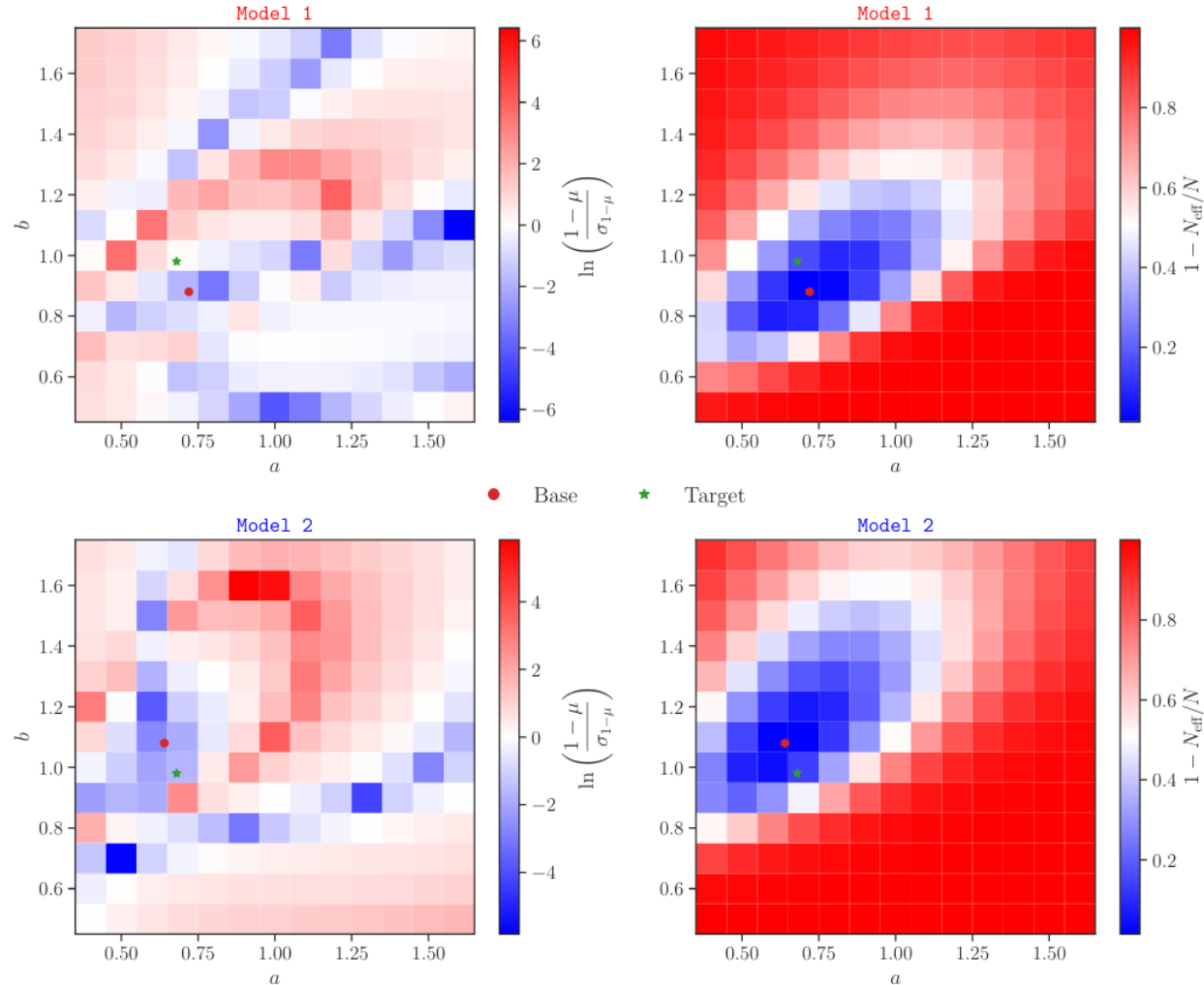
$$\mathcal{L}_{\chi^2}(\mathbf{y}_{\text{sim}}, \mathbf{y}_{\text{exp}}; \mathbf{w}) = \sum_{i=1}^{n_{\text{bins}}} \frac{(y_{\text{sim}}^{(i)} - y_{\text{exp}}^{(i)})^2}{\sigma_{\text{sim},i}^2 + \sigma_{\text{exp},i}^2}$$



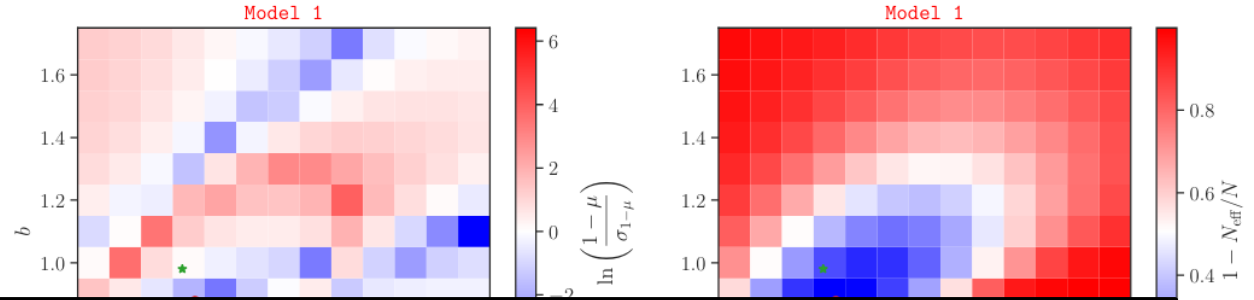
No free lunch

Statistical power drops off quickly as you move away from the base parameterization

$$\mu \equiv \sum_{i=1}^N \frac{w_i}{N}, \quad N_{\text{eff}} = \frac{\left(\sum_{i=1}^N w_i\right)^2}{\sum_{i=1}^N w_i^2}$$



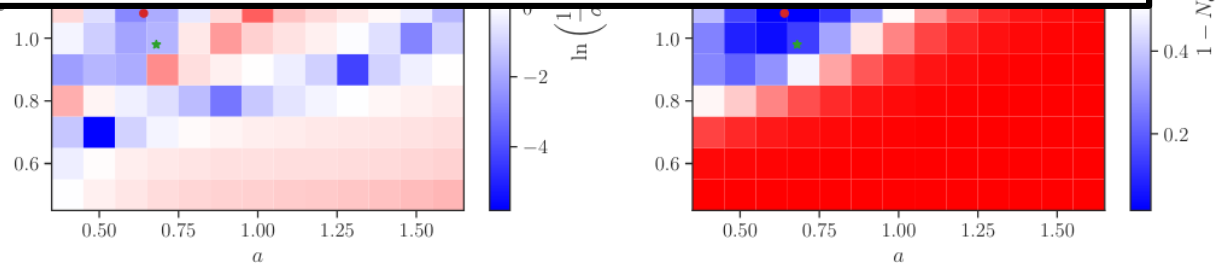
No free lunch



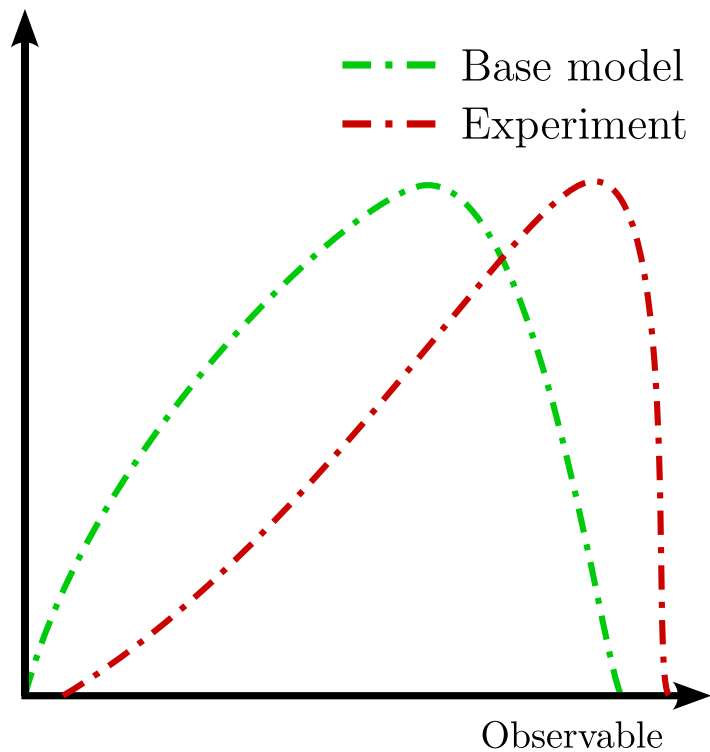
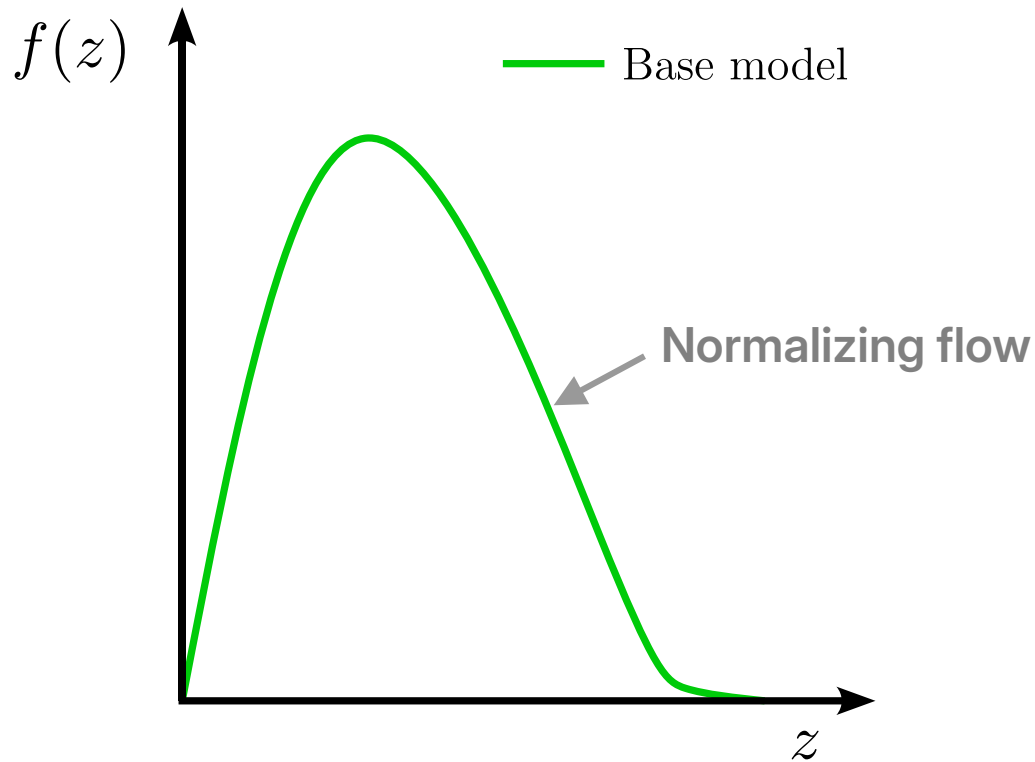
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Road to differentiable Pythia?

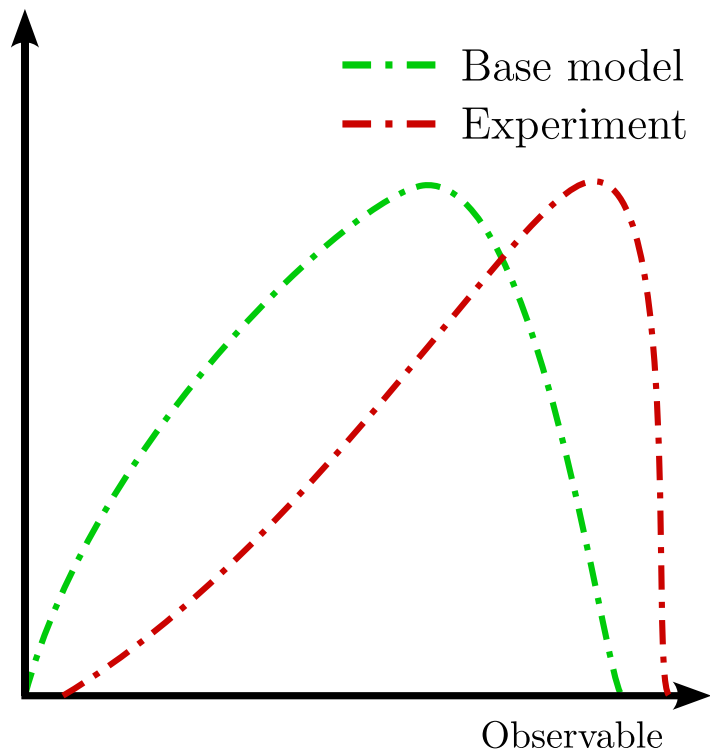
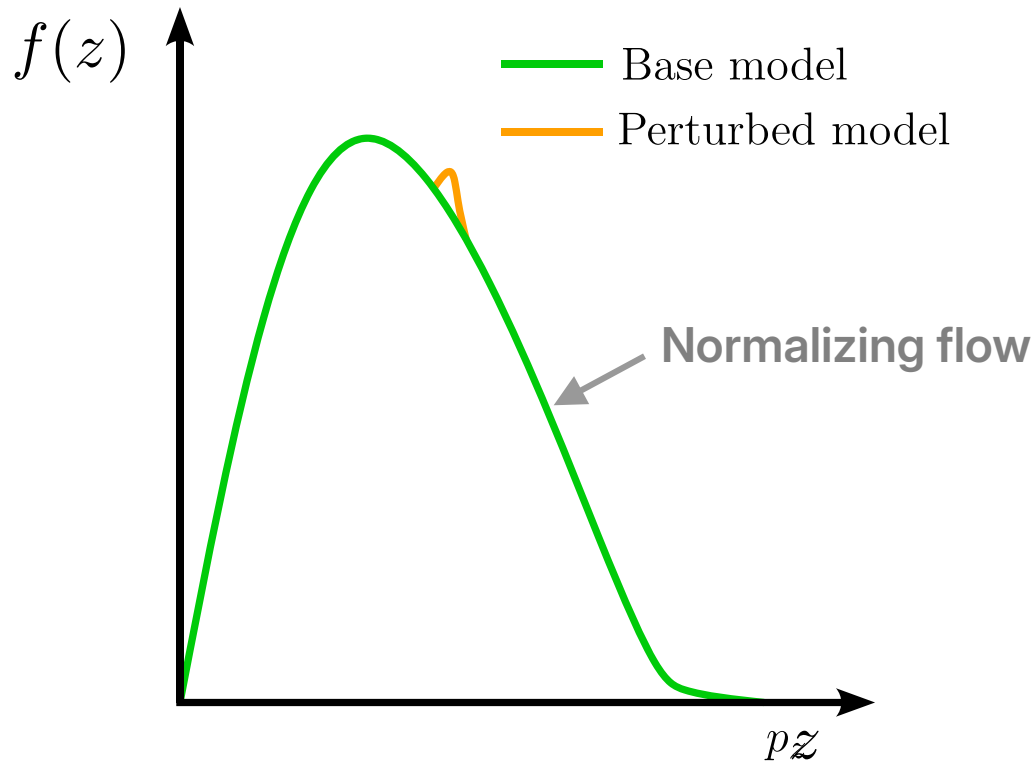
$\mu \equiv \sum_{i=1}^I$



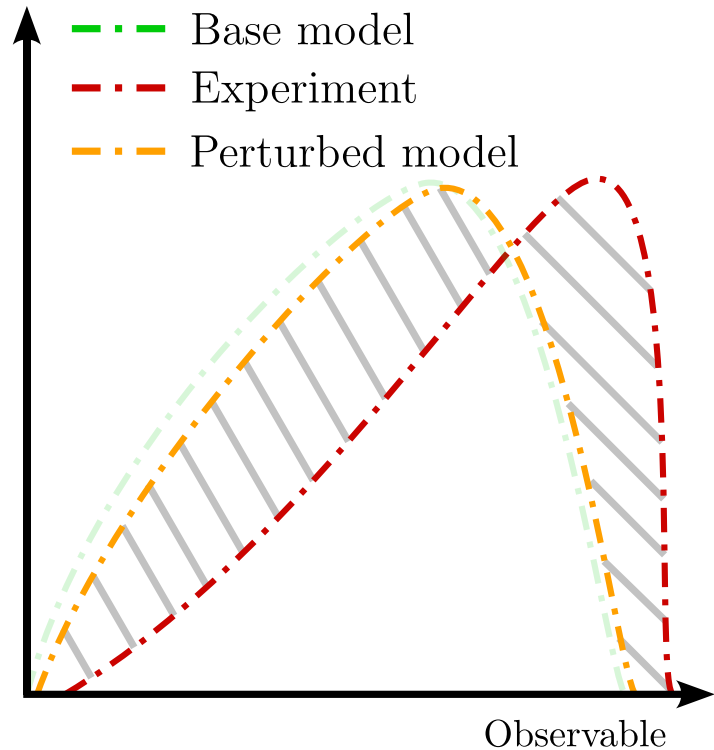
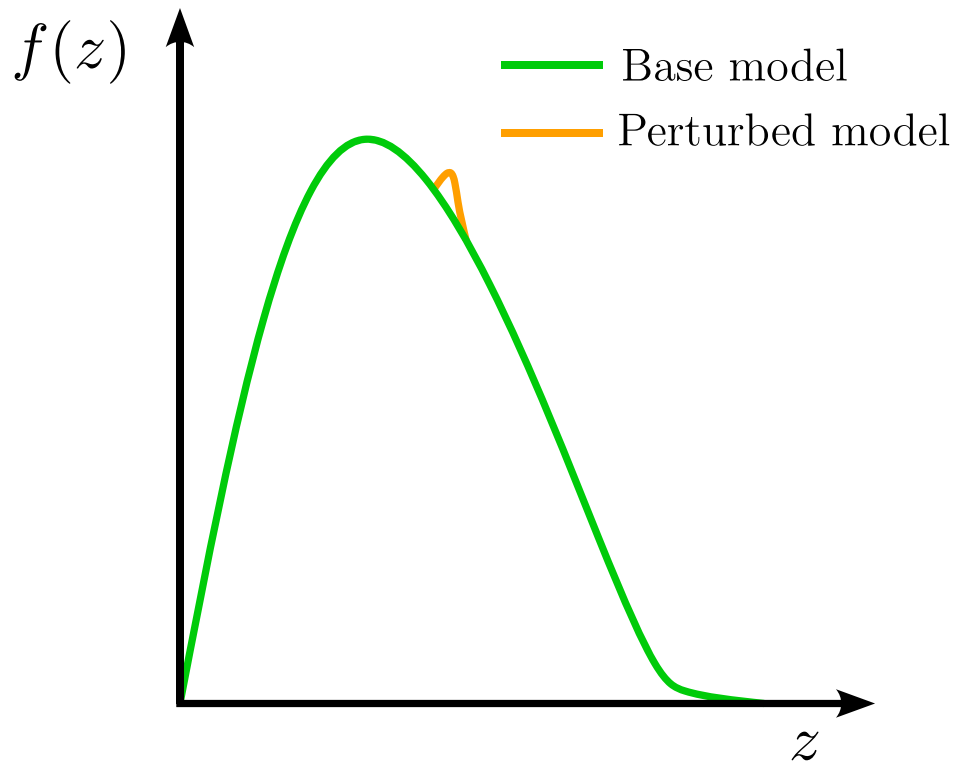
MLHAD efforts: **MAGIC**



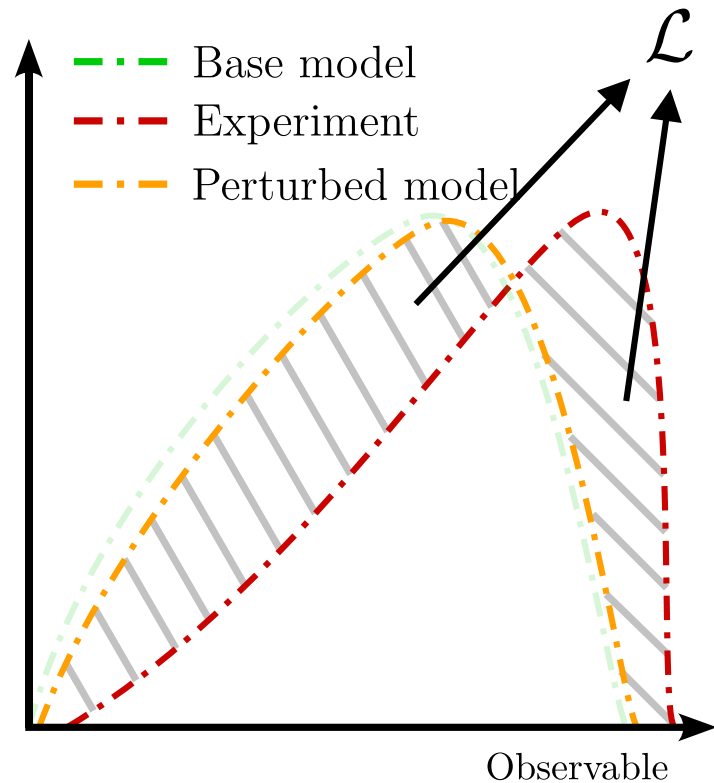
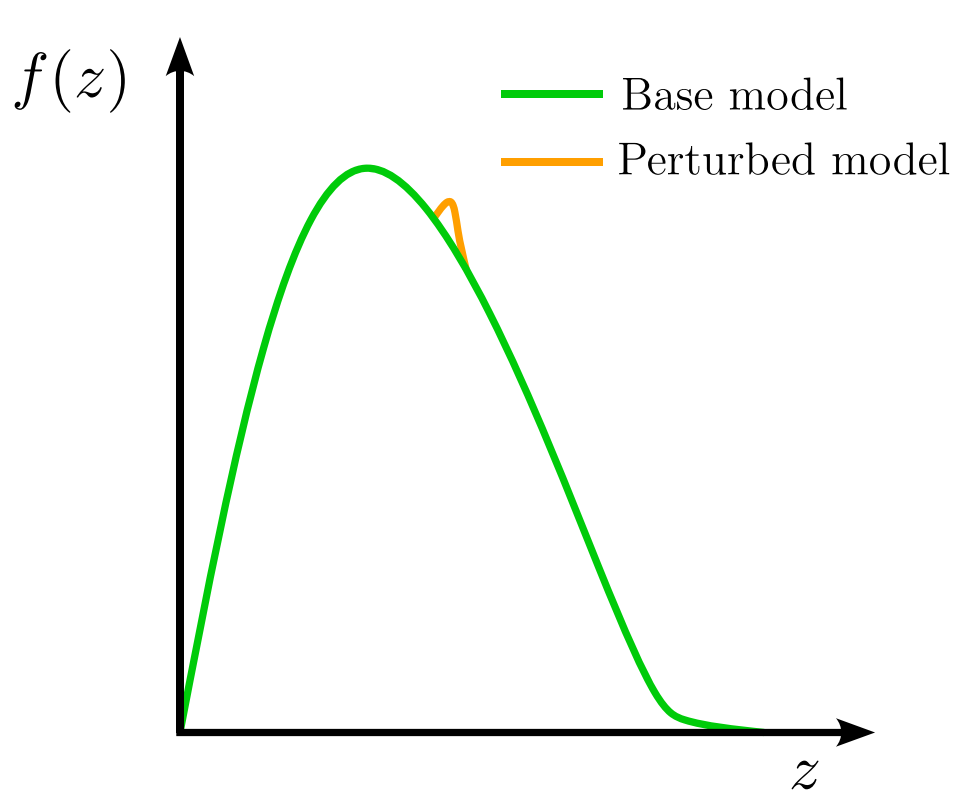
MLHAD efforts: **MAGIC**



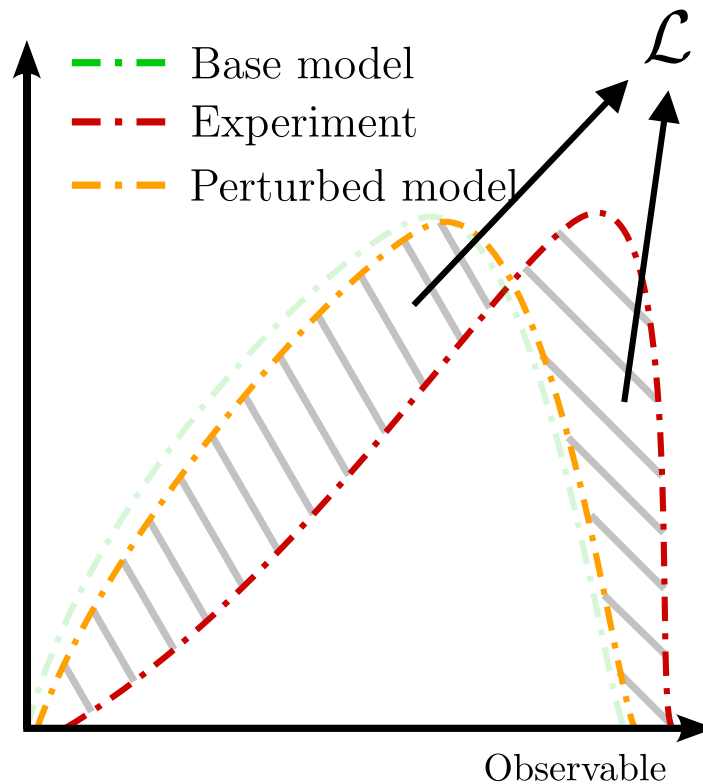
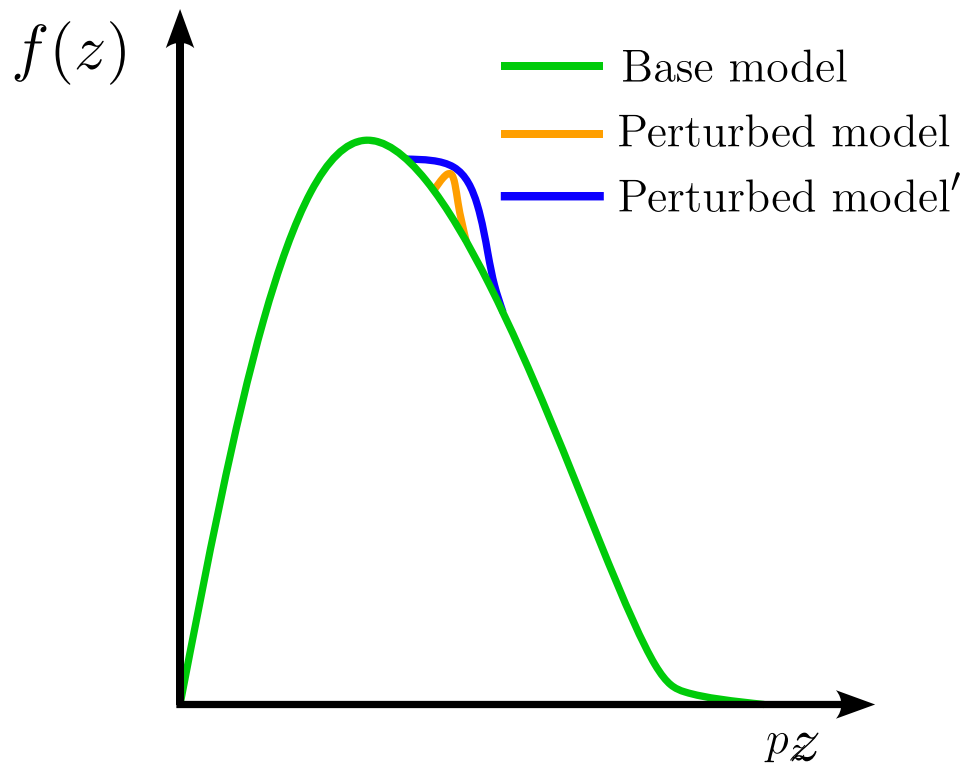
MLHAD efforts: **MAGIC**



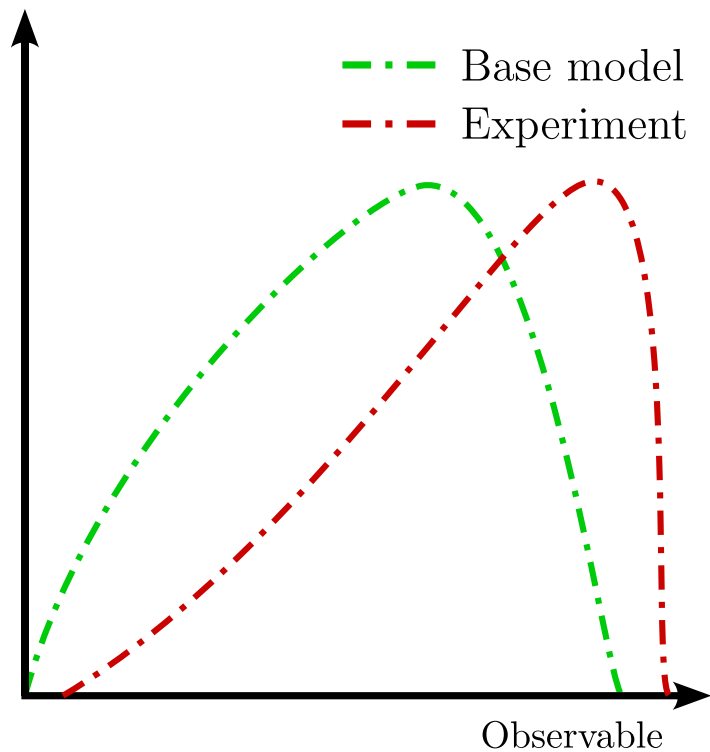
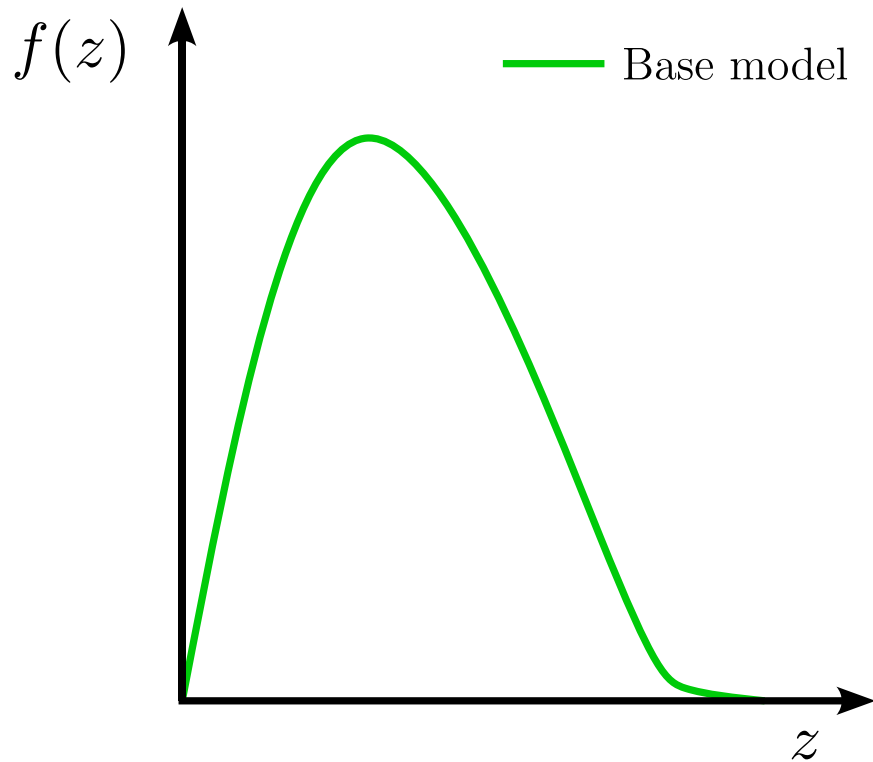
MLHAD efforts: **MAGIC**



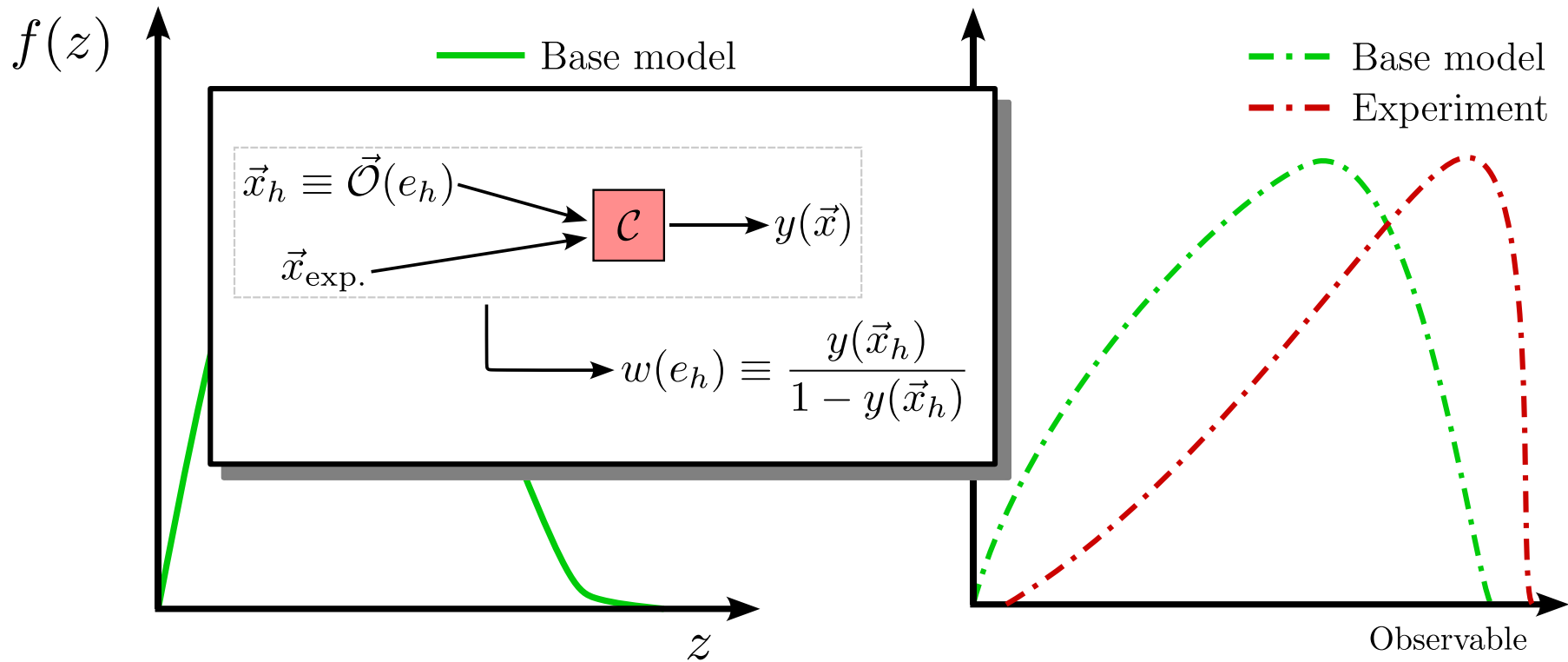
MLHAD efforts: **MAGIC**



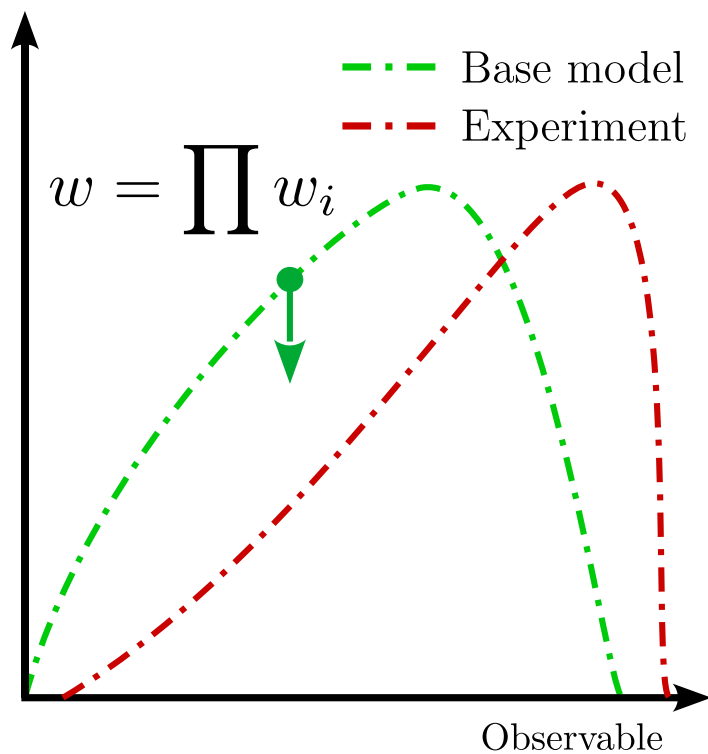
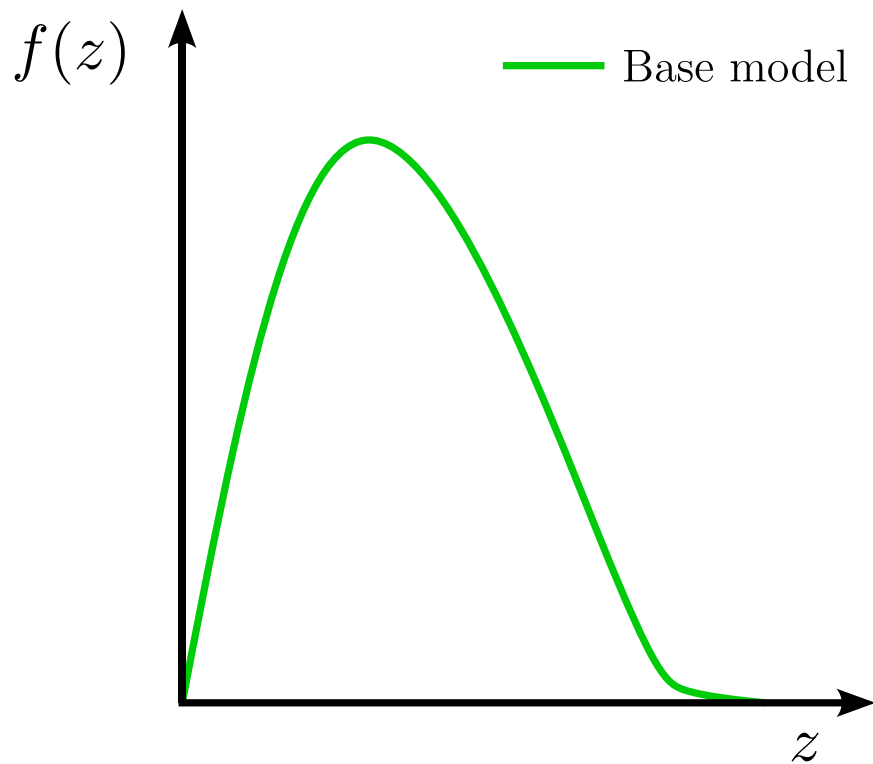
MLHAD efforts: HOMER



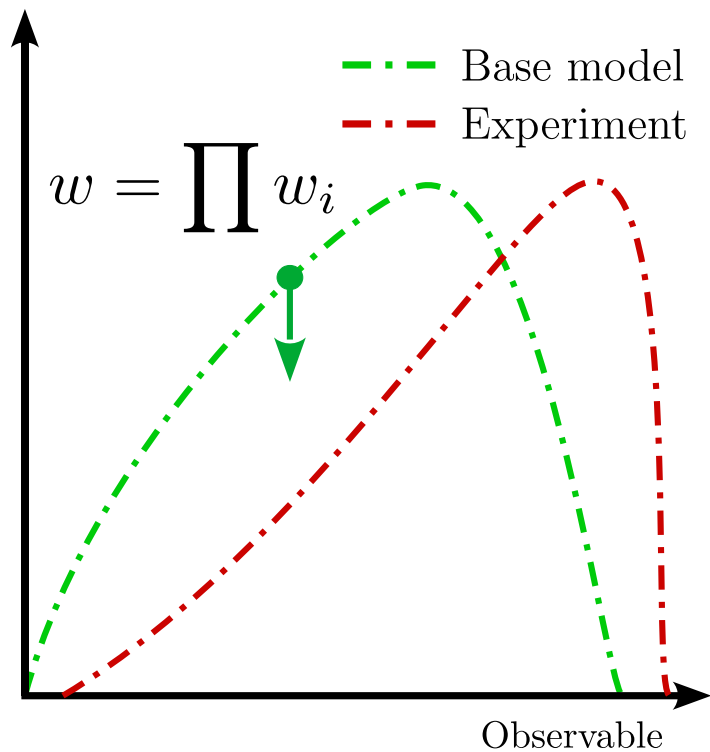
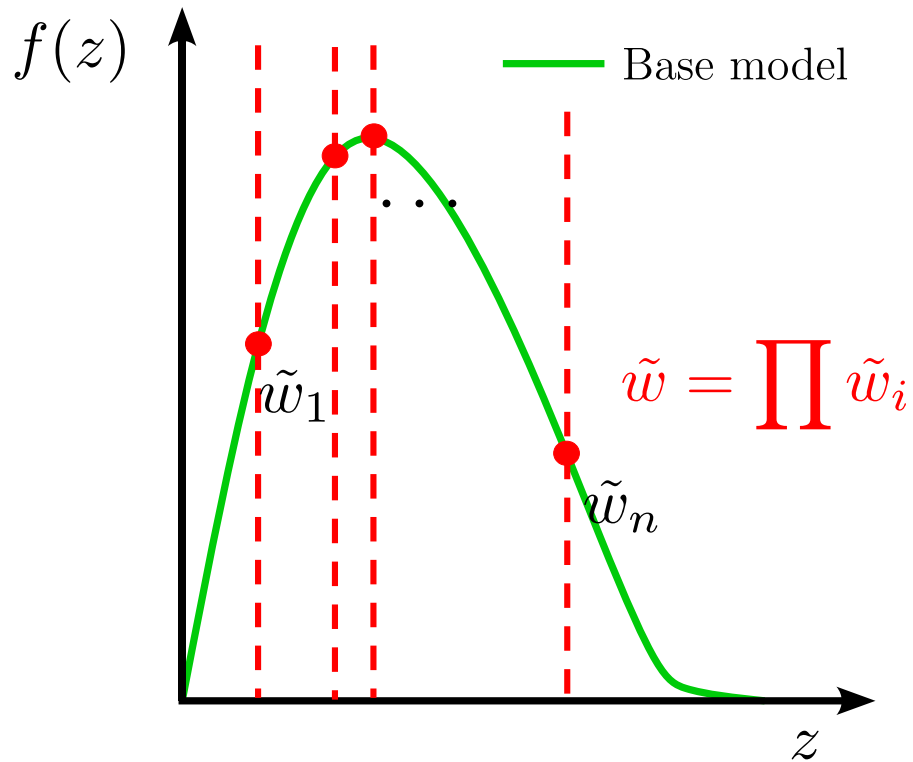
MLHAD efforts: HOMER



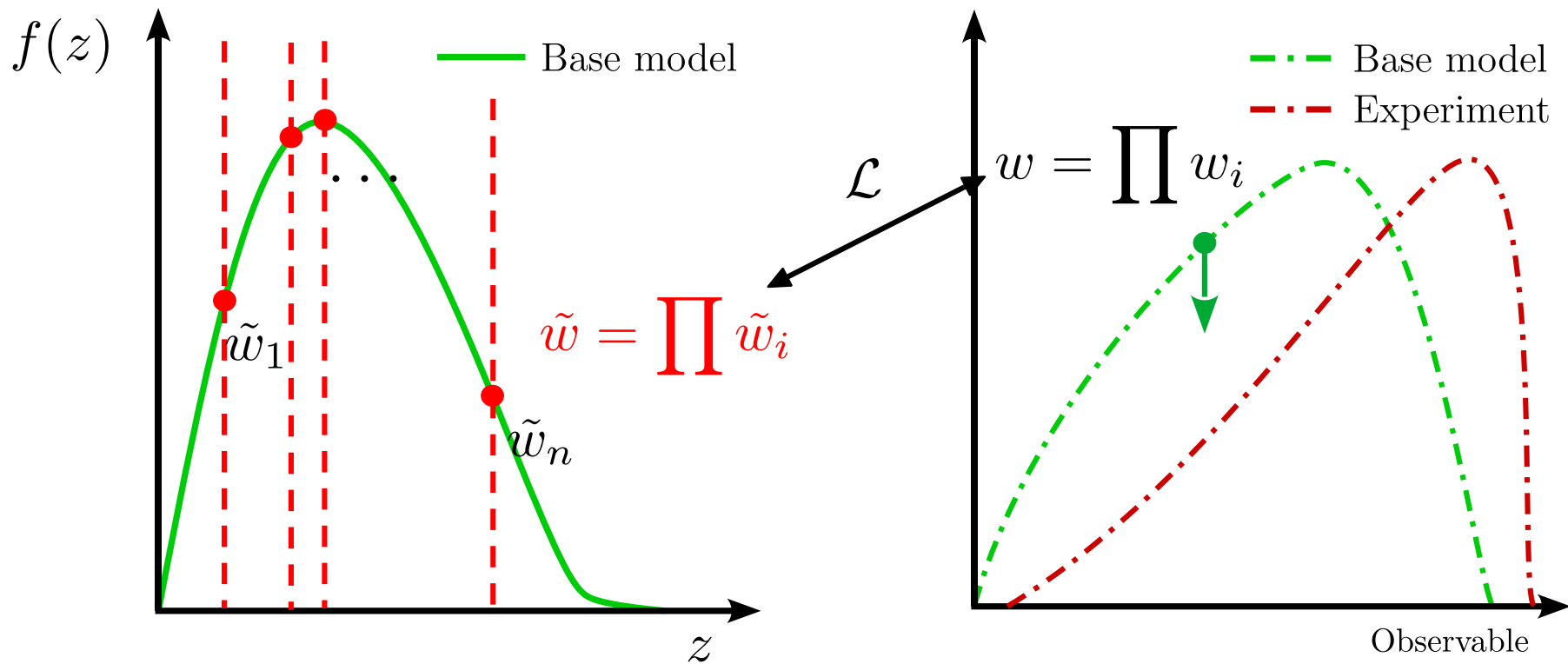
MLHAD efforts: HOMER



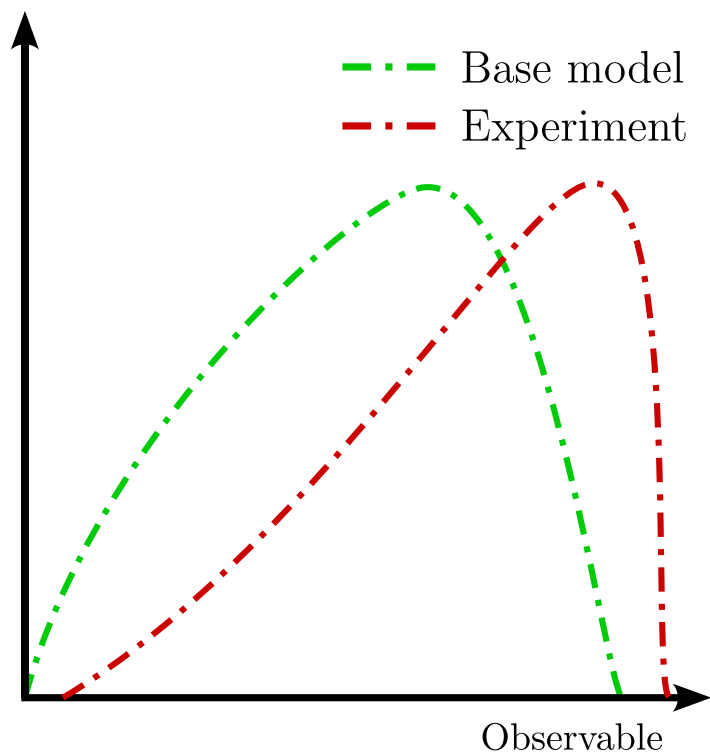
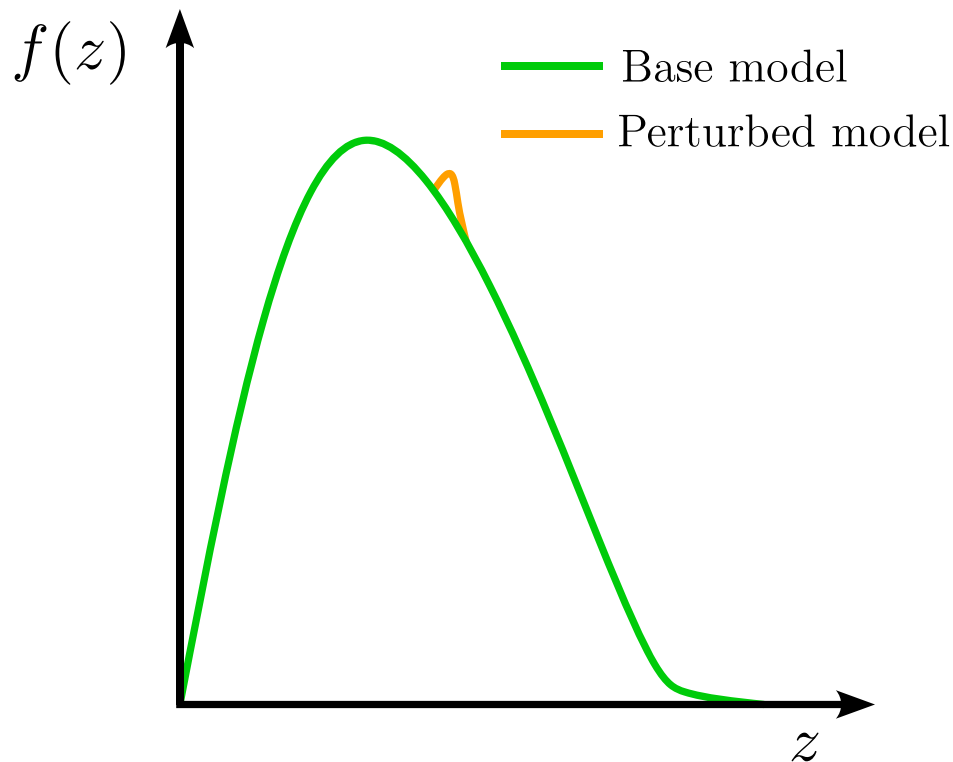
MLHAD efforts: HOMER



MLHAD efforts: HOMER



MLHAD efforts: HOMER



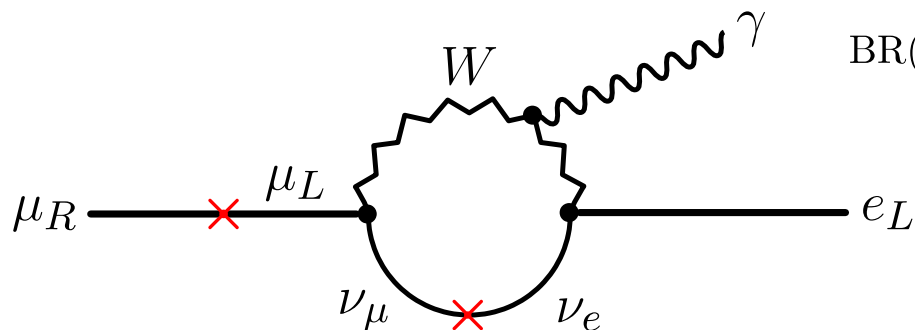
Heavy and light new physics in rare lepton decays

Standard lore - CLFV

- The Standard Model (SM) has an accidental global flavor symmetry

$$U(1)_e \times U(1)_\mu \times U(1)_\tau$$

- Because $m_\nu \neq 0$ charged-lepton-flavor violation (CLFV) can occur at one-loop



$$\text{BR}(\mu \rightarrow e\gamma) \simeq \frac{3\alpha}{32\pi M_W^4} |U_{\mu 3} U_{e 3}^* \Delta m_{31}^2 + U_{\mu 2} U_{e 2}^* \Delta m_{21}^2|^2 \simeq 10^{-54}$$

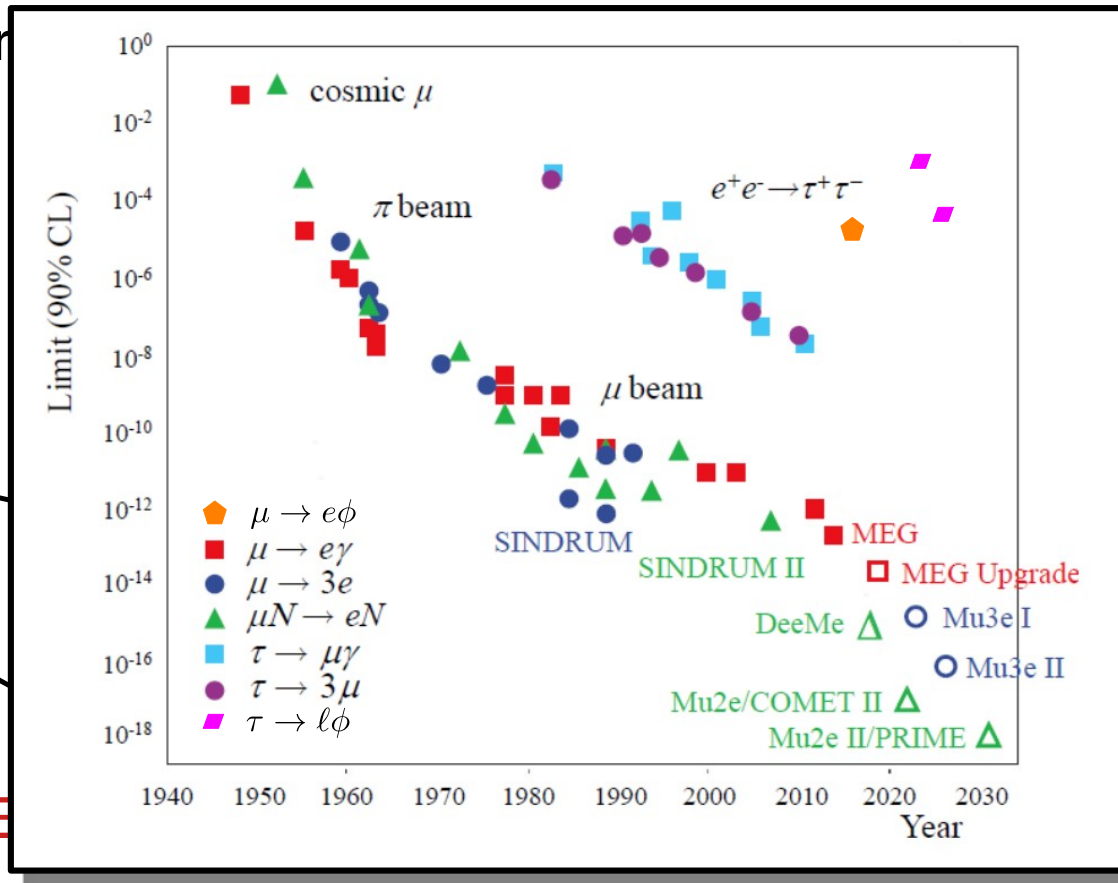
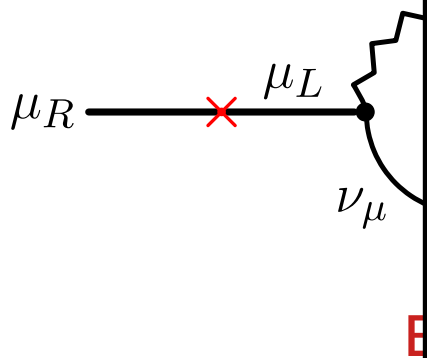
$$\text{BR}(\mu \rightarrow eee) \simeq \frac{\alpha}{3\pi} \left(\log \frac{m_\mu^2}{m_e^2} - 3 \right) \times \text{BR}(\mu \rightarrow e\gamma)$$

$$\text{CR}(\mu N \rightarrow eN) \simeq \alpha \times \text{BR}(\mu \rightarrow e\gamma)$$

Bottom line: Observing CLFV = new physics

Standard lore - CLFV

- The Standard Model
- Because m_ν is tiny, CLFV processes occur at one-loop



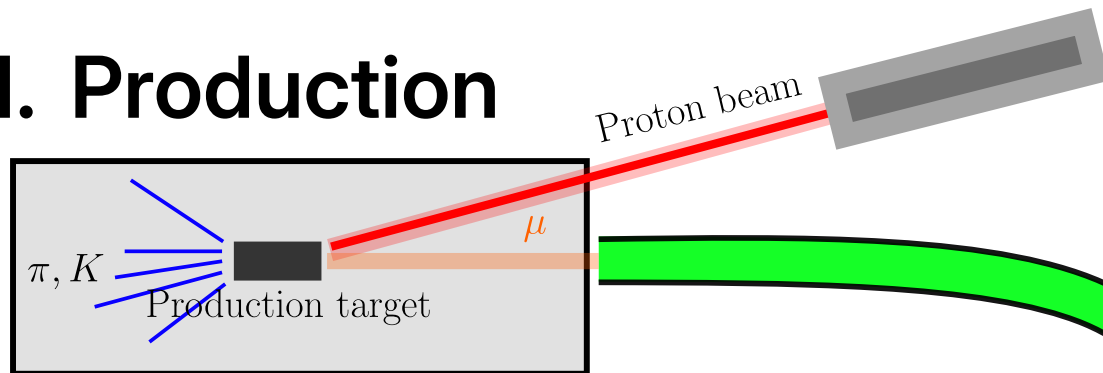
symmetry

occur at

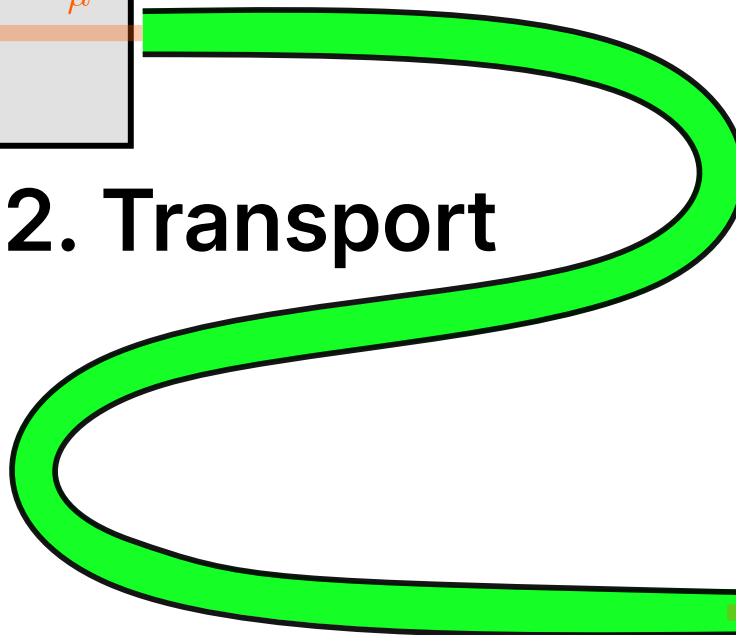
$$|J_{\mu 2} U_{e 2}^* \Delta m_{21}^2|^2 \simeq 10^{-54}$$

$$\left. \begin{array}{l} 3 \\ \times \text{BR}(\mu \rightarrow e \gamma) \end{array} \right) \text{BR}(\mu \rightarrow e \gamma)$$

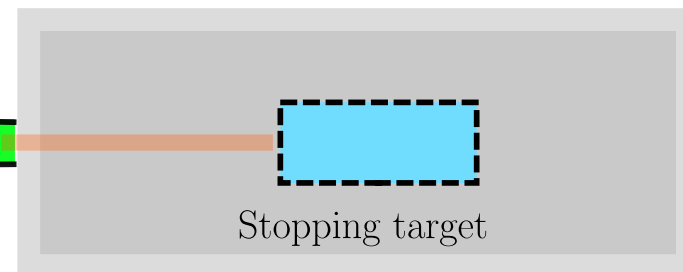
1. Production



2. Transport



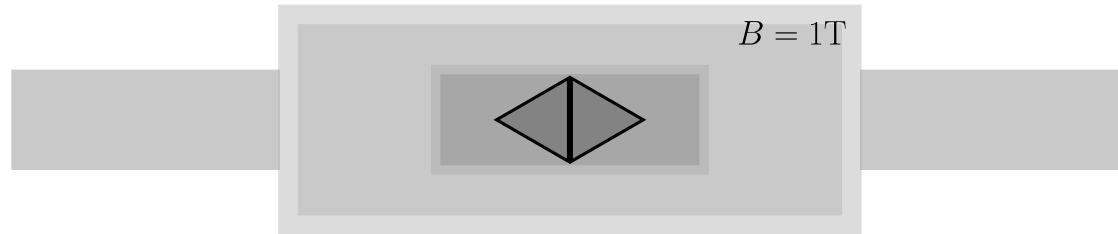
3. Stopping



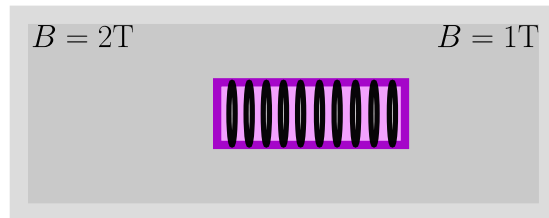
4. Detection (phenomenology is influenced by detector)

Mu3e (μ^+)

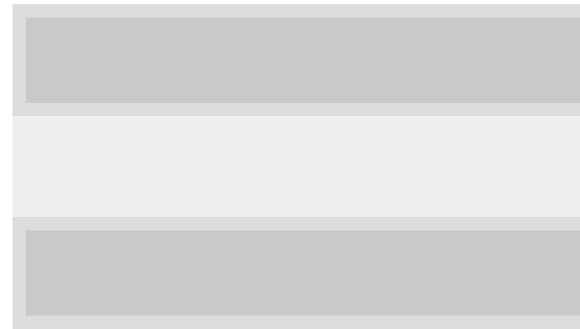
* requires $p_T > 10$ MeV

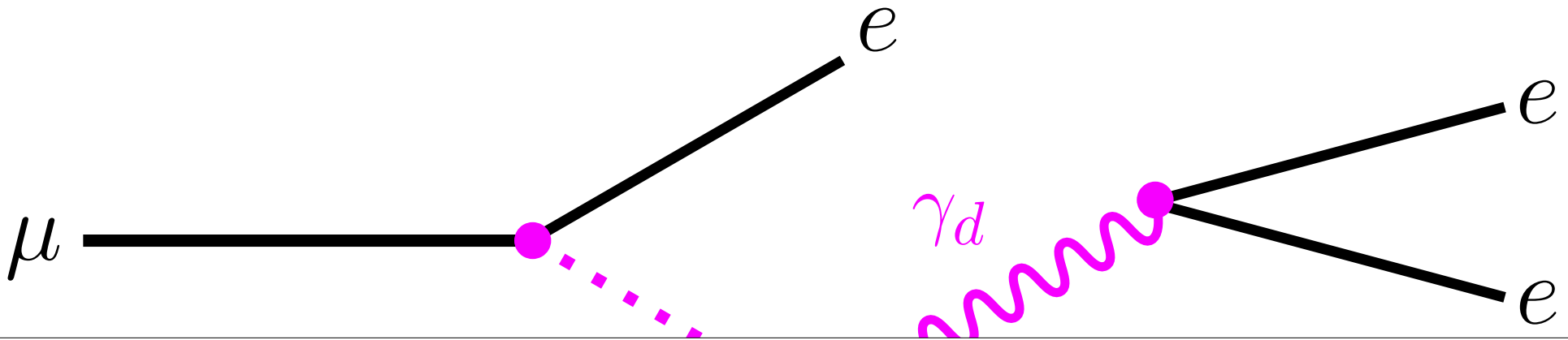


Mu2e (μ^-)



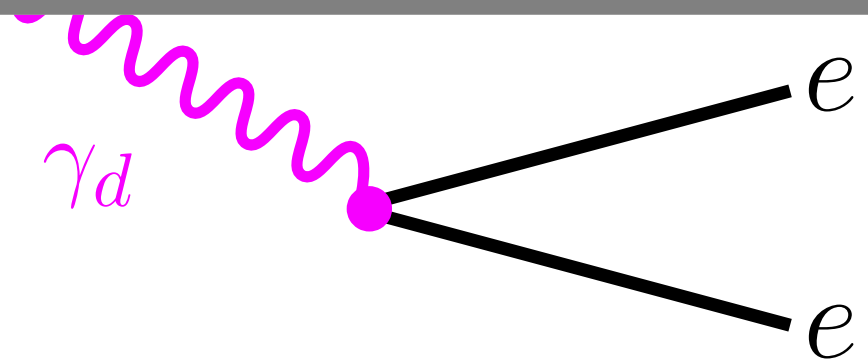
* requires $p_T > 90$ MeV





Multi-electron muon decays

JHEP 10 (2023) 006, 2306.15631



Mu3e \rightarrow Mu5e

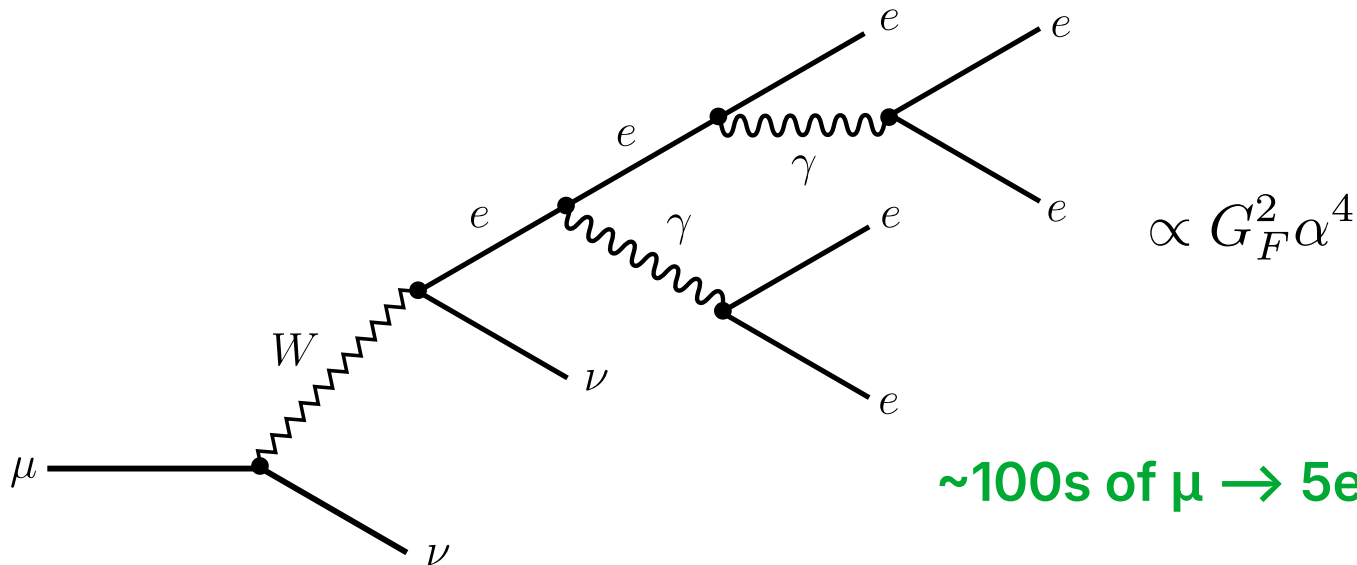
- Mu3e will see $\sim 10^{15}$ total muon decays from the stopping target.
- With these statistics are there any other interesting channels?
 - What about $\mu \rightarrow 5e$?

$\mu \rightarrow e e e e e \nu \nu$

- SM background for Mu5e

From MG5: $\mathcal{B}(\mu^+ \rightarrow e^+ e^- e^+ e^- e^+ \nu_e \bar{\nu}_\mu) = (3.929 \pm 0.001) \times 10^{-10}$

$$\mathcal{B}(\mu^+ \rightarrow e^+ e^- e^+ e^- e^+ \nu_e \bar{\nu}_\mu | \text{all } p_{e^\pm}^{\text{T,true}} > 10 \text{ MeV}) = (1.4 \pm 0.1) \times 10^{-14}$$



~100s of $\mu \rightarrow 5e\nu\nu$ events after cuts!

$\mu \rightarrow e e e e e$

- Higgsed $U(1)_D$ + SM portal via kinetic mixing

$$\mathcal{L}_{DS} = (D_\mu \phi)^\dagger D^\mu \phi - \frac{1}{4} F_d^{\mu\nu} F_{d\mu\nu} - \frac{\varepsilon}{2} F_d^{\mu\nu} F_{\mu\nu} - \mu^2 (\phi^\dagger \phi) - \lambda (\phi^\dagger \phi)^2$$

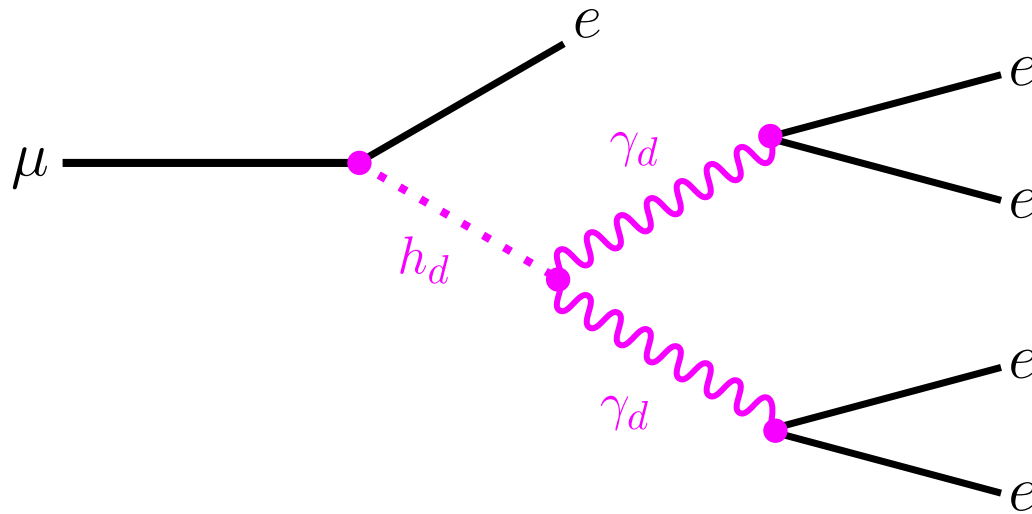
$$\mathcal{L}_{LFV} = -\frac{C_{ij}}{\Lambda} \phi (\bar{L}_i H) \ell_j + \text{h.c.}$$

$$\downarrow \quad y_{ij} \simeq \frac{Cv}{\Lambda}$$

$$\mathcal{L} \supset -m_{\ell_i} \bar{\ell}_{Li} \ell_{Ri} \left(1 + \frac{h}{v}\right) - y_{ij} \bar{\ell}_{Li} \ell_{Rj} h_d \left(1 + \frac{h}{v}\right) + \text{h.c.},$$

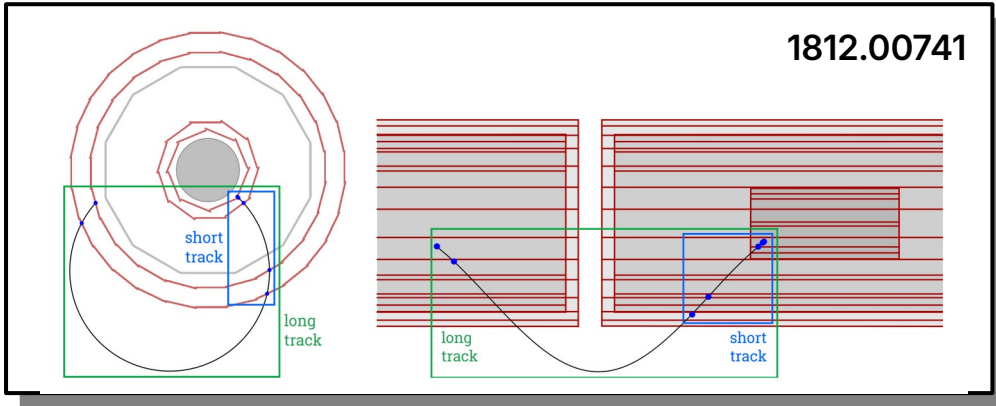
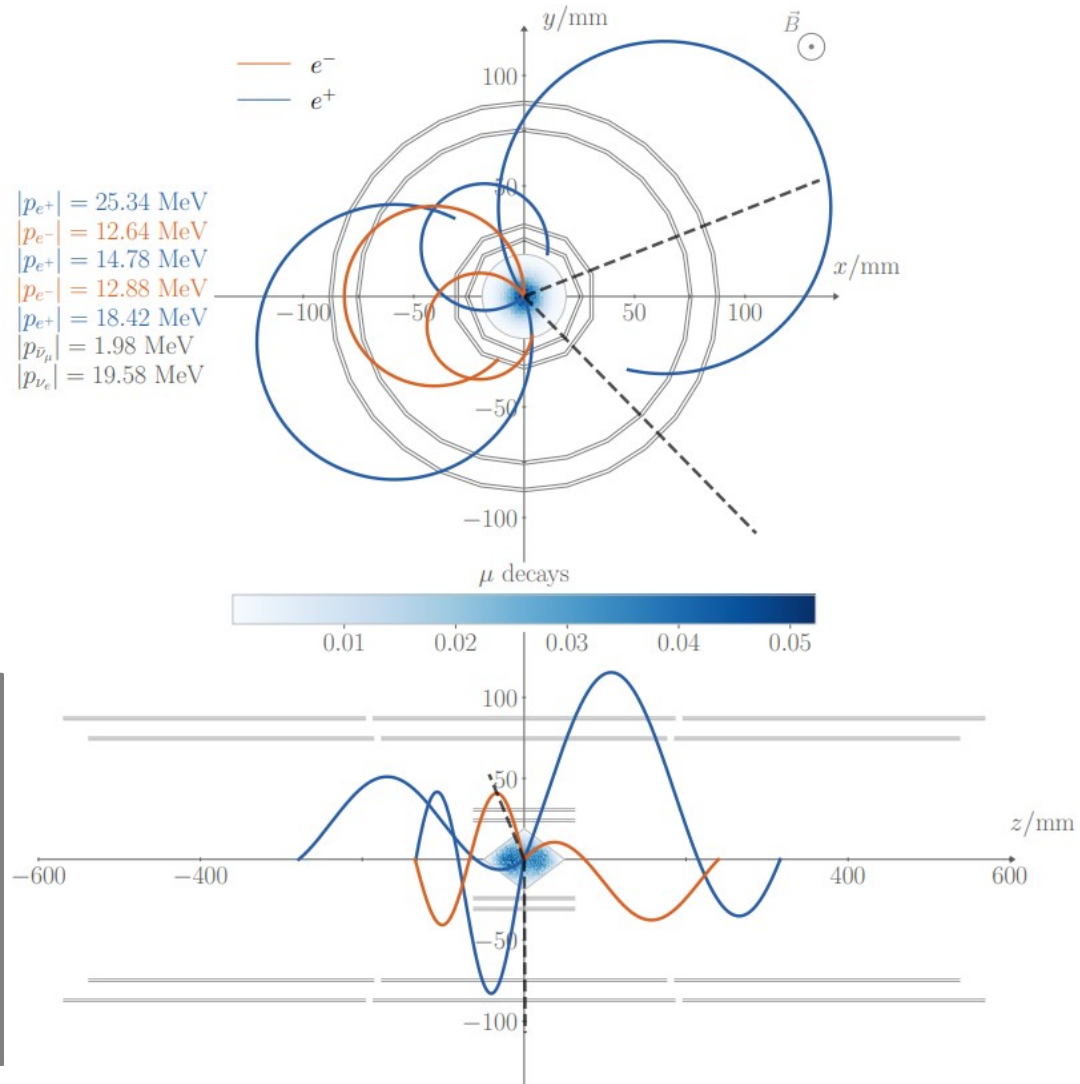
$$\mu \rightarrow e e e e e e$$

- Leads to cascade decay

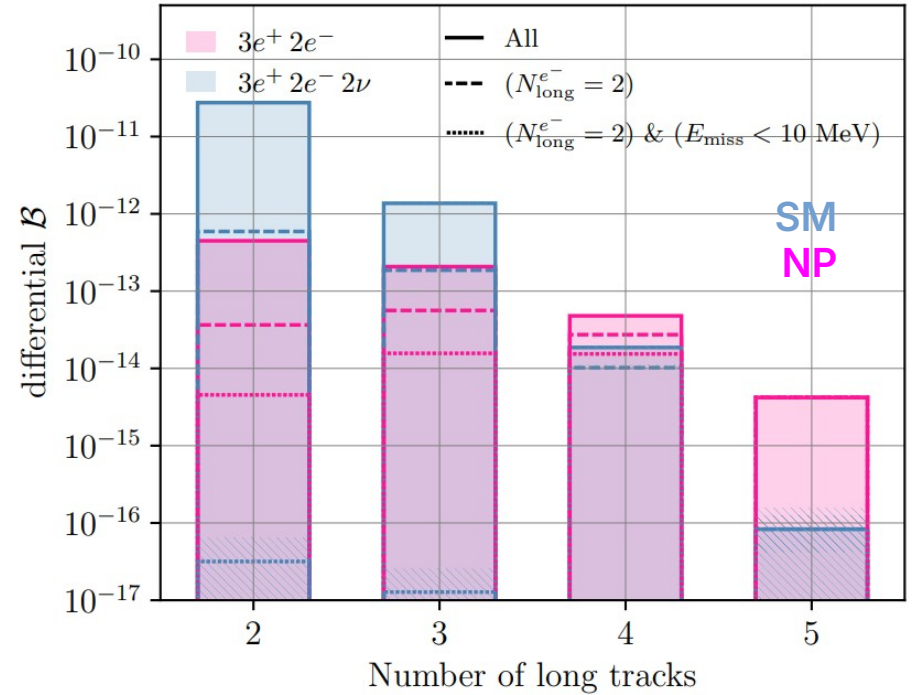
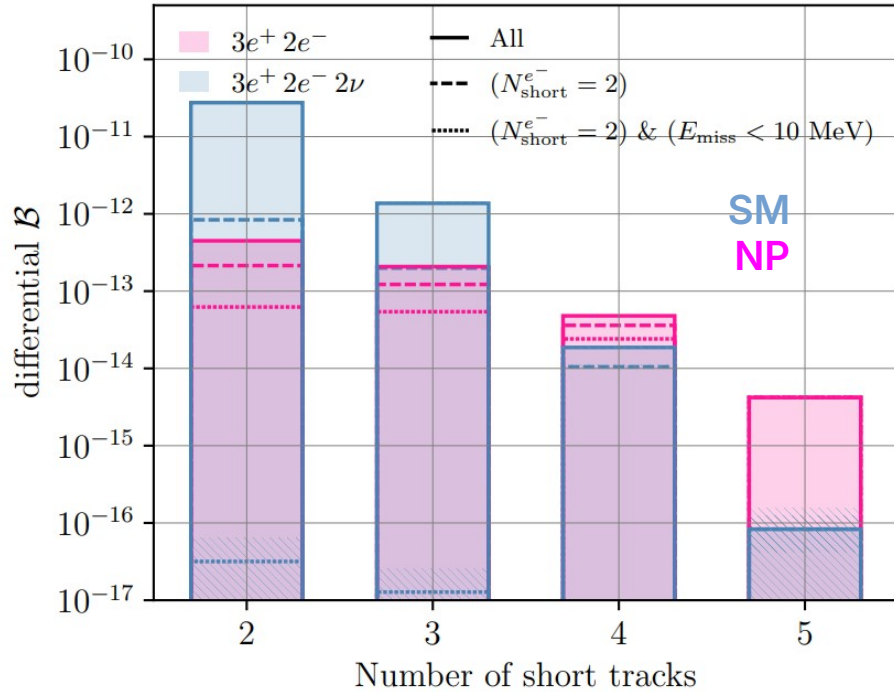


Signatures

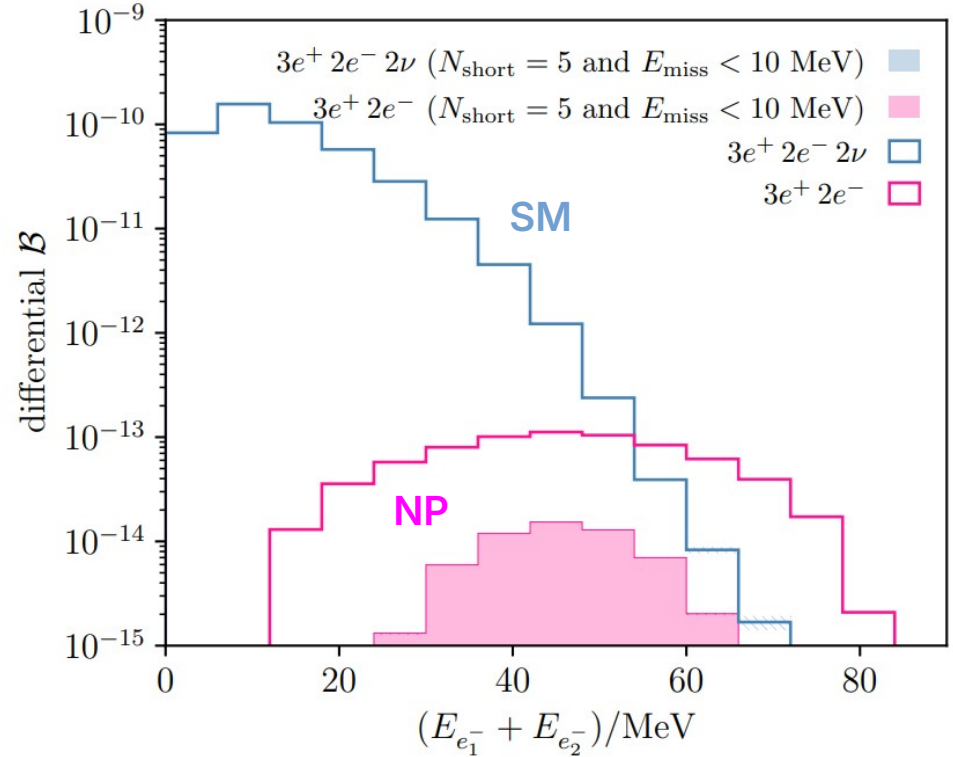
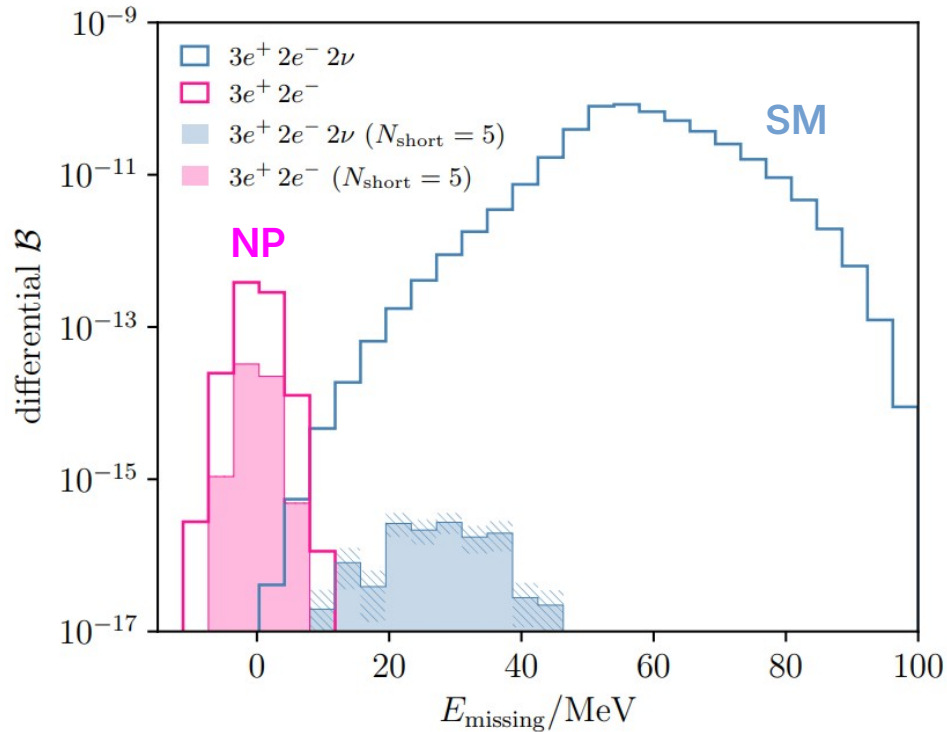
- Momentum of tracks must be reconstructed from energy deposits or 'hits' in the layers of the detectors.
- 4 hits = short track
- 6+ hits = long track



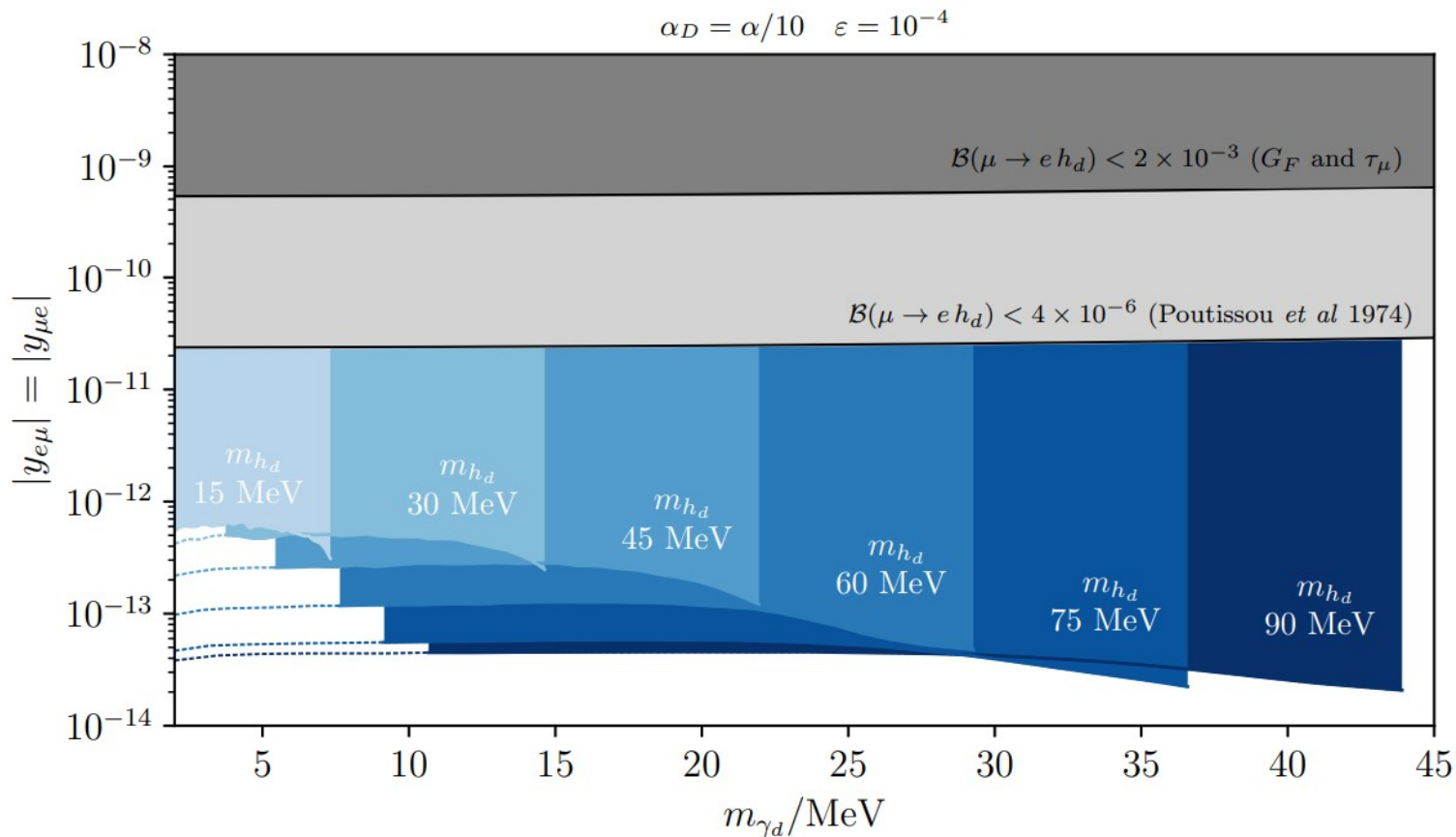
Signatures



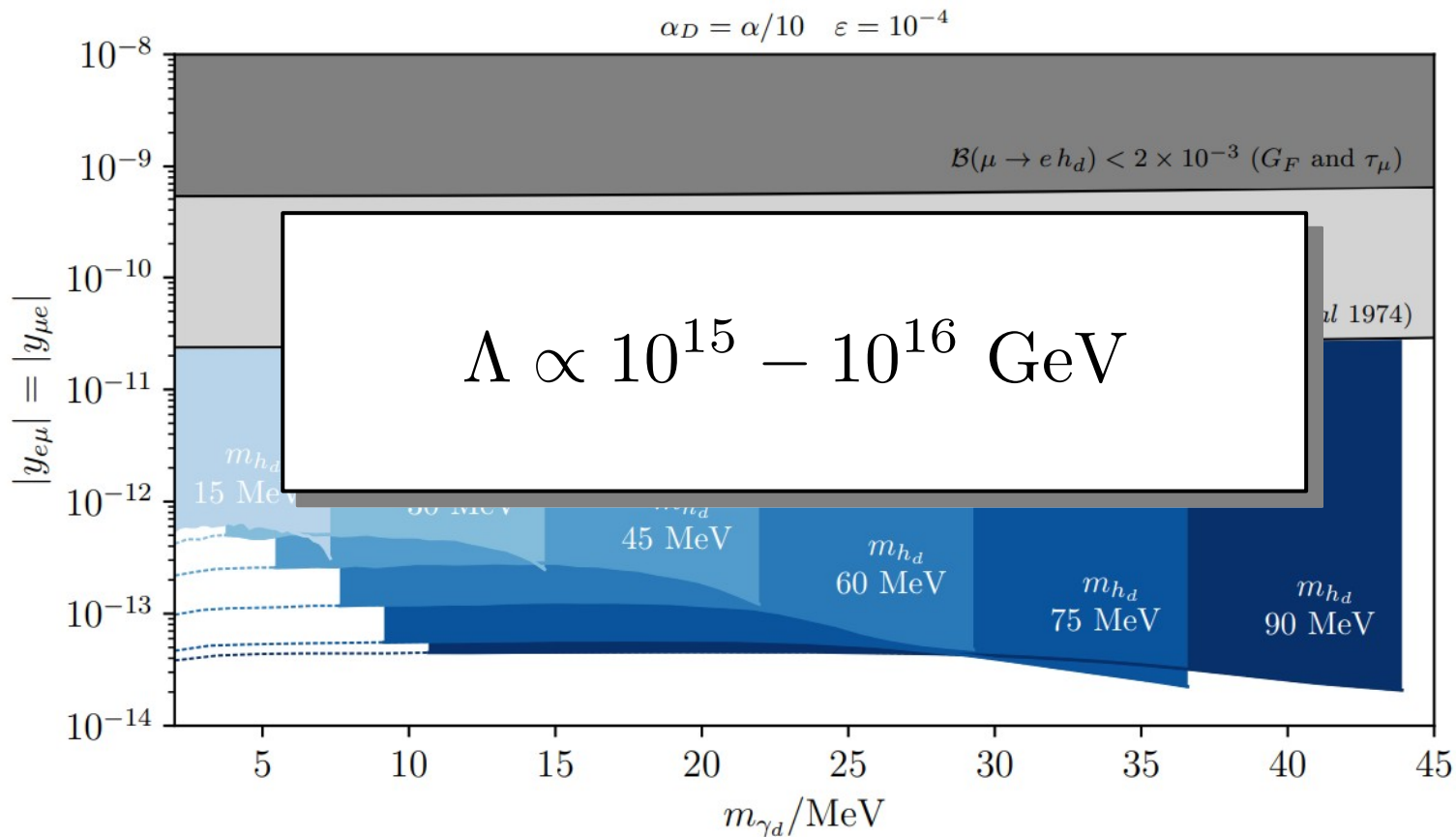
Signatures

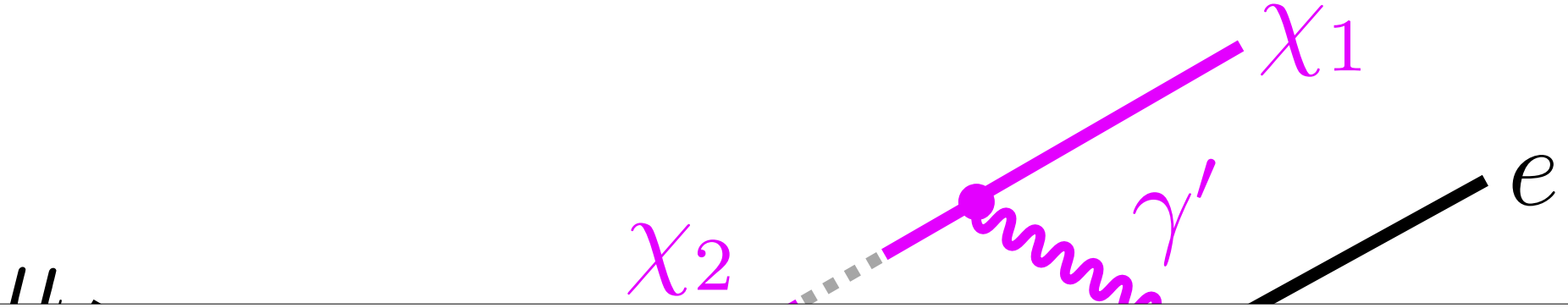


Reach



Reach





Muon-induced baryon number violation

2407.03450



Widening the view

- Can the CLFV signal @ Mu2e stem from non-CLFV physics?
- NP scenarios where $E_e > 105$ MeV?
 - Can we take advantage of the huge energy reservoir that is the nucleus?



- Three dark states + $U(1)_D$ + SM portal via kinetic mixing

$$\mathcal{L}_{\mu p} = G_{\mu p} (\bar{p} \chi_2) (\bar{\mu} \chi_0) + \text{h.c.}$$

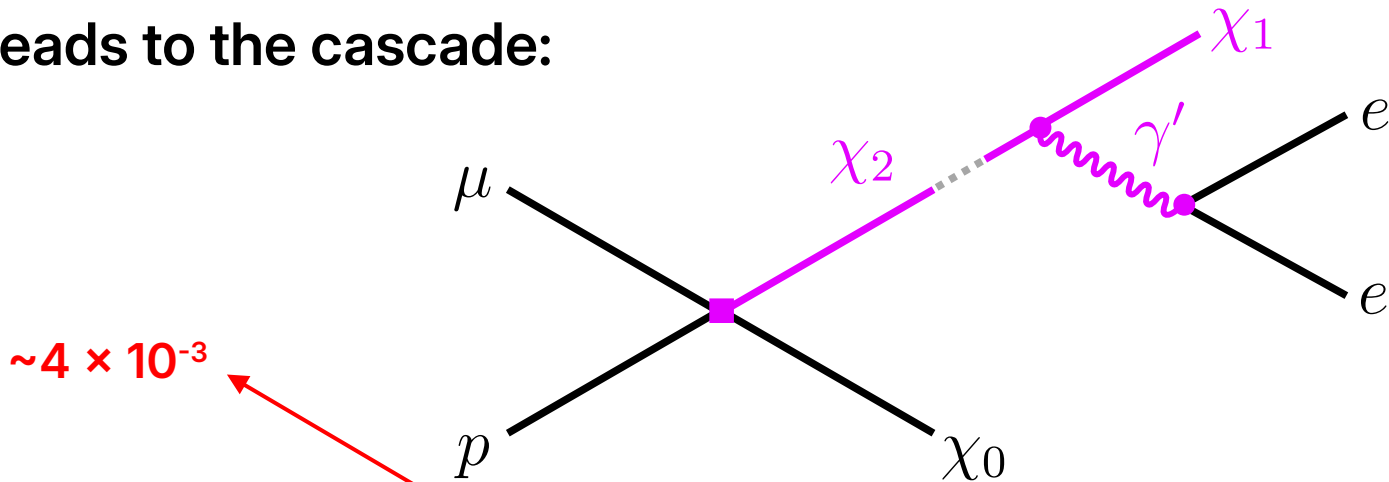
$$\mathcal{L}_{e A'} = e \varepsilon A'_\mu J_{\text{EM}}^\mu$$

$$\mathcal{L}_\chi = g_D (\bar{\chi}_2 \gamma^\mu \chi_1) A'_\mu + \text{h.c.}$$



Benchmark: $m_2 = 1030 \text{ MeV}$, $m_1 = 900 \text{ MeV}$, $m_0 = 0$, $m_{A'} = 20 \text{ MeV}$,
 $G_{\mu p} = (300 \text{ TeV})^{-2}$, $\varepsilon = 10^{-4}$, $\alpha_D = 10^{-3}$

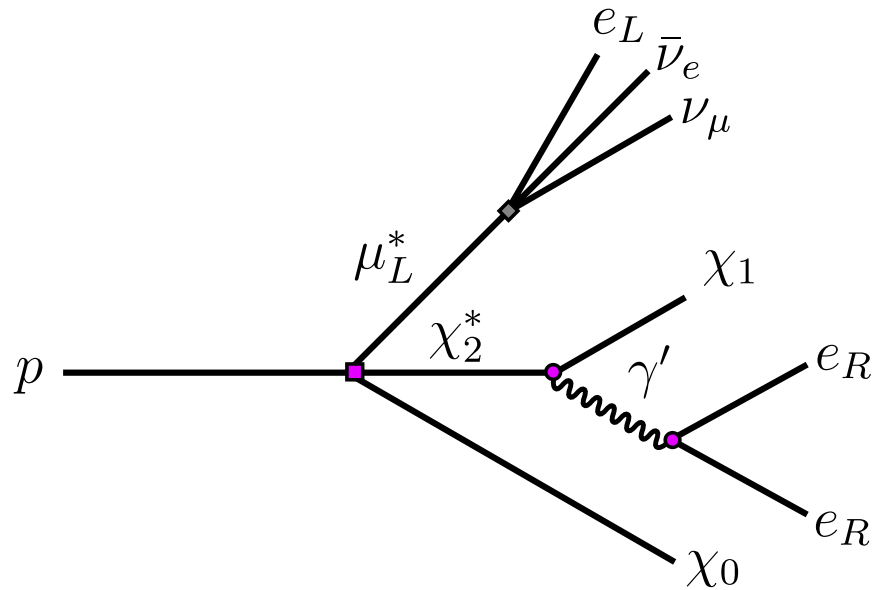
- Leads to the cascade:



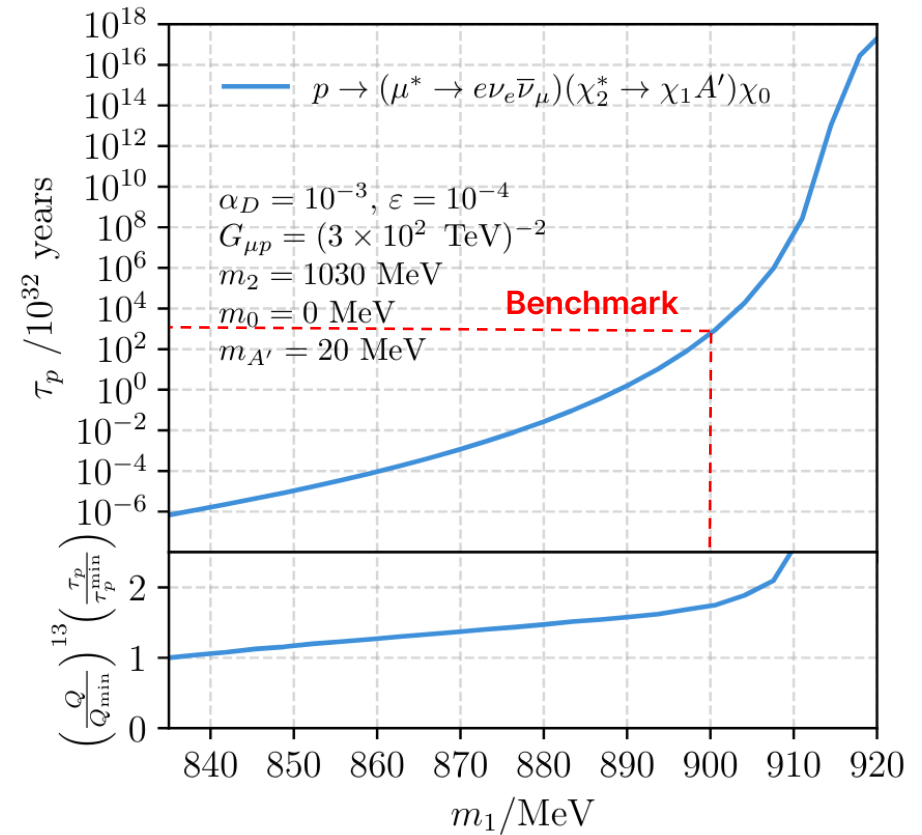
$\sim 4 \times 10^{-3}$

$$\Gamma(\mu \text{}^{27}\text{Al} \rightarrow \chi_0 \chi_2 \text{}^{26}\text{Mg}) \simeq r_{\text{p.s.}} \frac{G_{\mu p}^2}{G_F^2} \Gamma(\mu \text{}^{27}\text{Al} \rightarrow \nu_\mu n \text{}^{26}\text{Mg}) \longrightarrow R \equiv \frac{\Gamma_{\text{exotic}}}{\Gamma_{\mu\text{Al}}} \sim 10^{-15}$$

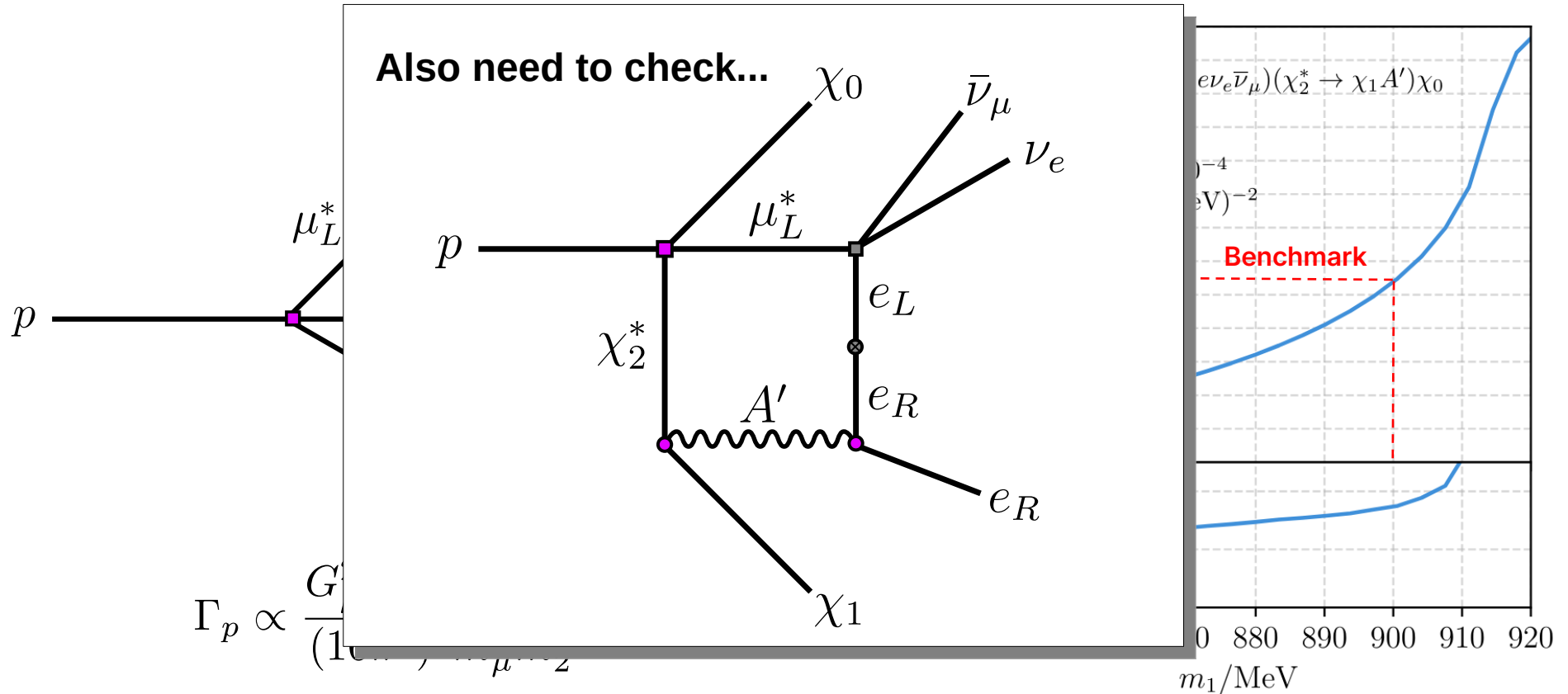
Tug-of-war with nucleon decay



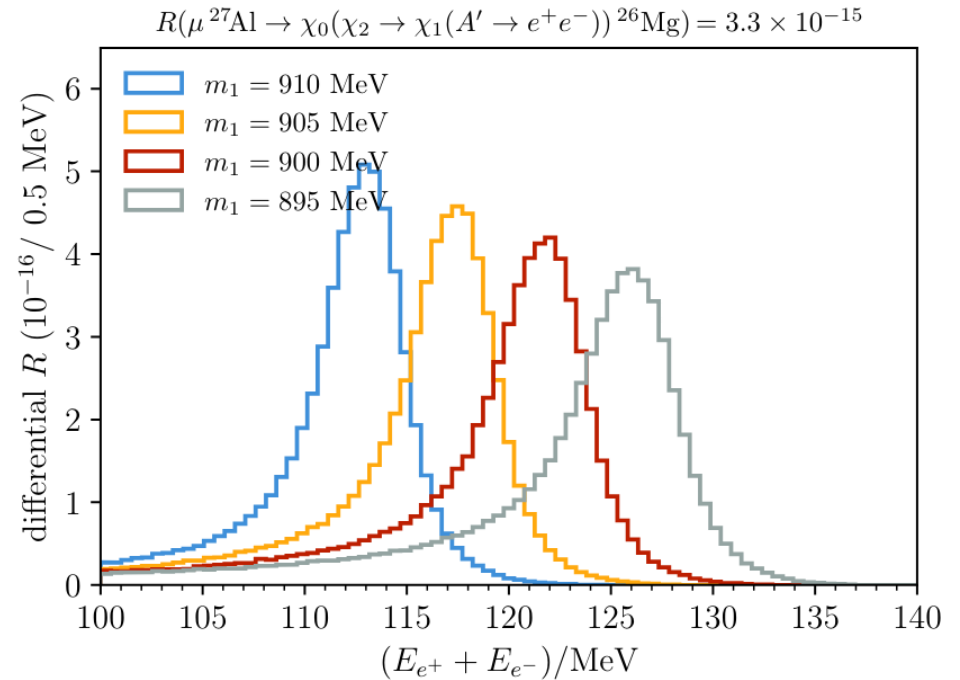
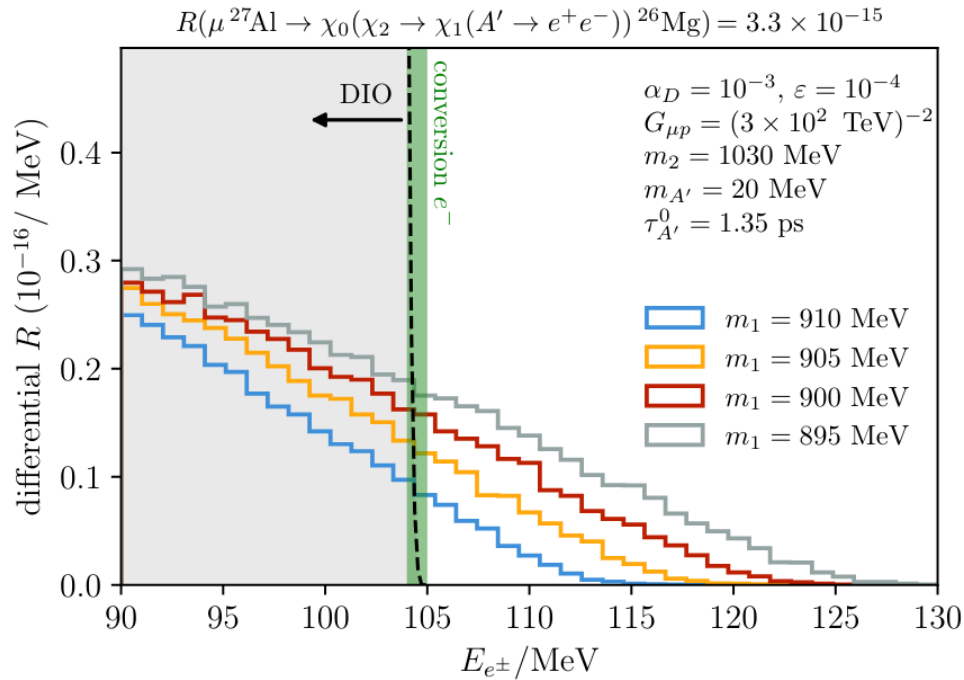
$$\Gamma_p \propto \frac{G_{\mu p}^2 G_F^2 \alpha_D Q^{13}}{(16\pi^2)^4 m_\mu^2 m_2^2}$$



Tug-of-war with nucleon decay



Signature



A UV completion

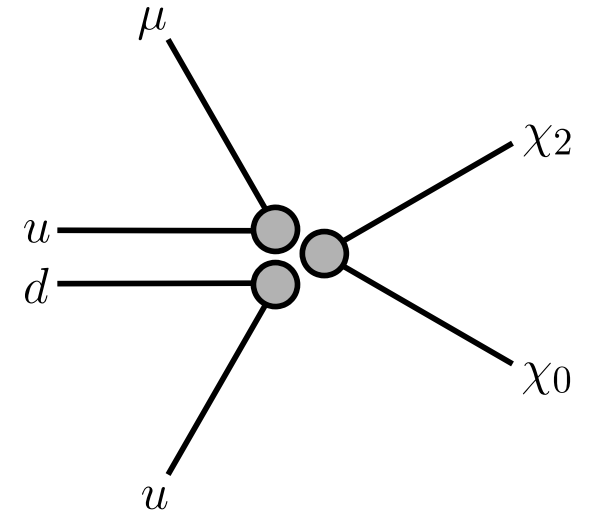
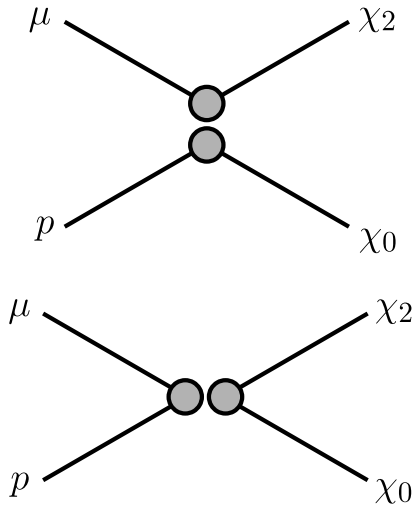
$$\mathcal{L}_{\mu p} = G_{\mu p} (\bar{p}\chi_2)(\bar{\mu}\chi_0) + \text{h.c.}$$

- **What's the new physics reach? Naively:**

$$G_{\mu p} \simeq 10^{-8} G_F \rightarrow \Lambda \simeq 10^3 \text{ TeV}$$

but Λ isn't fundamental...

$$\frac{1}{\Lambda^2} \simeq \frac{\Lambda_{\text{QCD}}^3}{\Lambda_{\text{col.}}^{d_{\text{col.}}} \Lambda_{\text{sing.}}^{5-d_{\text{col.}}}}$$

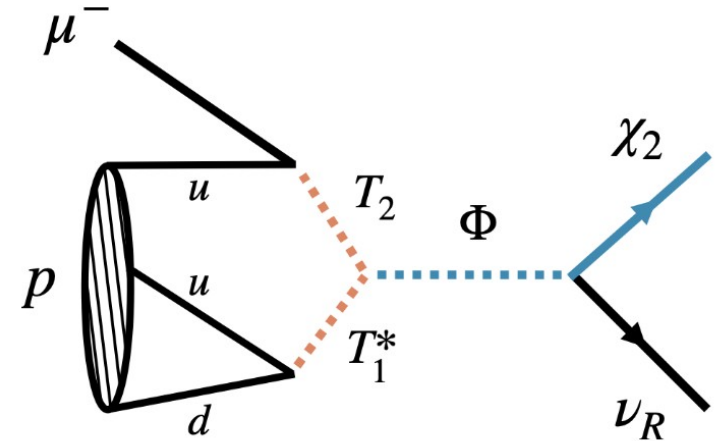


A UV completion

Introduce two colored scalars - a diquark T_1 and a leptoquark T_2 as well as a complex scalar Φ .

$$\mathcal{L} \supset -y_{ud}(\overline{u_R^{iC}} d_R^j)\epsilon_{ijk}T_1^k - y_{\mu u}(\overline{u_R^{iC}} \mu_R)(T_2^*)_i + \text{h.c.}$$

$$\mathcal{L} \supset \rho T_1^{k*} T_{2k} \Phi^* + y_\chi \Phi(\overline{\nu_R} \chi_{2R}^C) + \text{h.c.}$$



A UV completion

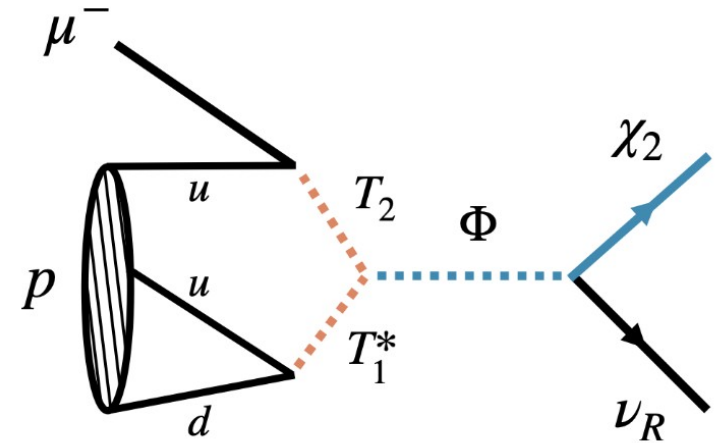
Introduce two colored scalars - a diquark T_1 and a leptoquark T_2 as well as a complex scalar Φ .

Integrate out T_1 and T_2 , for off-shell Φ :

$$\mathcal{L}_{\text{eff}} = \frac{1}{\Lambda_{\text{col}}^3} \frac{1}{\Lambda_{\text{sin}}^2} (\overline{u_R^{iC}} \mu_R) \epsilon_{ijk} (\overline{u_R^{jC}} d_R^k) (\overline{\nu_R} \chi_{2R}^C) + \text{h.c.}$$

$$\frac{1}{\Lambda_{\text{col}}^3} = y_{ud} y_{\mu u} \frac{\rho}{m_{T_1}^2 m_{T_2}^2}$$

$$\frac{1}{\Lambda_{\text{sin}}^2} = \frac{y_\chi}{m_\Phi^2}$$



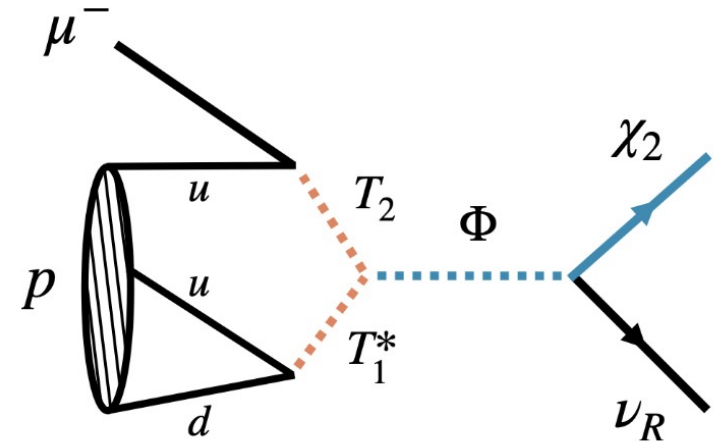
A UV completion

Introduce two colored scalars - a diquark T_1 and a leptoquark T_2 as well as a complex scalar Φ .

Integrate out T_1 and T_2 , for off-shell Φ :

$$\tilde{G}_{\mu p} \simeq 10^{-6} G_F$$

$$\sim 10^{-6} G_F y_{ud} y_{\mu u} y_\chi \left(\frac{1 \text{ TeV}}{\sqrt{m_{T_1} m_{T_2}}} \right)^4 \left(\frac{\rho}{4 \text{ TeV}} \right) \left(\frac{2 \text{ GeV}}{m_\Phi} \right)^2$$



A UV completion

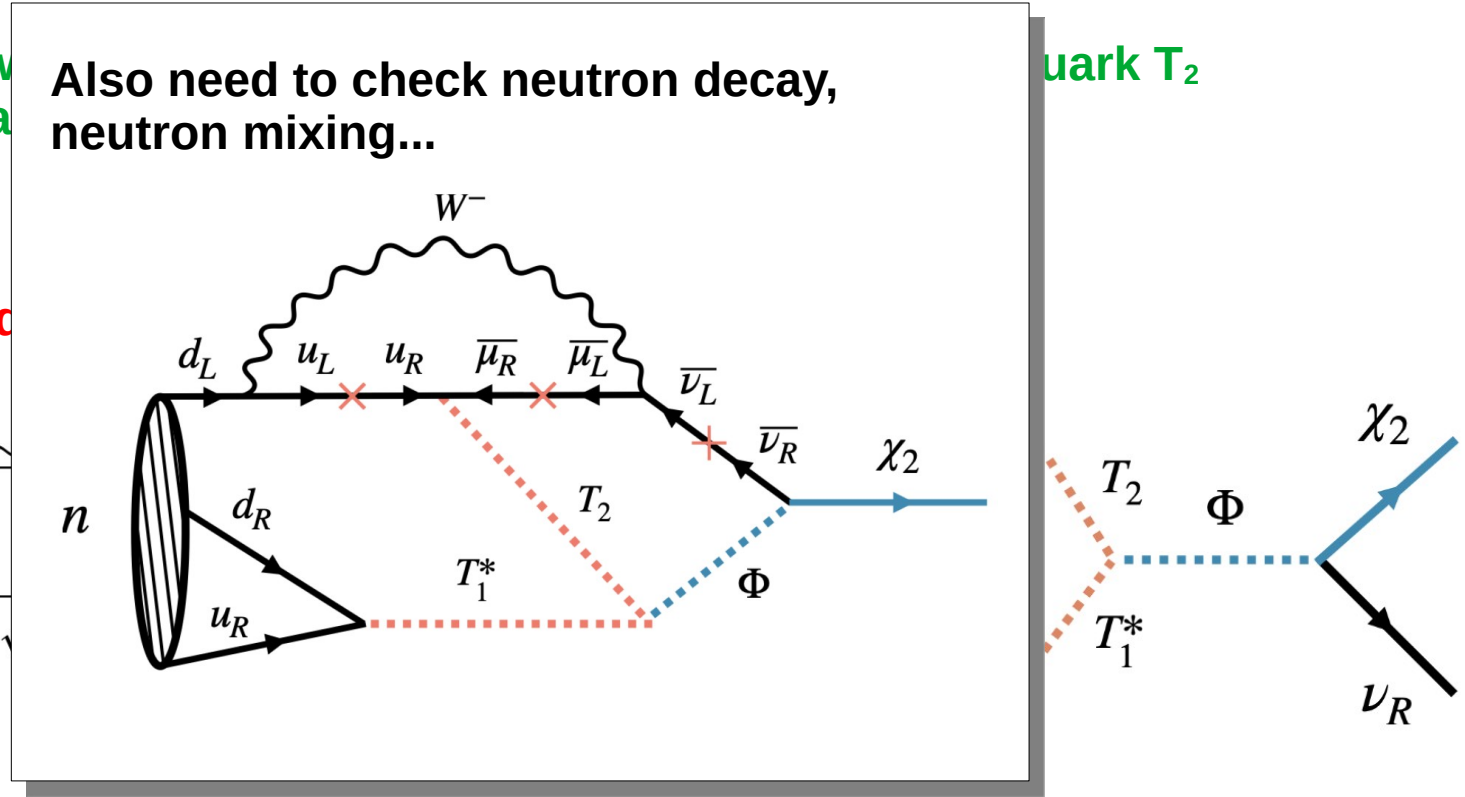
Introduce two new particles
as well as a new interaction

Integrate out T_1 and T_2

Also need to check neutron decay,
neutron mixing...

quark T_2

$$\tilde{G}_{\mu p} \supseteq \sim 10^{-6} G_F y_{ud} y_{\mu u} y_{\chi} \left(\dots \right)$$

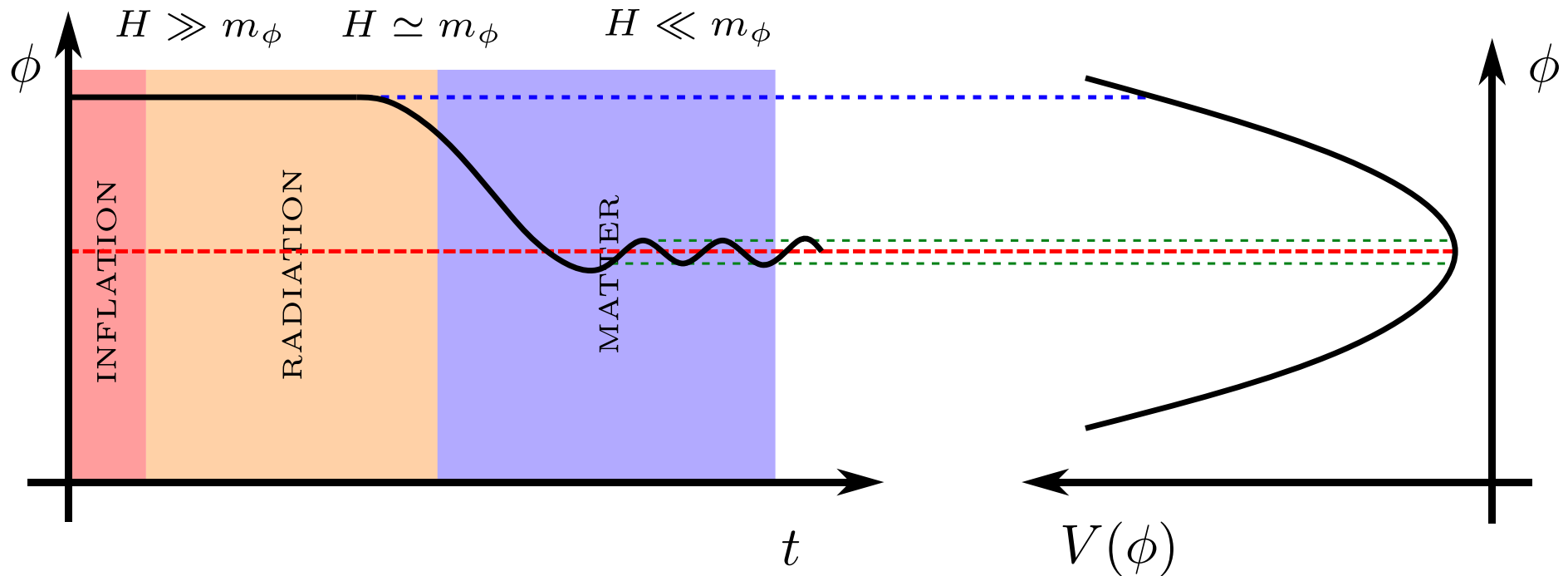


Direct detection of ultralight DM via CLFV

2407.03450

Standard lore - ULDM

EOM:
$$\int d^4x \sqrt{-g} \left(\frac{1}{2} (\partial_\mu \phi)^2 - \frac{1}{2} m_\phi^2 \phi^2 \right) \rightarrow \ddot{\phi} + 3H\dot{\phi} - \frac{\nabla^2 \phi}{a^2} + m_\phi^2 \phi = 0$$



Standard lore - ULDM

For $m_\phi \lesssim \text{eV}$ the de Broglie volume admits a huge occupation number

$$N \sim n \lambda_{\text{dB}}^3 \sim \frac{\rho_\phi}{m_\phi} \left(\frac{1}{m_\phi v} \right)^3 \simeq 10^3 \left(\frac{1 \text{ eV}}{m_\phi} \right)^4 \left(\frac{10^{-3}}{v} \right)^3 \left(\frac{\rho_\phi}{10^{-42} \text{ GeV}^4} \right)$$

ULDM can be accurately described as a classical wave

$$\phi_c(\mathbf{x}, t) = \phi_0(\mathbf{x}) \cos(m_\phi t + \delta)$$

$$\phi_0 = \frac{\sqrt{2\rho_\phi}}{m_\phi} \quad \rho_\phi = 0.4 \text{ GeV}/\text{cm}^3 \simeq 10^{-42} \text{ GeV}^4$$

Standard lore - ULDM

For $m_\phi \lesssim eV$ the de Broglie volume admits a huge occupation number

$$N \sim n\lambda_{\text{dB}}^3$$

ULDM ca

Each mass has an associated “timescale”

$$\tau_\phi = \frac{2\pi}{m_\phi} \simeq 4\text{s} \left(\frac{10^{-15}\text{eV}}{m_\phi} \right)$$

$$\left(\frac{\rho_\phi}{42 \text{ GeV}^4} \right)$$

wave

$$\phi_0 = \frac{\sqrt{2\rho_\phi}}{m_\phi}$$

$$\rho_\phi = 0.4 \text{ GeV}/\text{cm}^3 \simeq 10^{-42} \text{ GeV}^4$$

Why ULDM + CLFV?

CLFV experiments probe extremely high NP scales

Detecting a CLFV signal does not immediately imply DM is the source.

Detecting a *time-dependent* CLFV signal is a smoking gun signal of DM.

Minimal analysis can convert intensity frontier experiments into dark matter detectors

ULDM + CLFV

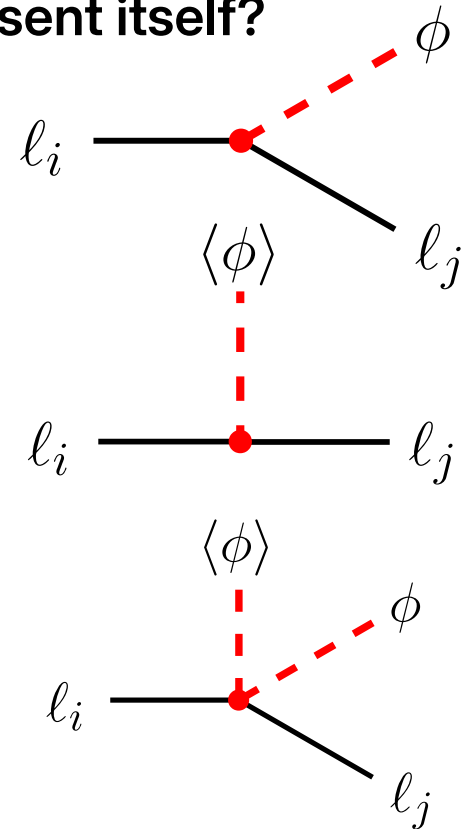
How and where does time-modulation present itself?

Two types of time-dependence:

- Manifestation in mass matrix

$$m_{ij} = \text{diag}(m_e, m_\mu, m_\tau) + y_{ij}\phi_c$$

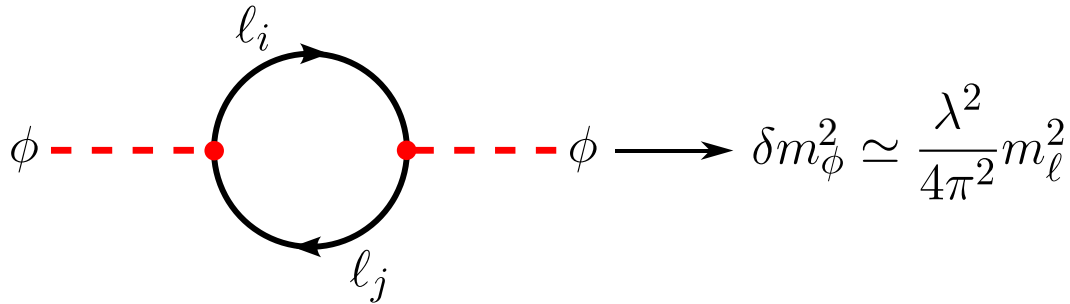
- Manifestation in decay/scattering rates
 - Inherently higher dimensional



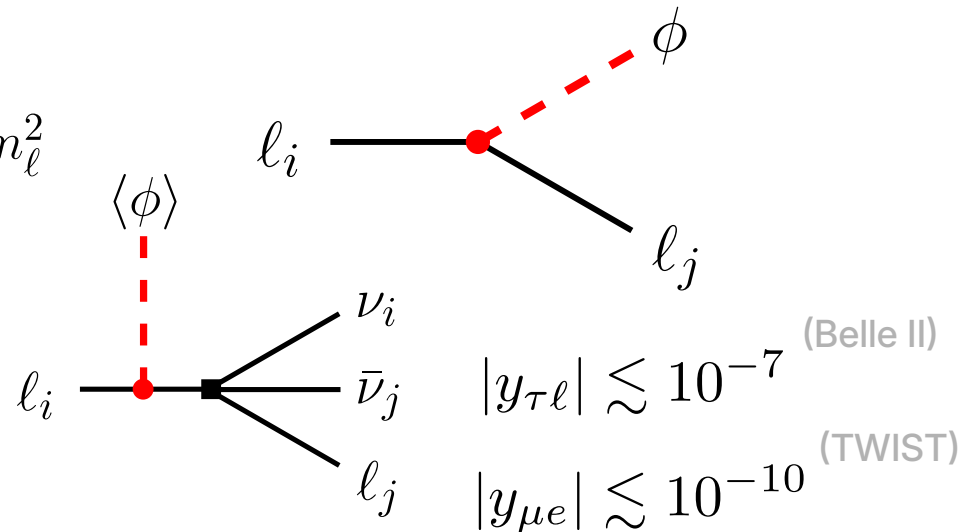
Journey towards phenomenologically viable operator

First guess: $\frac{C_{ij}}{\Lambda} \phi H \bar{L}_i l_j \rightarrow y_{ij} \phi \bar{l}_i l_j$

1. Fine-tuning



2. No time-modulation



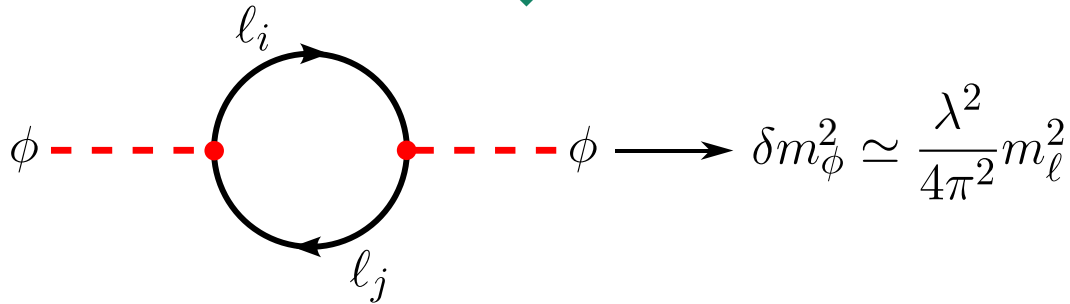
3. SM rate-modulation

Journey towards phenomenologically viable operator

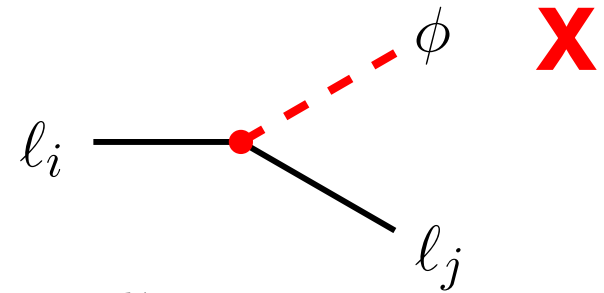
First guess: $\frac{C_{ij}}{\Lambda} \phi H \bar{L}_i l_j \rightarrow y_{ij} \phi \bar{l}_i l_j$

1. Fine-tuning

“✓”

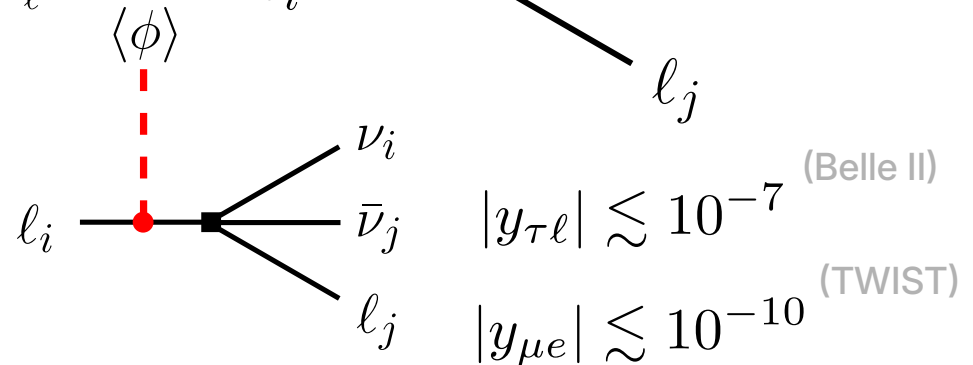


2. No time-modulation



3. SM rate-modulation

“X”



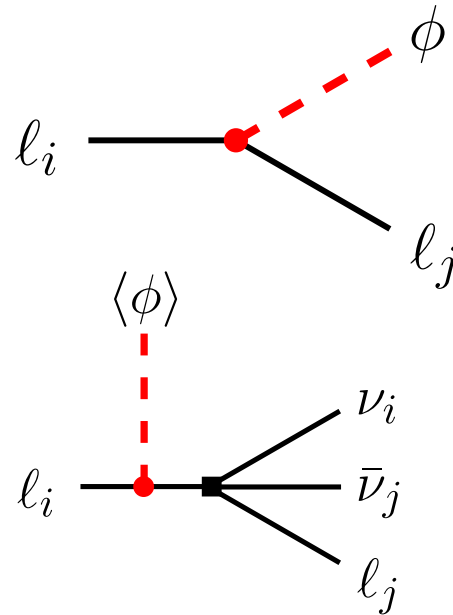
Journey towards phenomenologically viable operator

Second guess: $\frac{C_{ij}}{\Lambda} \partial_\mu \phi (\bar{l}_i \gamma^\mu \gamma^5 l_j)$

1. Fine-tuning: ✓

2. Time-modulation: ✗

3. SM-modulation: ✗



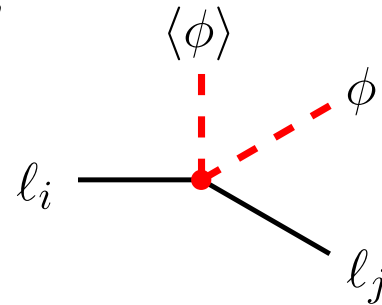
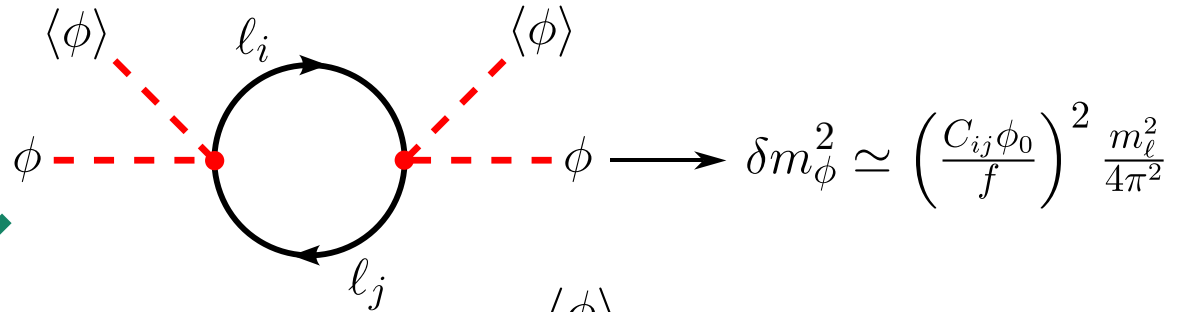
Journey towards phenomenologically viable operator

Third guess: $\frac{C_{ij}}{f} \phi^2 (\bar{l}_i l_j)$

1. Fine-tuning: **X**

2. Time-modulation: **✓**

3. SM-modulation: **✓**



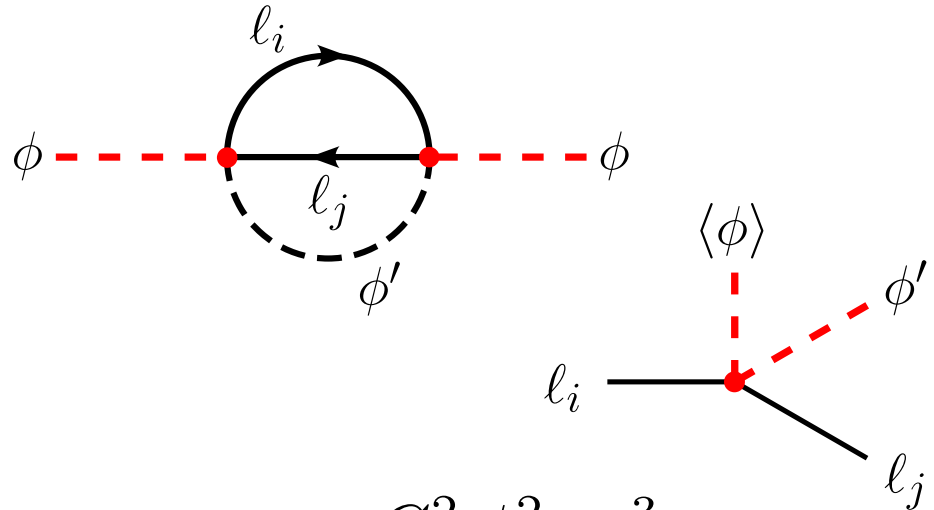
Journey towards phenomenologically viable operator

Fourth guess: $\frac{C_{ij}}{f} \phi \phi' (\bar{l}_i l_j)$ or $\frac{C_{ij}}{f^2} \phi \partial_\mu \phi' (\bar{l}_i \gamma^\mu l_j)$

1. Fine-tuning: "✓"/✓

2. Time-modulation: ✓

3. SM-modulation: ✓



$$\mathcal{B}(l_i \rightarrow l_j \phi) = \frac{C_{ij}^2 \phi_0^2}{64\pi f^4} \frac{m_{l_i}^3}{\Gamma_{l_i}} \cos^2(m_\phi t + \delta)$$

Explicit realization

- Non-abelian pseudo-NGB + $U(1)_D$

$$SU(3)_L \times SU(3)_R \times U(1)_D \rightarrow SU(3)_V \times U(1)_D$$

$$D_\mu \equiv \partial_\mu + ig_D A'_\mu [Q, U] \quad U = \exp \left(i\sqrt{2}\Pi_D / f_D \right)$$

$$\Pi_D = \begin{pmatrix} \frac{\pi_D^0}{\sqrt{2}} + \frac{\eta_{D8}}{\sqrt{6}} & \pi_D^+ & K_D^+ \\ \pi_D^- & -\frac{\pi_D^0}{\sqrt{2}} + \frac{\eta_{D8}}{\sqrt{6}} & K_D^0 \\ K_D^- & \bar{K}_D^0 & -\sqrt{\frac{2}{3}}\eta_{D8} \end{pmatrix}$$

Explicit realization

- Non-abelian pseudo-NGB + $U(1)_D$

$$\mathcal{L}_D \supset \frac{f_{UV}^2}{8} (D_\mu U D^\mu U^\dagger) + \frac{f_{UV}^2}{8} (\chi^\dagger U + U^\dagger \chi) + \dots$$

$$Q \rightarrow q_U \lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & q_U \\ 0 & q_U & 0 \end{pmatrix}$$

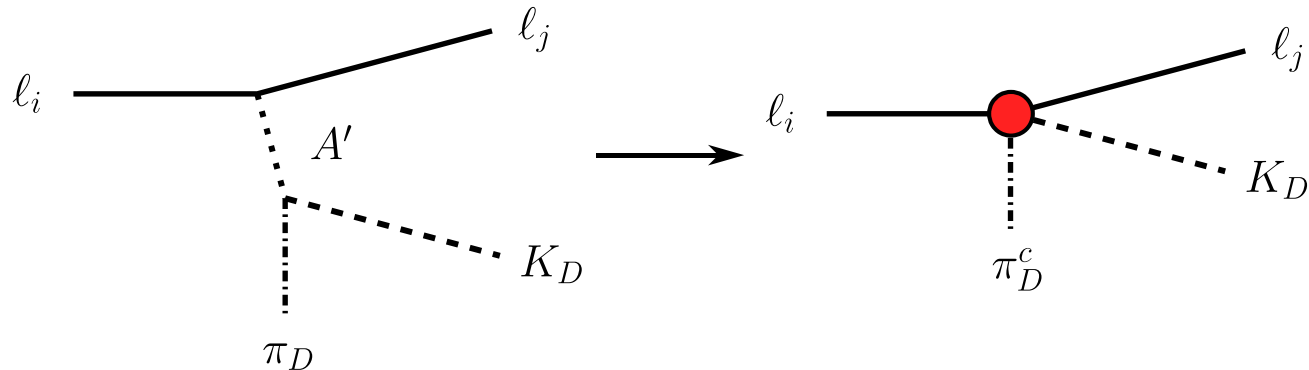
$$\supset -g' q_U A'_\mu (\pi_D^+ i \partial^\mu K_D^- - K_D^- i \partial^\mu \pi_D^+ + \text{h.c.}) + \dots$$

Explicit realization

- Non-abelian pseudo-NGB + $U(1)_D$

$$\supset -g' q_U A'_\mu (\pi_D^+ i\partial^\mu K_D^- - K_D^- i\partial^\mu \pi_D^+ + \text{h.c.}) + \dots$$

Charge the SM under $U(1)_D$: $-\mathcal{L}_{\text{portal}} = ig' c_{ij} \ell_i \gamma^\mu A'_\mu \ell_j + \text{h.c.}$



Explicit realization

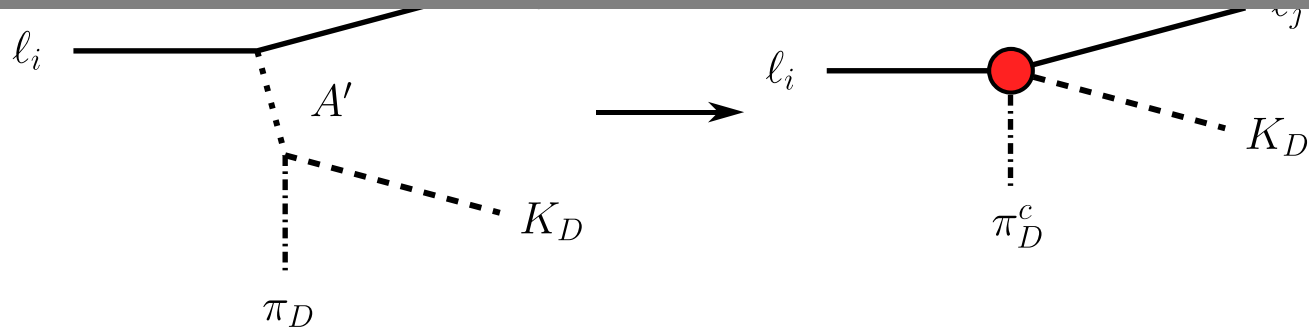
- Non-abelian pseudo-NGB + $U(1)_D$

$$\supset -g' q_U A'_\mu (\pi_D^+ i \partial^\mu K_D^- - K_D^- i \partial^\mu \pi_D^+ + \text{h.c.}) + \dots$$

Charge t

$$\supset \frac{c_{ij} q_U g'^2}{m_{A'}^2} (\bar{\psi}_i \gamma^\mu \psi_j) (\pi_D^+ \partial_\mu K_D^-) + \dots$$

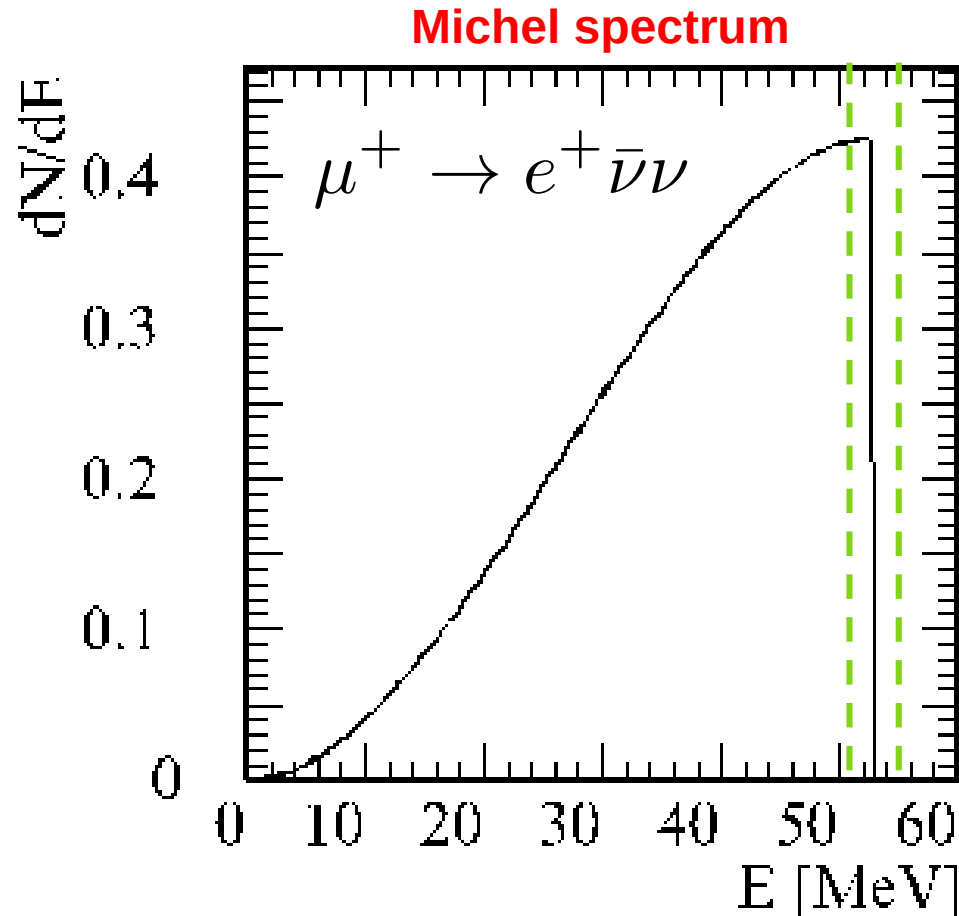
h.c.



Sensitivity (Mu3e)

Consider, for example, Mu3e.

- Will run for $T \sim 300$ days
- $\sim 10^{15}$ muon decays
- $\sim 10^{13}$ of which will lie in the final kinematic bin
- $\sim 10^8$ muon decays/s



Sensitivity

- Sensitivity can essentially be determined by:

$$m_\phi, \quad T, \quad N_{\text{total}}$$

- Statistical uncertainty + systematic uncertainty maximally correlated across all time bins

$$\sigma_{\text{stat}} = \sqrt{N_{\text{bg}}/n_{\text{bin}}} \quad , \quad \sigma_{\text{sys}} = \alpha N_{\text{bg}}/n_{\text{bin}}$$

$$\chi^2 = \sum_{k,p=1}^{n_{\text{bin}}} S_k C_{kp}^{-1} S_p = \frac{1}{N_{\text{bg},1}} \sum_{k=1}^{n_{\text{bin}}} S_k^2 - \frac{\alpha^2}{1 + \alpha^2 n_{\text{bin}} N_{\text{bg},1}} \left(\sum_{k=1}^{n_{\text{bin}}} S_k \right)^2$$

Sensitivity

$$\chi^2 = \sum_{k,p=1}^{n_{\text{bin}}} S_k C_{kp}^{-1} S_p = \frac{1}{N_{\text{bg},1}} \sum_{k=1}^{n_{\text{bin}}} S_k^2 - \frac{\alpha^2}{1 + \alpha^2 n_{\text{bin}} N_{\text{bg},1}} \left(\sum_{k=1}^{n_{\text{bin}}} S_k \right)^2$$

$$S_k = 2\mathcal{B}_{\text{sig}} f_{\text{sig}} \frac{N_{\text{tot}}}{T} \int_{(k-1)\Delta t}^{k\Delta t} dt \cos^2(m_\phi t + \delta)$$

$$= \mathcal{B}_{\text{sig}} f_{\text{sig}} \frac{N_{\text{tot}}}{T} \left[\Delta t + \frac{\sin(2km_\phi \Delta t + 2\delta) - \sin(2(k-1)m_\phi \Delta t + 2\delta)}{2m_\phi} \right]$$

Three interesting limits:

1. Signal does not oscillate over the duration of the experiment
2. Signal oscillates but time bins cannot resolve the oscillations
3. Signal oscillates and time bins resolve the oscillations

Systematics
dominate

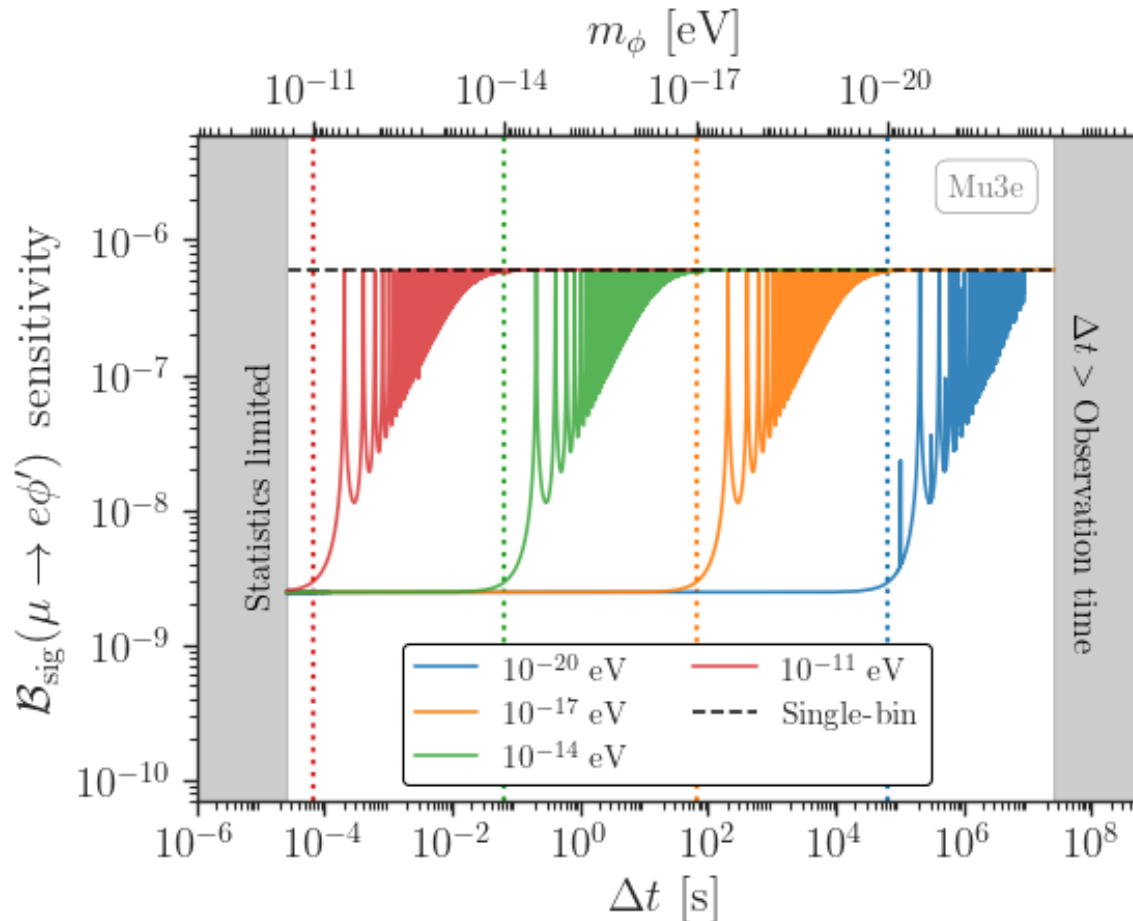
Statistics
dominate

Sensitivity

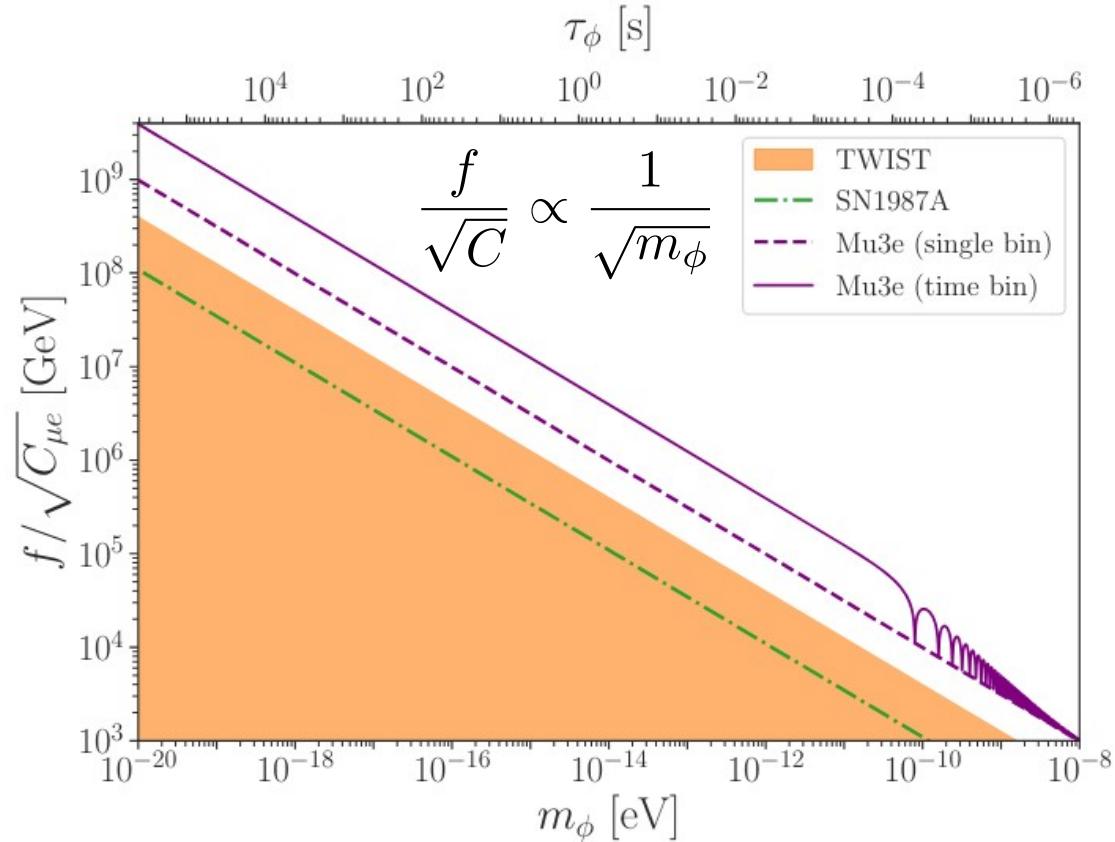
Assumptions:

- 1) Asimov dataset
- 2) Continuous running
- 3) Time-independent systematics

- **Notably, the sensitivity to ULDM mass is limited by statistics, not experimental time resolution**



Reach



Conclusions

- Interesting invasive and non-invasive avenues for making Pythia “fully” differentiable
- Interesting solutions for finalTwo? Parallels with constrained Brownian motion
- Fun CLFVing signals and model building at the intensity frontier.
- More natural models of ultralight scalar DM?

Conclusions

- Interesting invasive and non-invasive avenues for making Pythia “fully” differentiable
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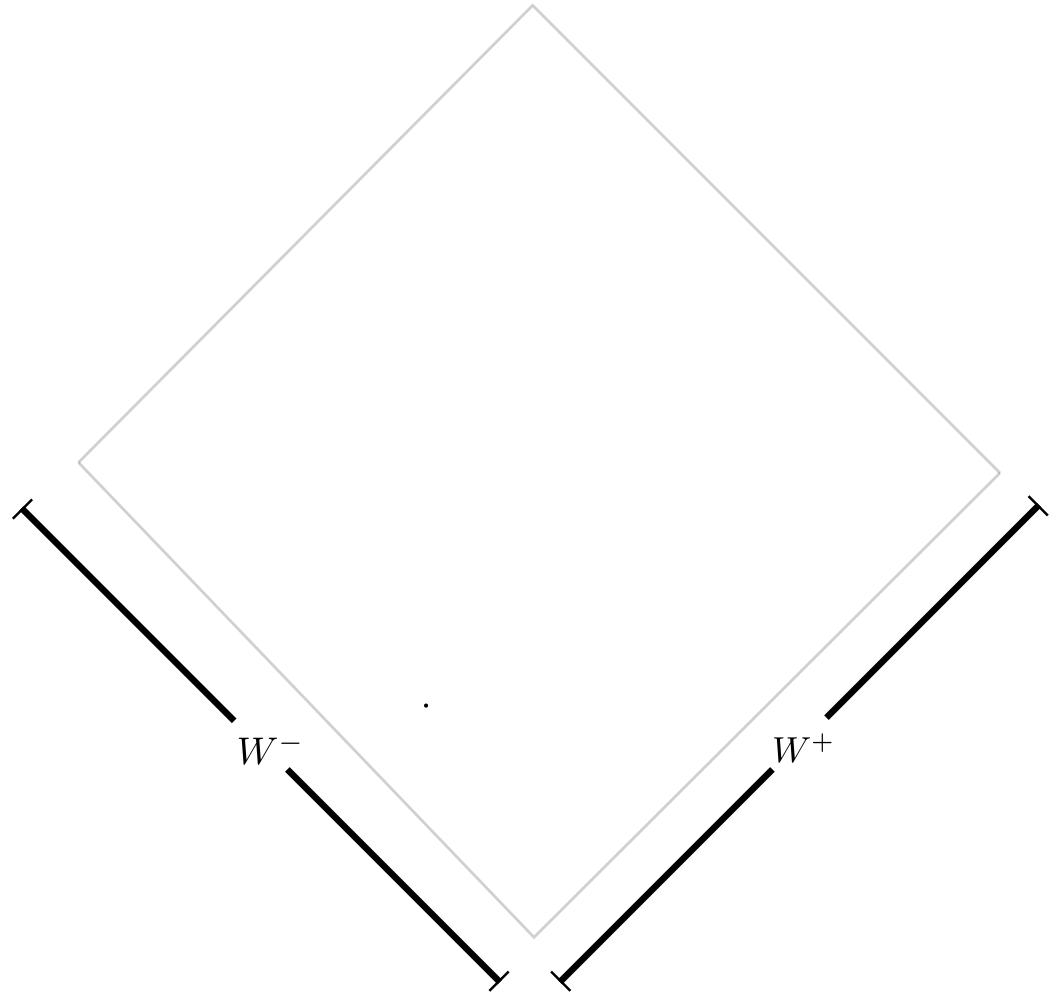
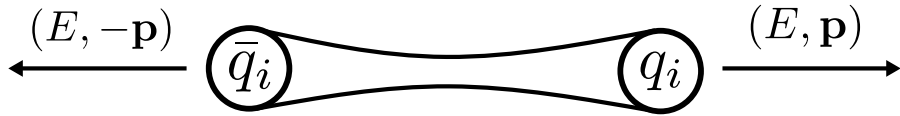
Thanks for your attention :)

Backups

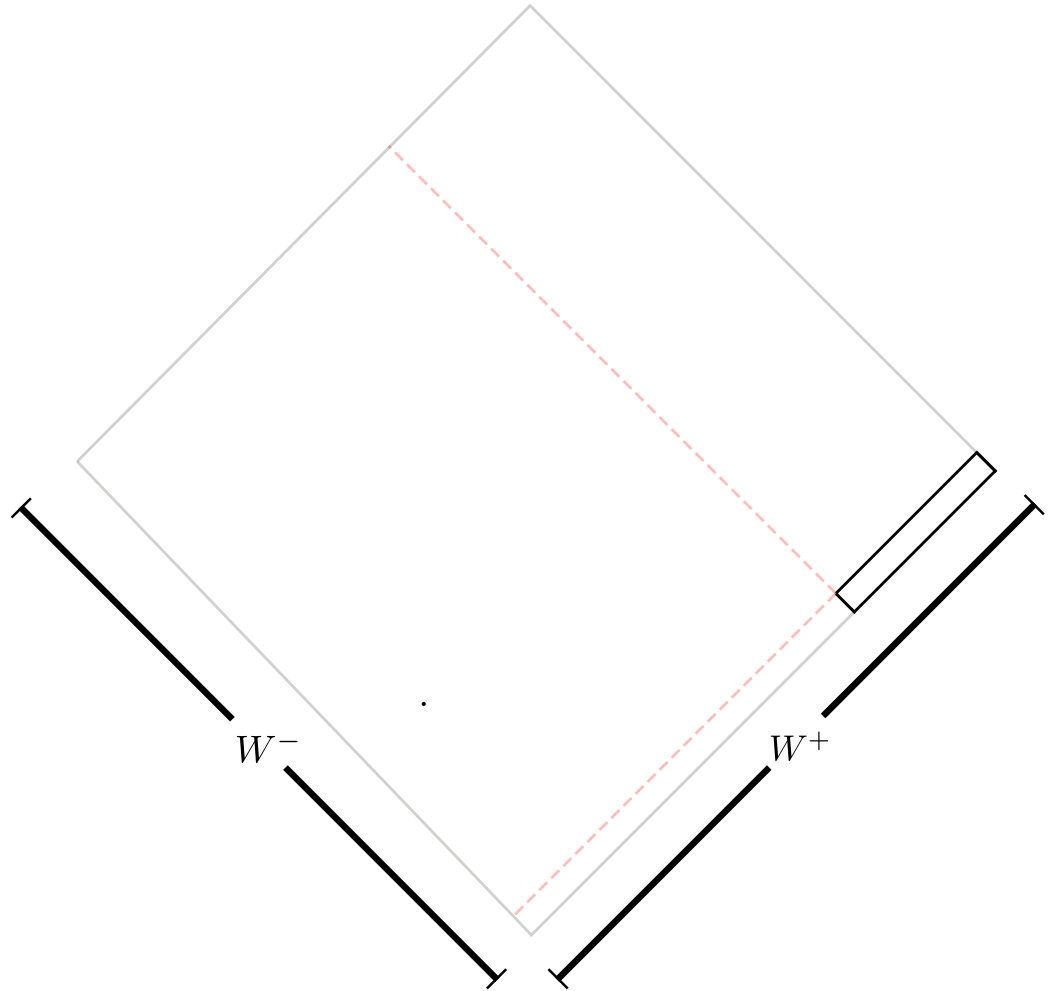
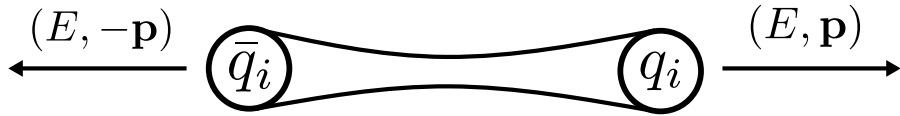
**Half-baked thoughts on the
interplay of “energy
conservation” in hadronization
and data-driven extraction of the
fragmentation function**

finalTwo

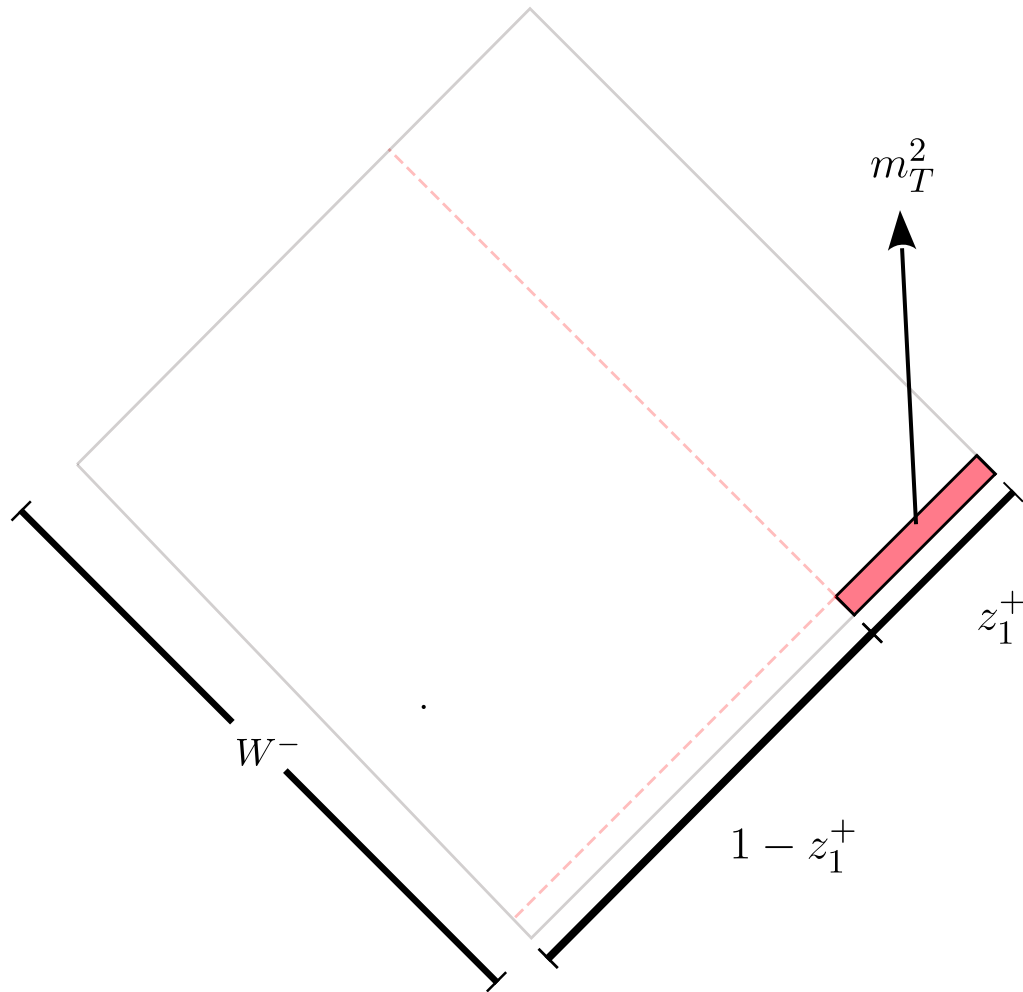
The algorithm ($q\bar{q}$)



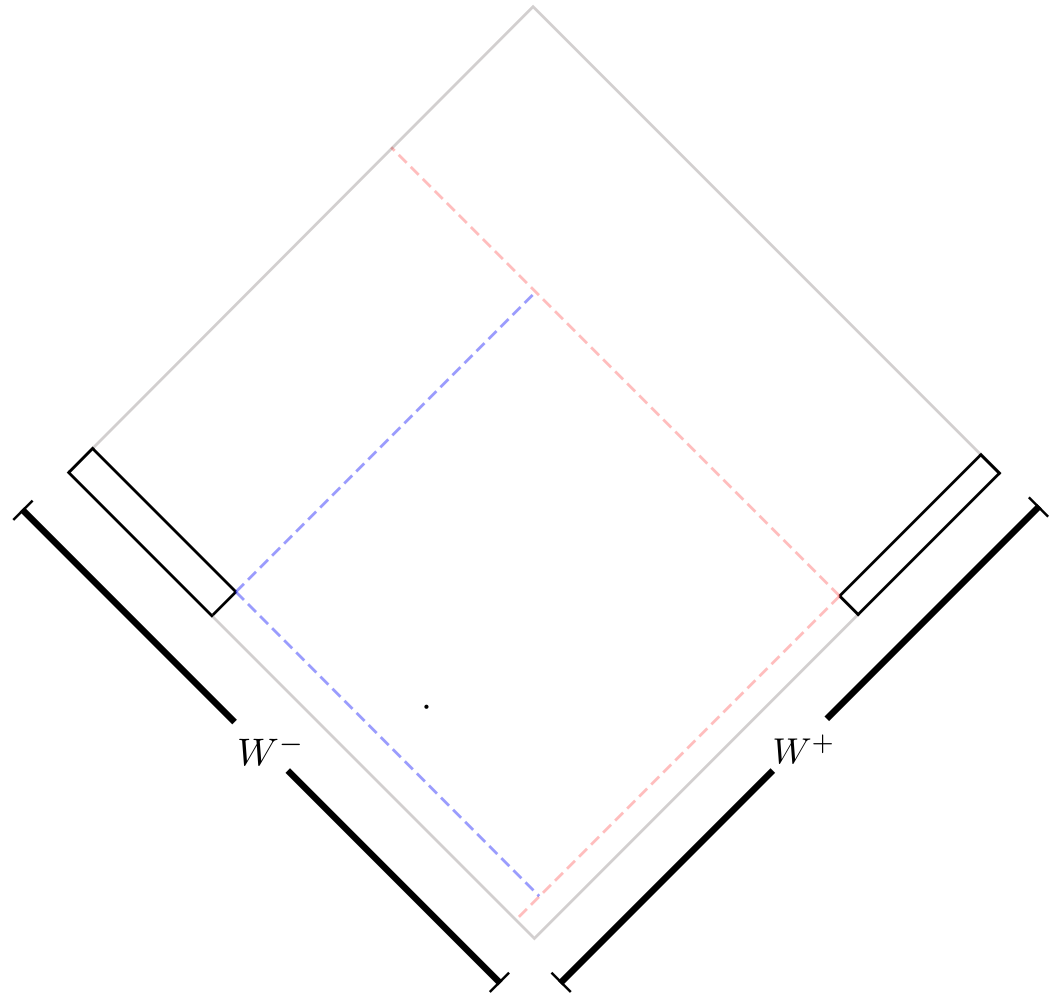
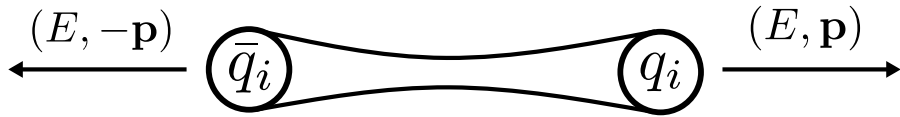
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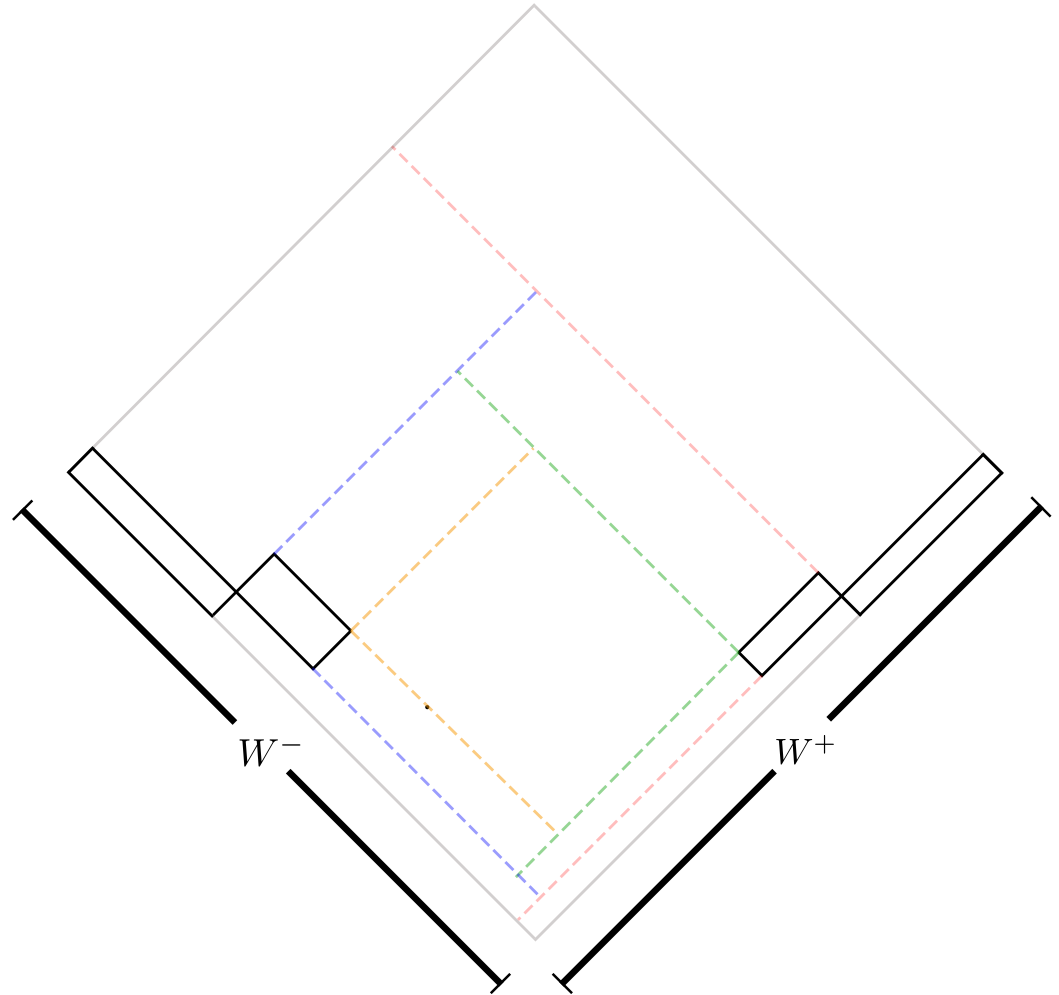
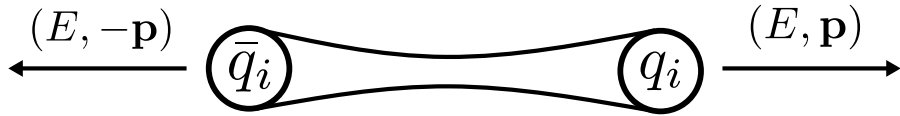
The algorithm ($q\bar{q}$)



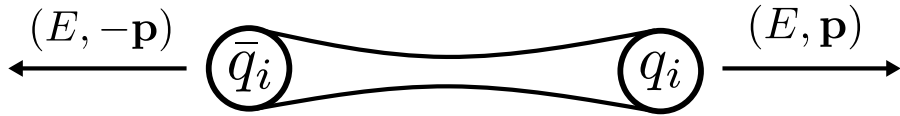
The algorithm ($q\bar{q}$)



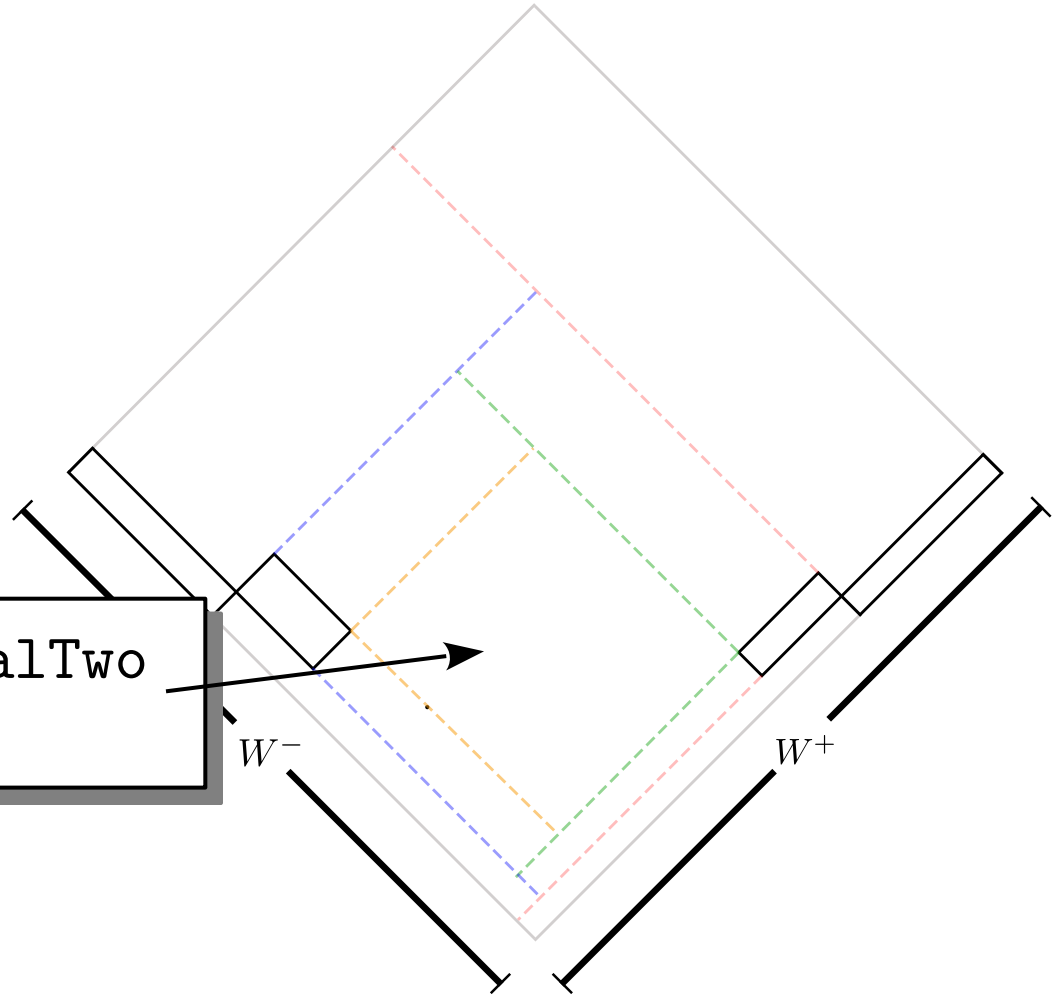
The algorithm ($q\bar{q}$)



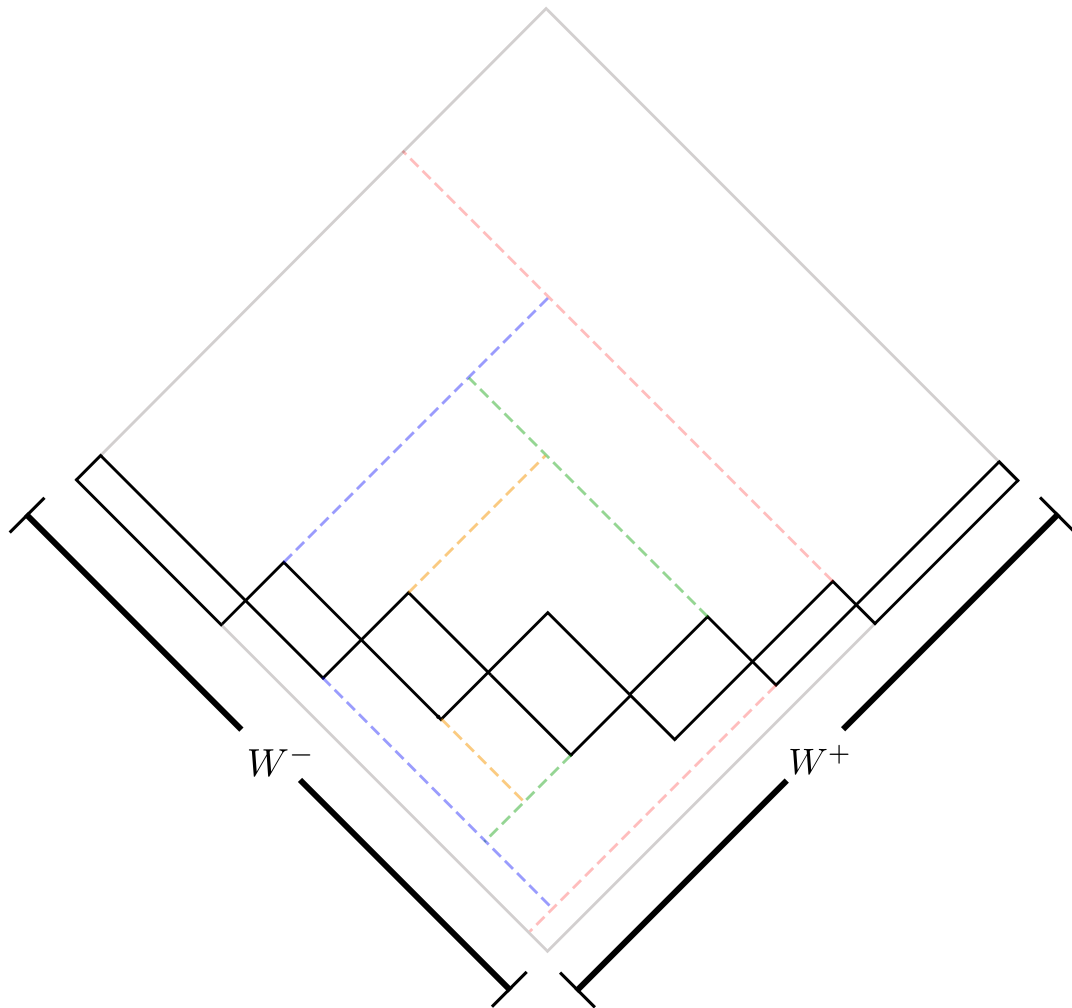
The algorithm ($q\bar{q}$)



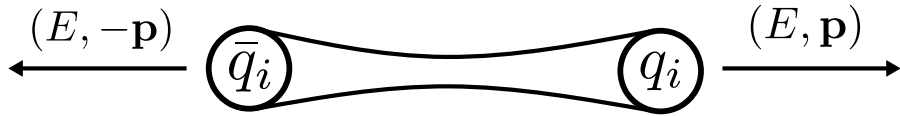
When E_{CM} goes below E_{cut} , `finalTwo` is called.



The algorithm ($q\bar{q}$)

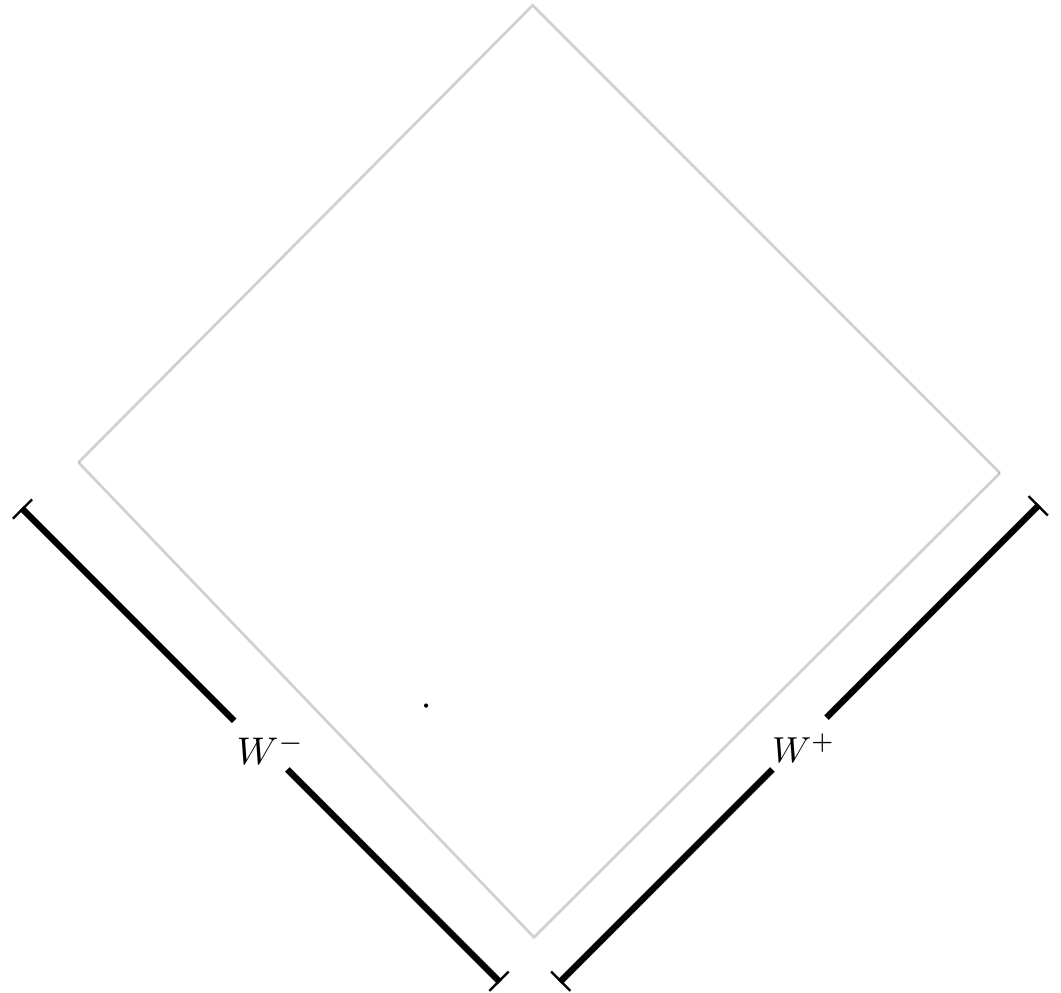


The algorithm ($q\bar{q}$)

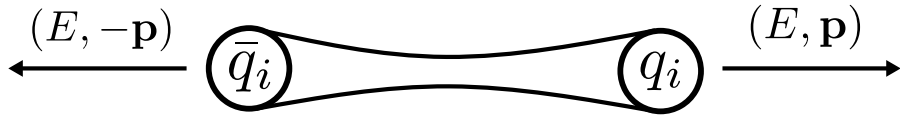


This system has two limits:

1. $E_{\text{CM}} \gg m_h$
- Random walk in $f(\mathbf{z})$

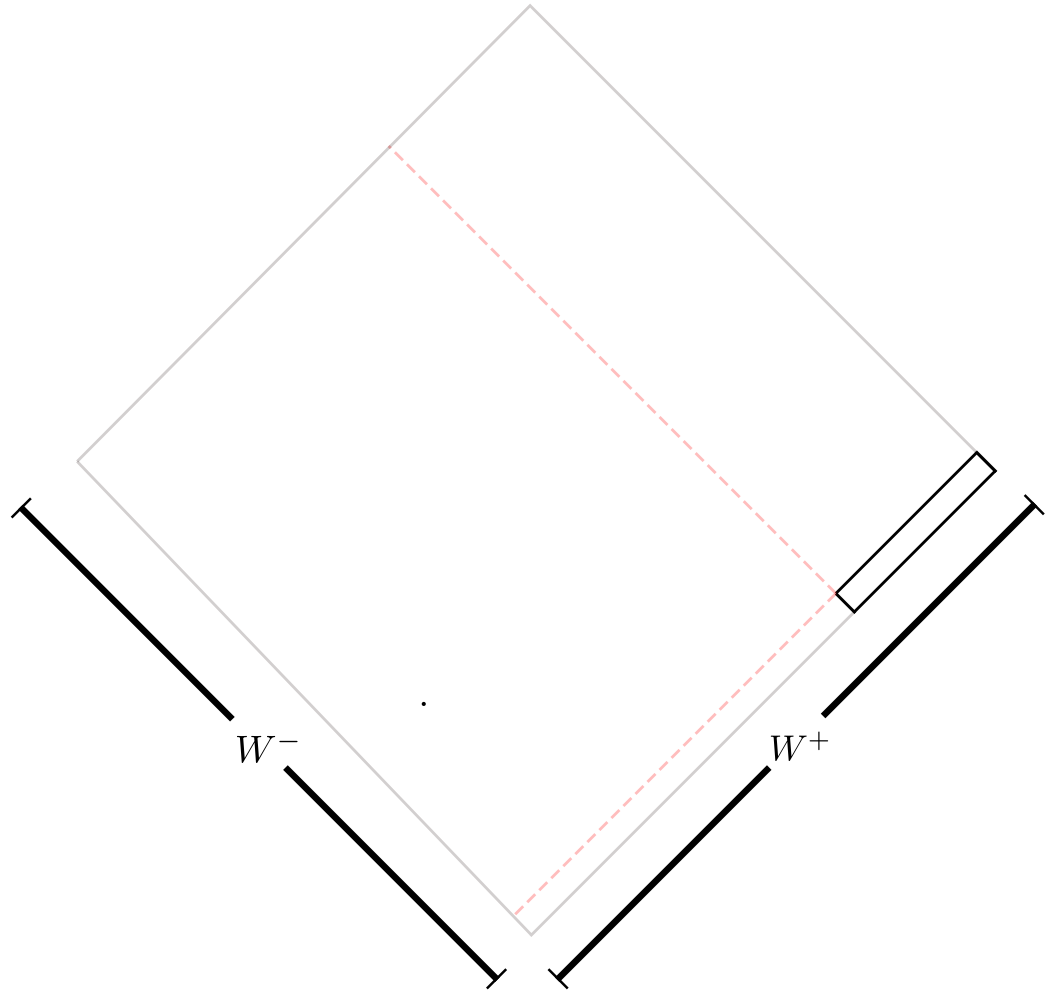


The algorithm ($q\bar{q}$)

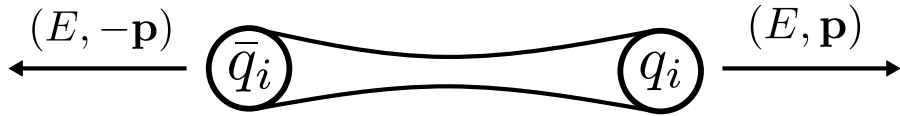


This system has two limits:

1. $E_{\text{CM}} \gg m_h$
- Random walk in $f(z)$

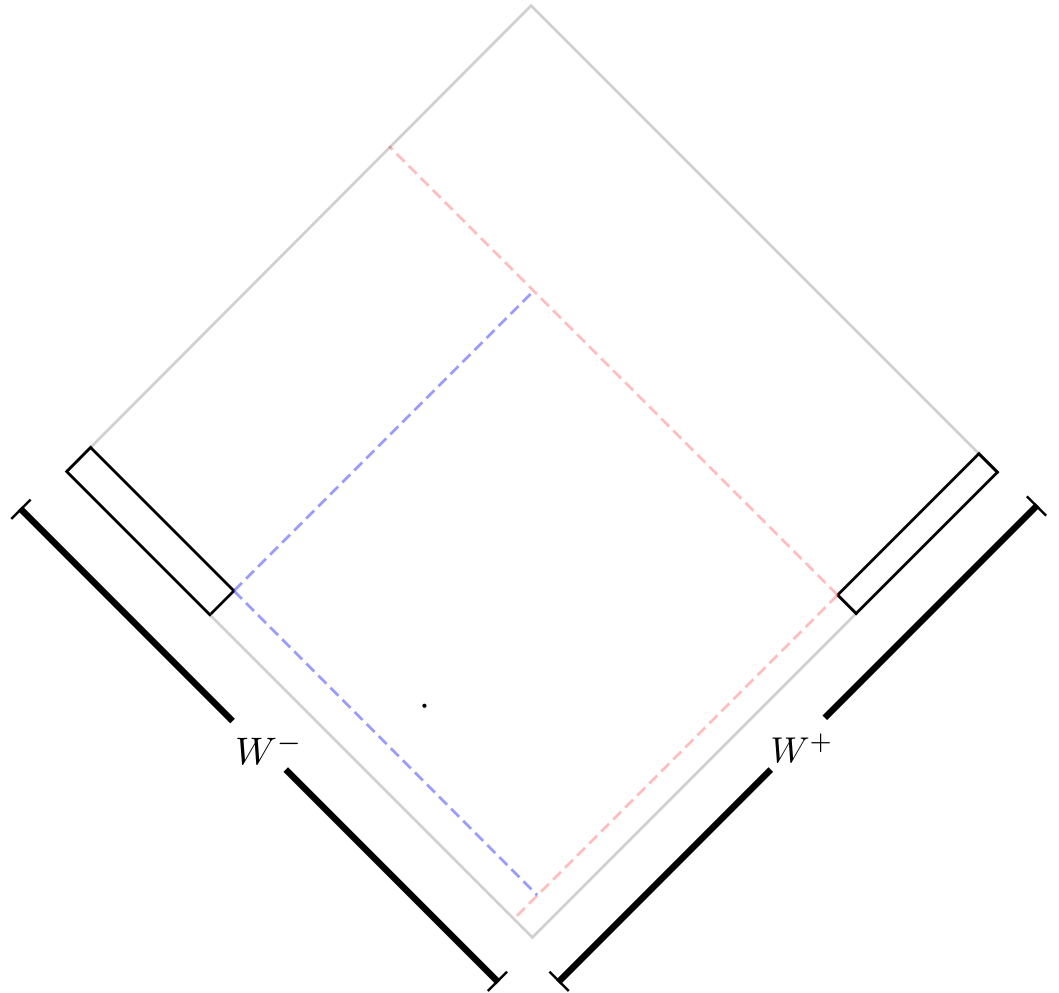


The algorithm ($q\bar{q}$)

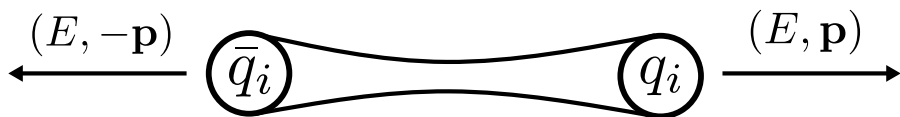


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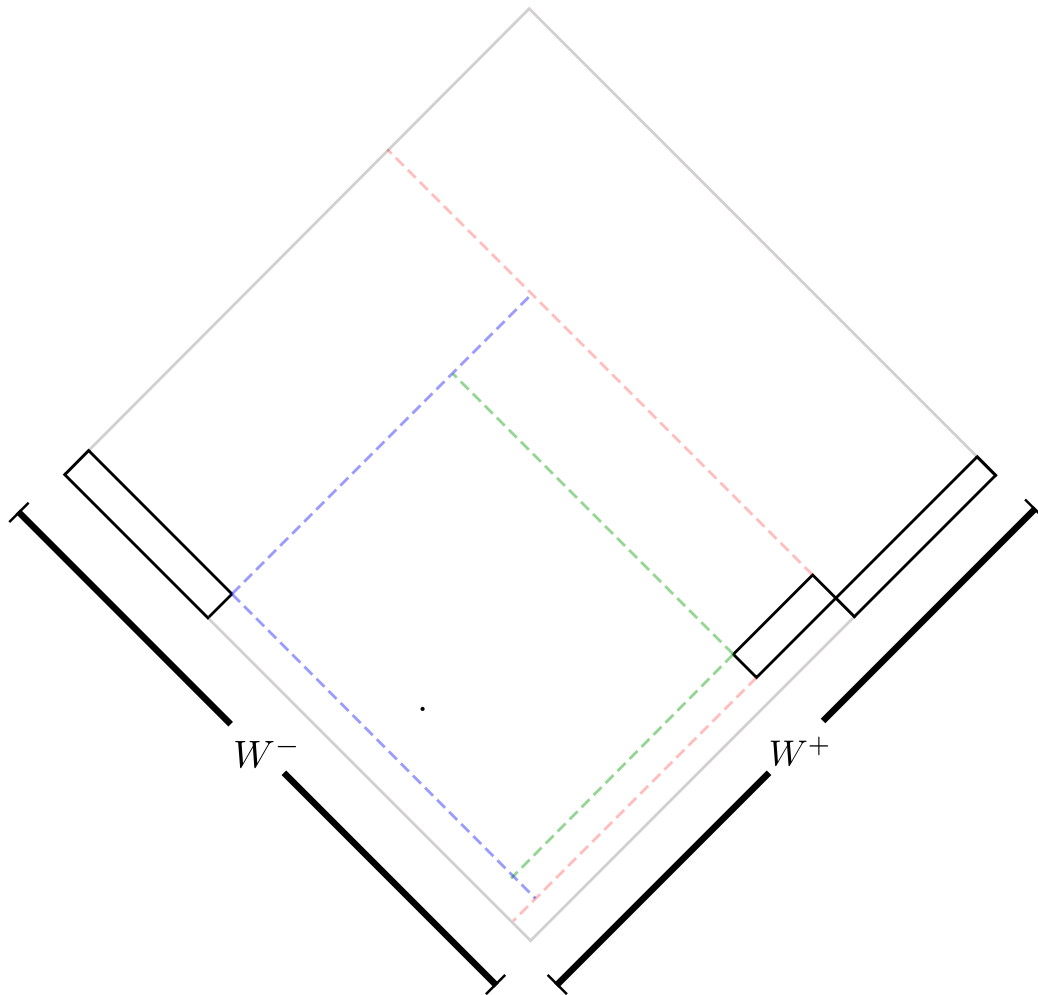


The algorithm ($q\bar{q}$)



This system has two limits:

1. $E_{\text{CM}} \gg m_h$
- Random walk in $f(z)$



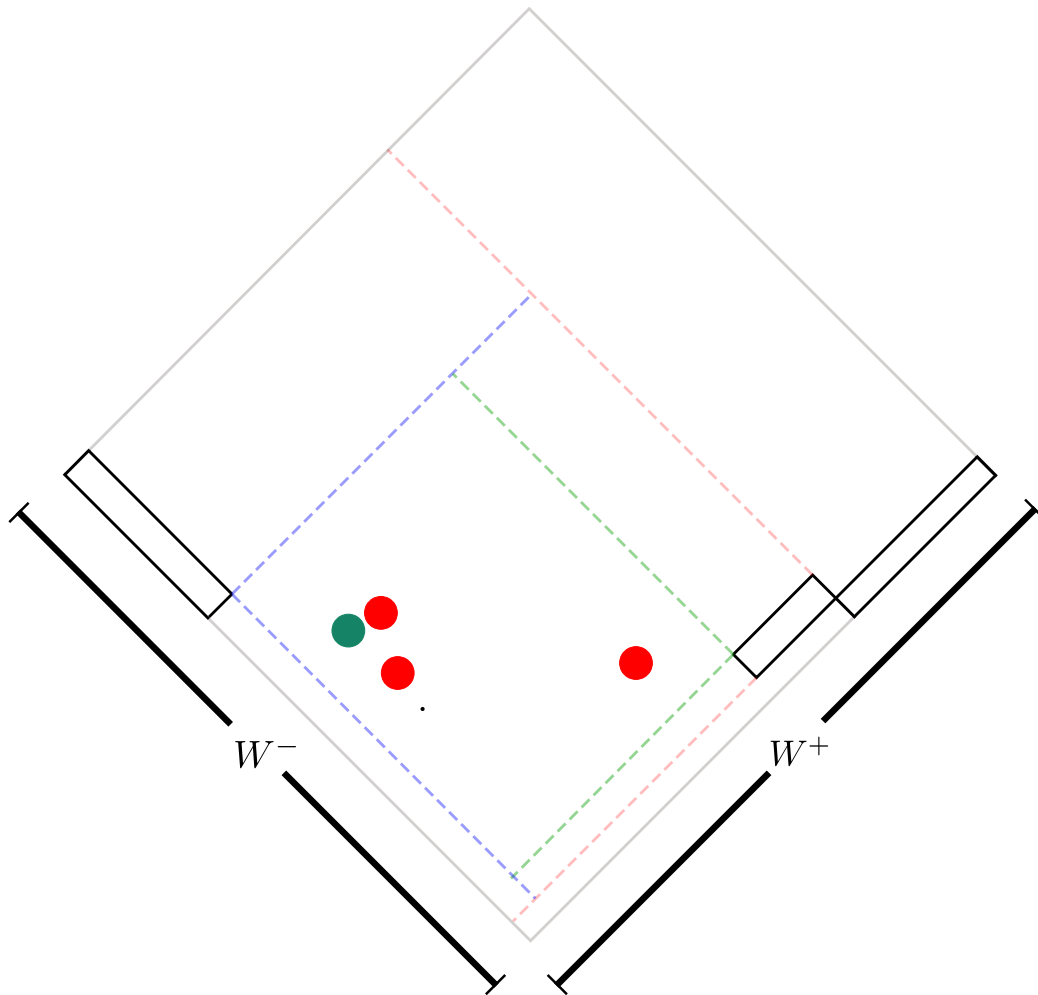
The algorithm ($q\bar{q}$)



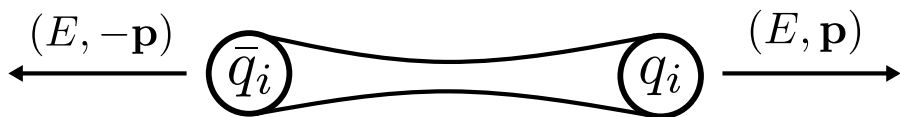
This system has two limits:

1. $E_{\text{CM}} \gg m_h$
- Random walk in $f(\mathbf{z})$

2. $E_{\text{CM}} \sim m_h$

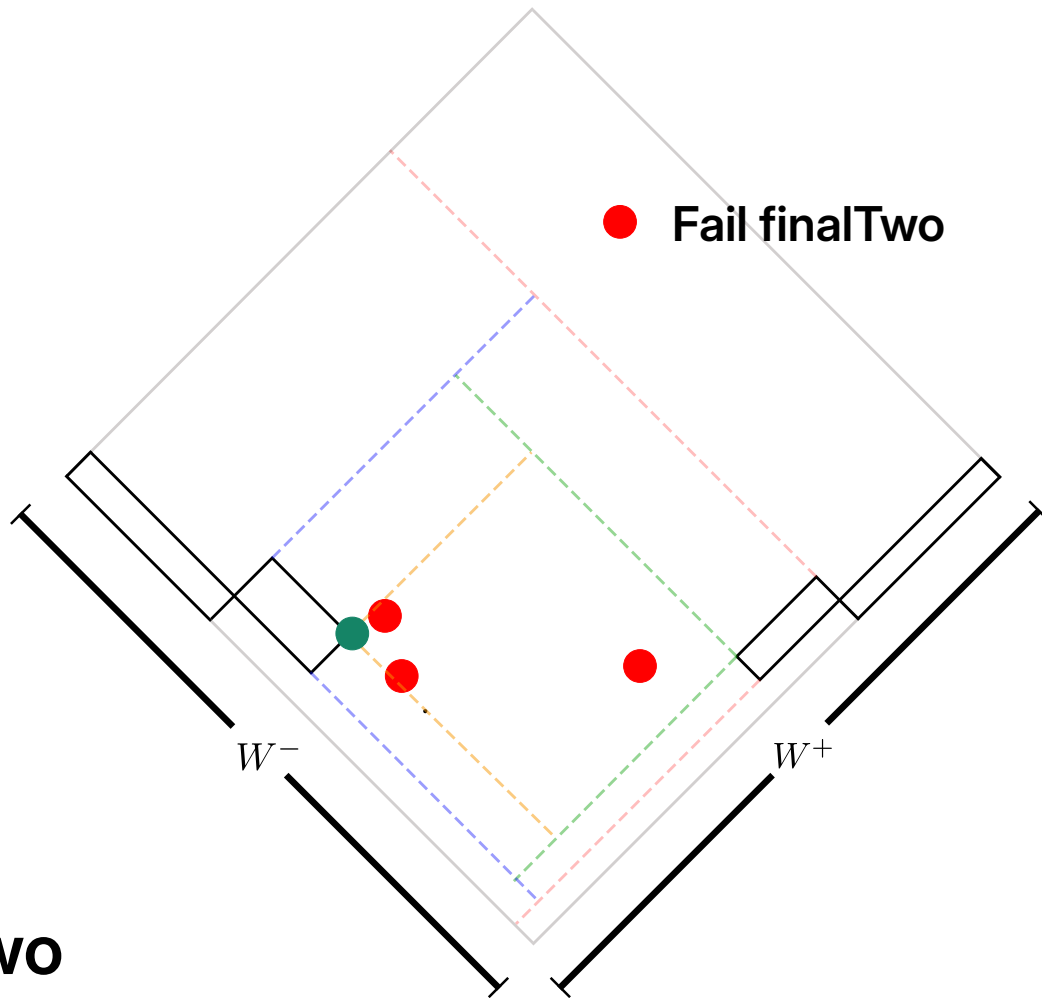


The algorithm ($q\bar{q}$)

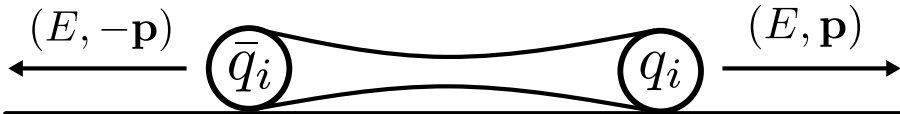


This system has two limits:

1. $E_{\text{CM}} \gg m_h$
 - Random walk in $f(\mathbf{z})$
2. $E_{\text{CM}} \sim m_h$
 - $f'(\mathbf{z})$ influenced by finalTwo



The algorithm ($q\bar{q}$)



● Fail finalTwo

We want to extract $f(\mathbf{z})$, not $f'(\mathbf{z})$ – cleaner extraction would factorize these two pieces.

How?

- $f'(\mathbf{z})$ influenced by finalTwo

Brownian bridge

- Gaussian random walk ($E_{CM} \gg m_h$)
 - Markovian
- Gaussian random walk with termination condition, i.e. Gaussian bridge
 - Pseudo-Markovian

Brownian bridge

- Gaussian random walk ($E_{CM} \gg m_h$)

$$dX_t = dW_t$$

- Gaussian bridge
 - Boundary conditions

$$X_0 = x_0 \quad X_T = x_T$$

$$dX_t = \frac{x_T - X_t}{T - t} dt + dW_t$$

Brownian bridge

- Gaussian random walk ($E_{CM} \gg m_h$)

$$dX_t = dW_t$$

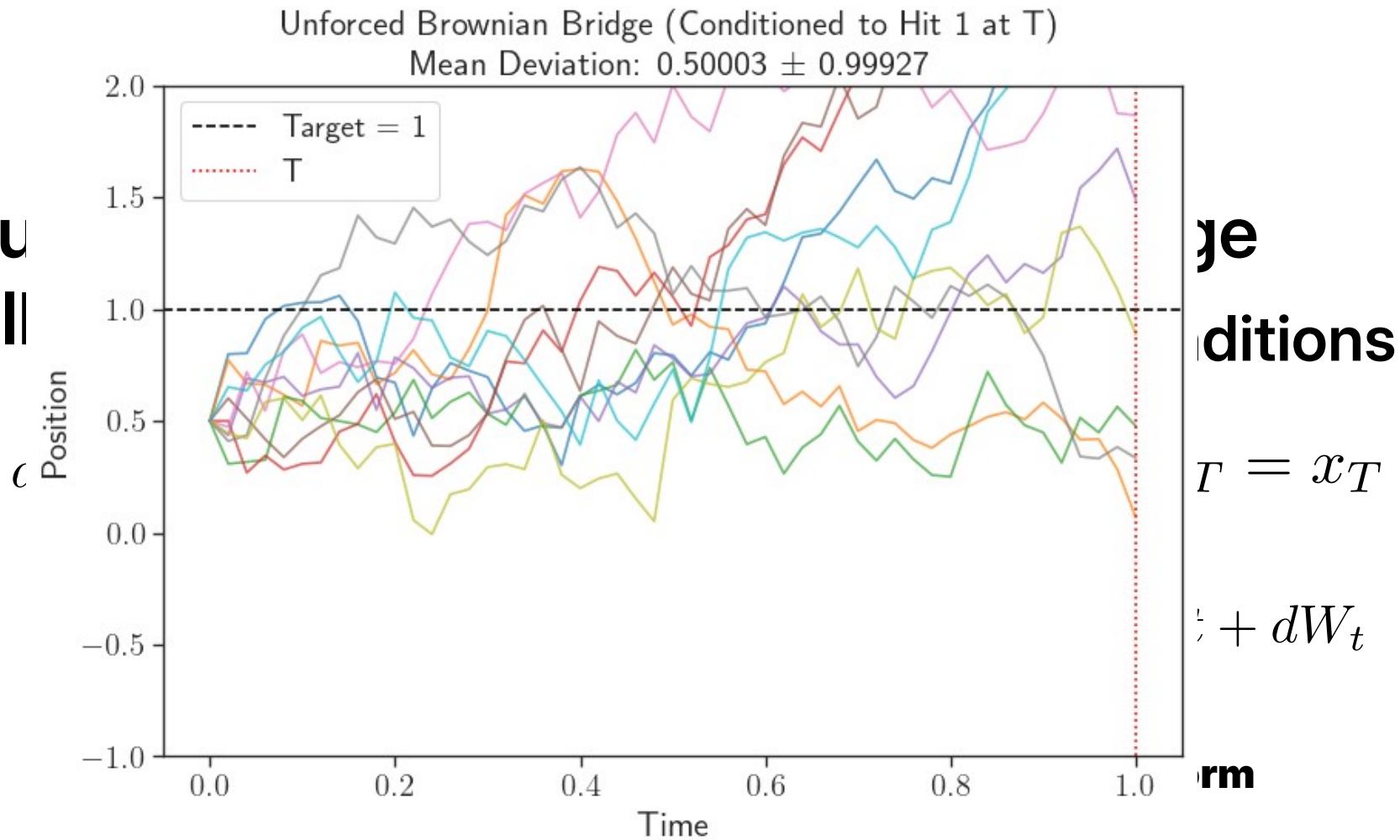
- Gaussian bridge
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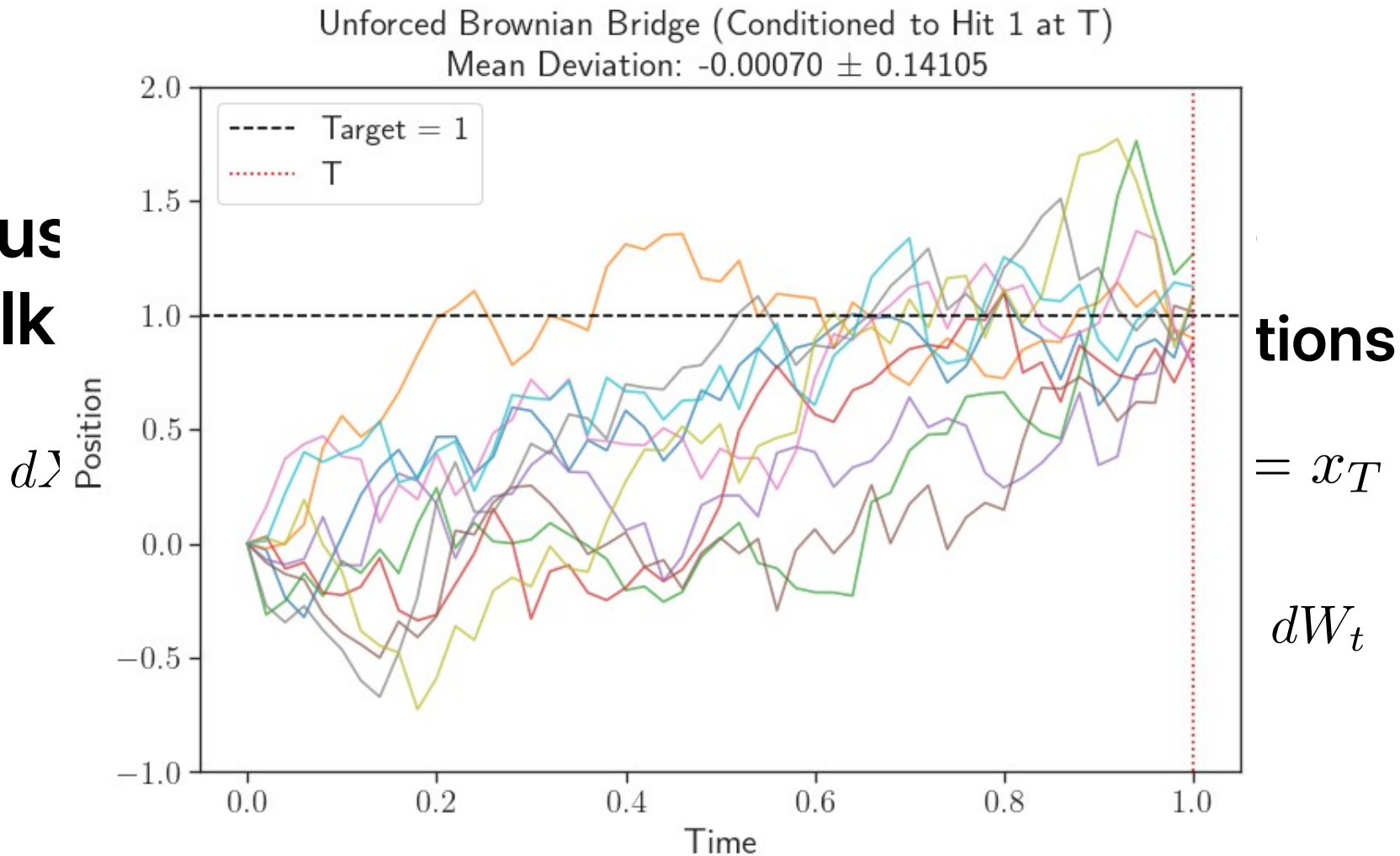
$$dX_t = \frac{x_T - X_t}{T - t} dt + dW_t$$

Doob h-transform

- **Gau**
wall

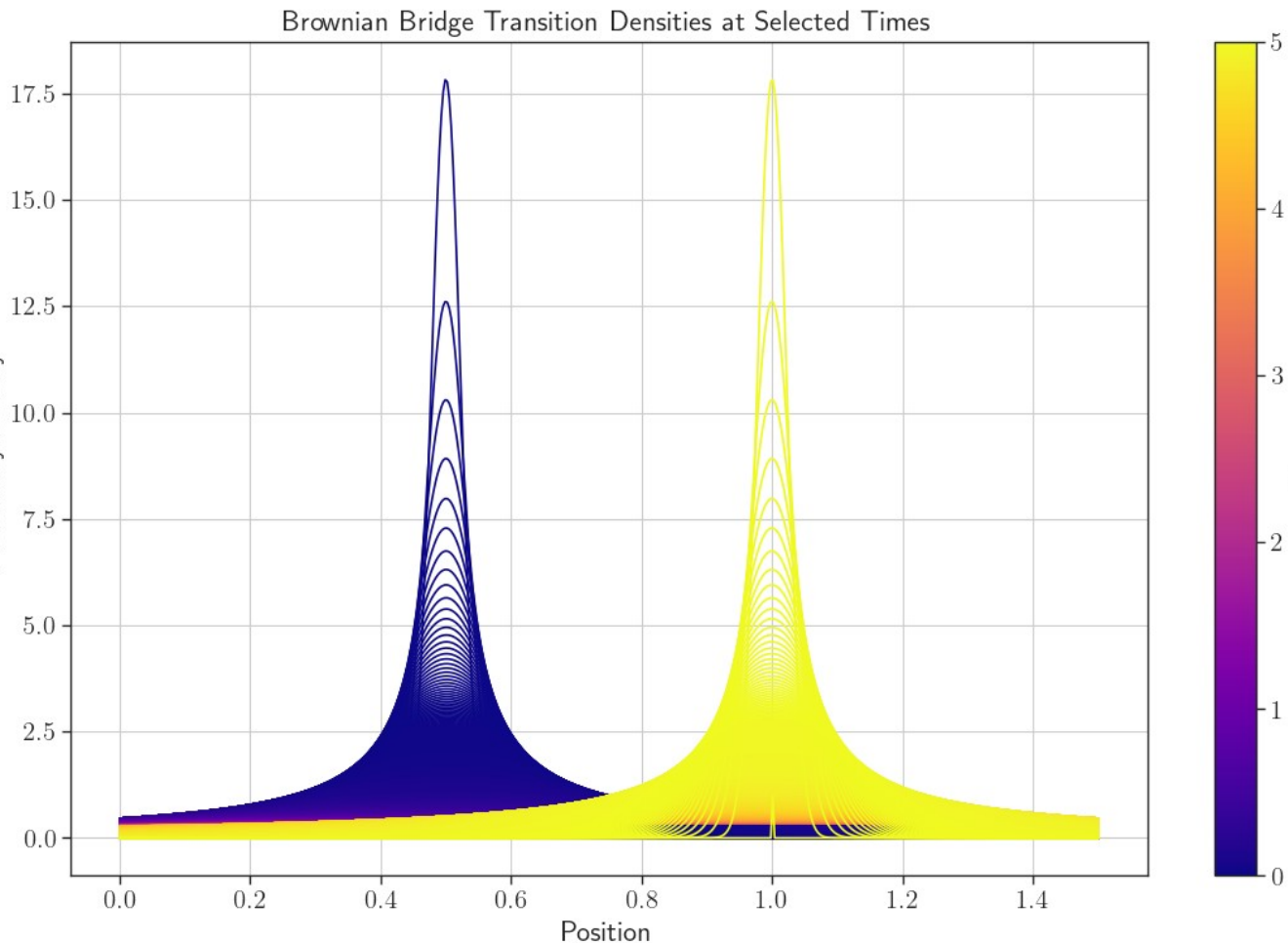


- **Gaus
walk**



- Gau
walk

d .



e
ditions

$$r = x_T$$

$$+ dW_t$$

'm

The algorithm (q \bar{q})

$(E, -\mathbf{p})$ \leftarrow \bar{q}

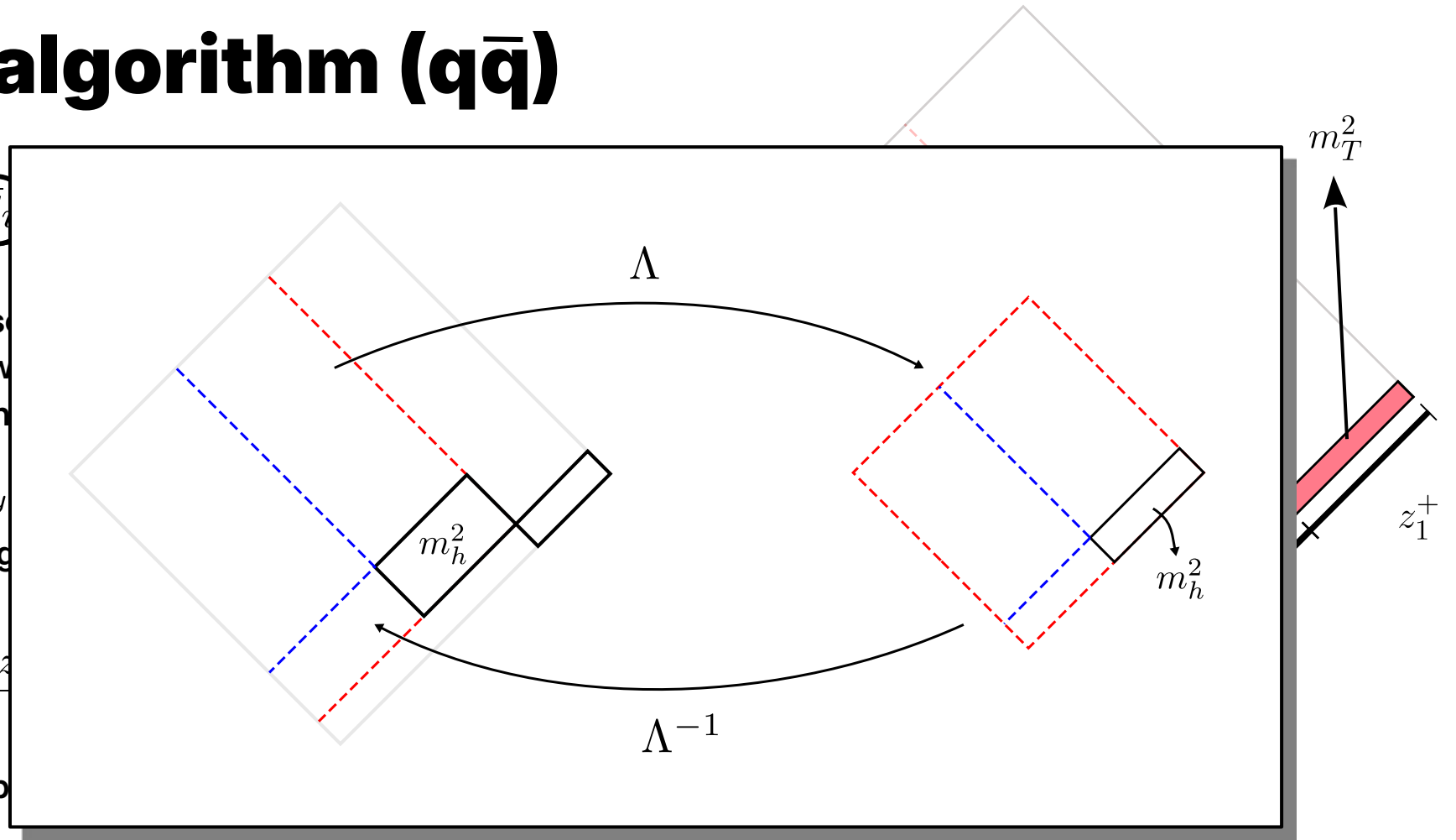
- 1) Randomly sample
- 2) Sample new
- 3) Sample trans

$$\mathcal{P}(p_x, p_y)$$

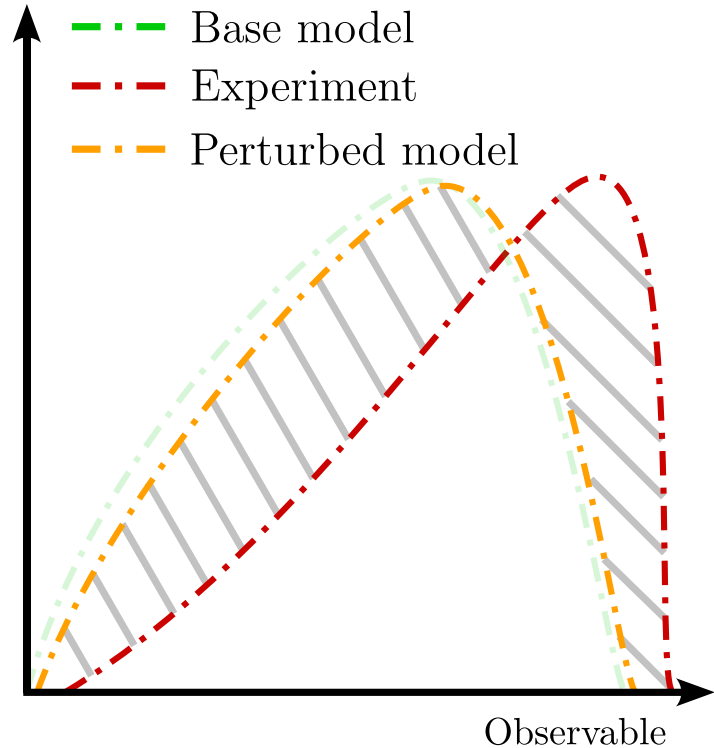
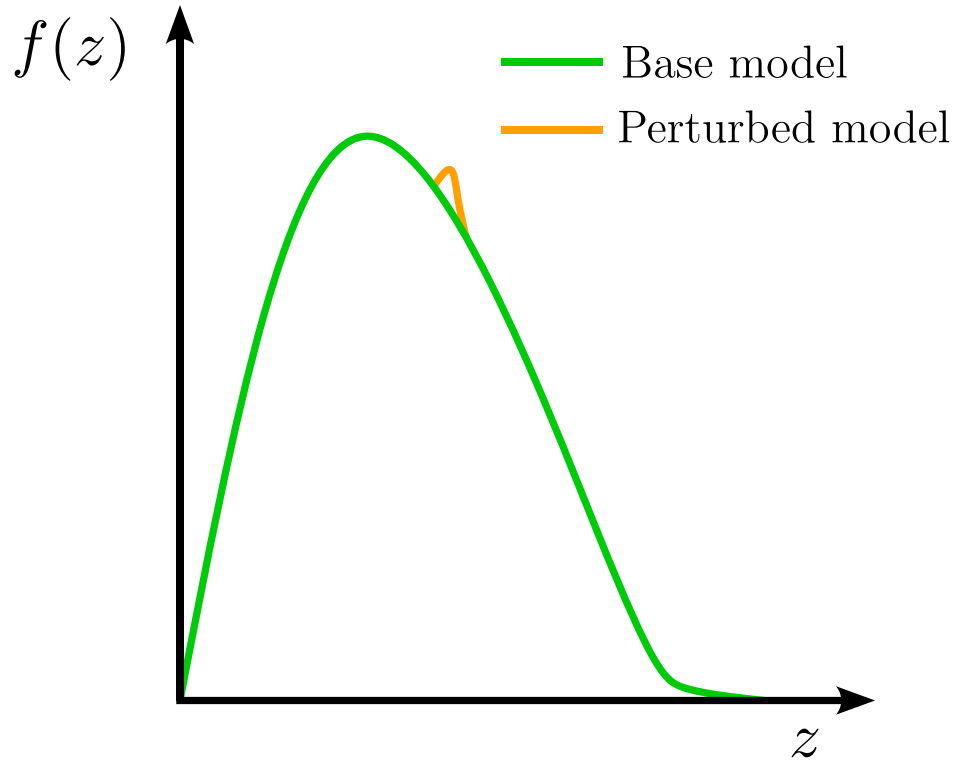
- 4) Sample long new hadron

$$f(z) \propto \frac{(1-z)}{z}$$

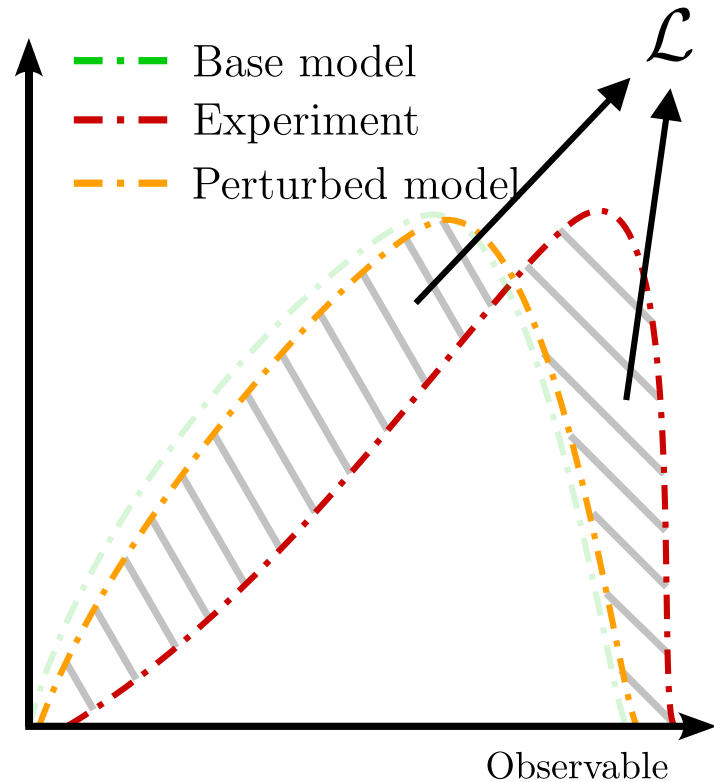
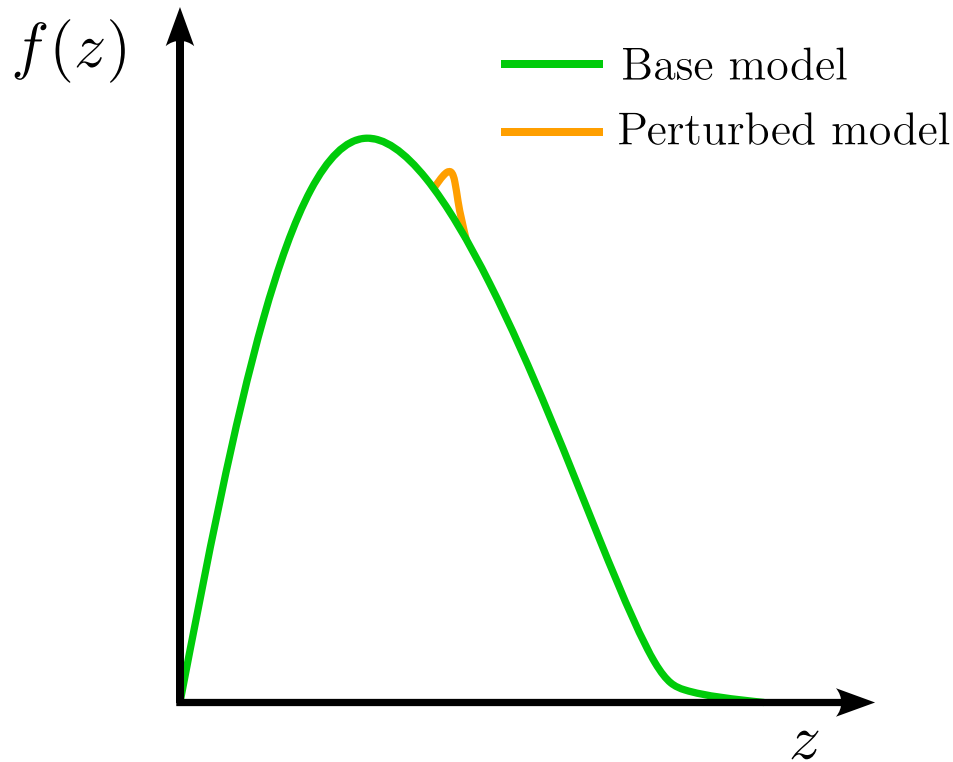
- 5) Repeat step



MLHAD efforts: big picture



MLHAD efforts: big picture



Applications

- Efficient means of exploring parameter space
- Inherently differentiable
- **Very useful for tuning!**
 - Embed the computation of weights into numerical autodifferentiation engine
 - Picking new parameters in the update step facilitated by well-developed optimizers (SGD, Adam, etc.)

Rejection sampling with Autodifferentiation (RSA)

WHAT is a differentiable simulator?

- All parameters are differential parameters
 - Differential with respect to what?

WHY a differentiable simulator?

- Parameter exploration (tuning)
- Uncertainty quantification
- Reproducibility

HOW to build a differentiable simulator?

- Many roads to a fully differentiable event generator. Varying levels of “intrusivity” (from a developers perspective).

The algorithm (q \bar{q})

$(E, -\mathbf{p})$ \leftarrow (\bar{q})

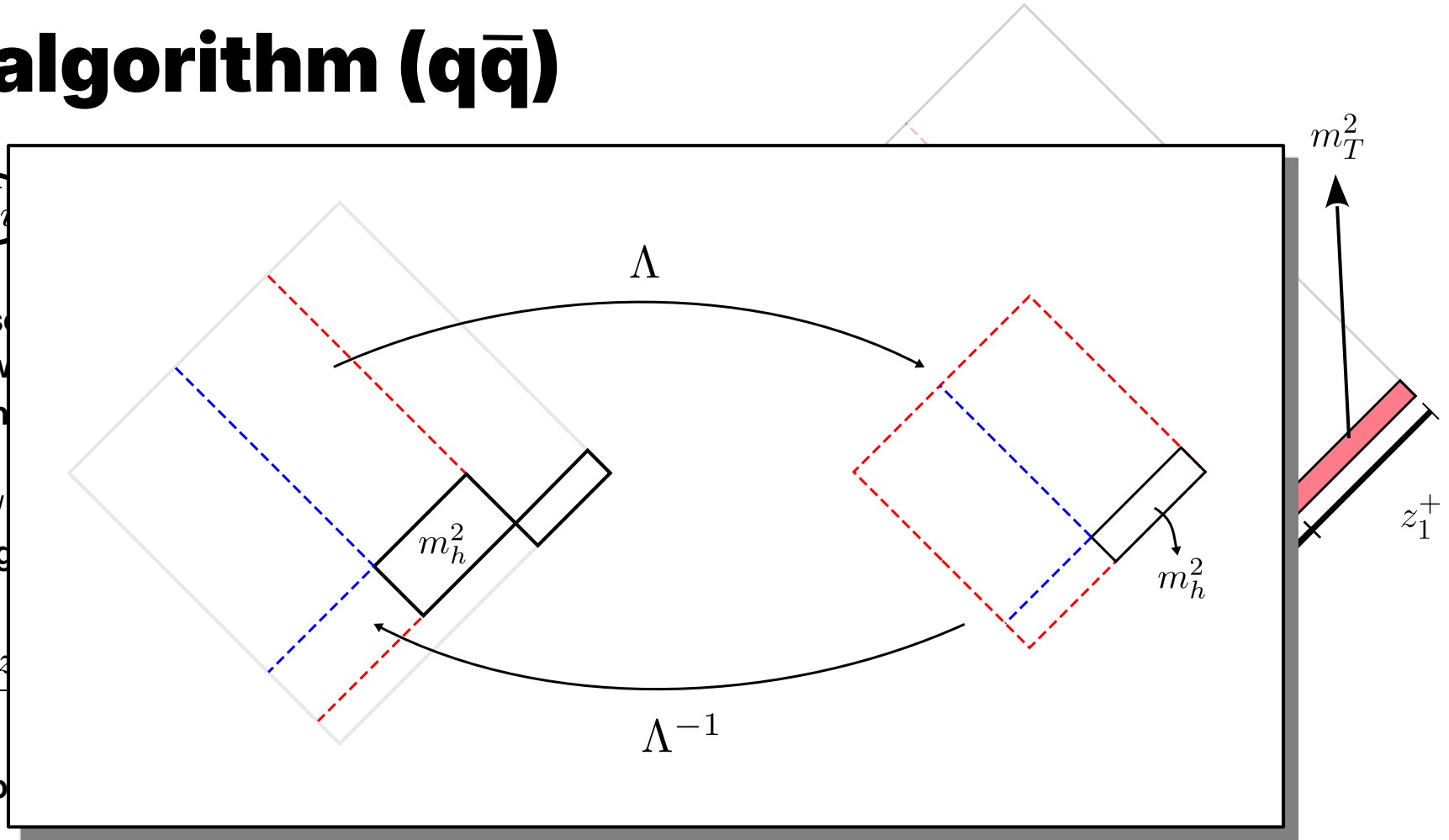
- 1) Randomly sample
- 2) Sample new
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$$\mathcal{P}(p_x, p_y)$$

- 4) Sample long
new hadron

$$f(z) \propto \frac{(1-z)}{z}$$

- 5) Repeat step



The "score" observable

- Train a deepsets classifier to distinguish simulation from experiment

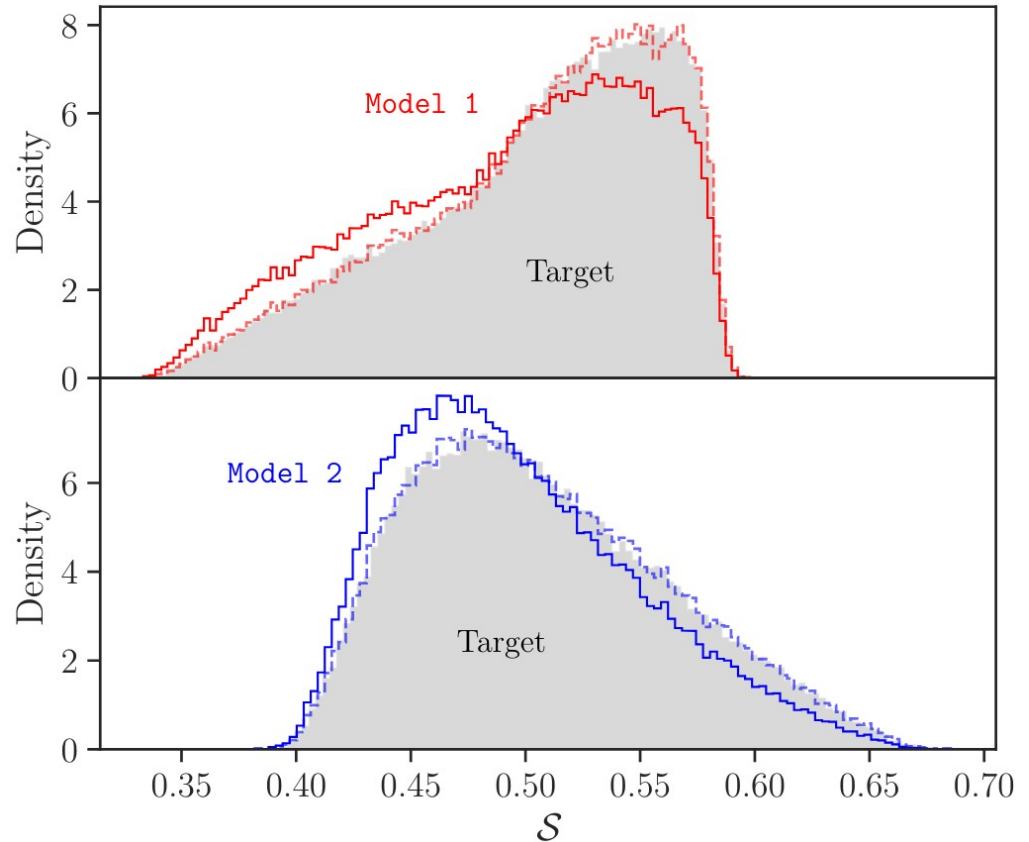
- Takes full event information as input

$(E, p_x, p_y, p_z)_1$

$(E, p_x, p_y, p_z)_2$

...

Use trained classifier output (score) as an observable

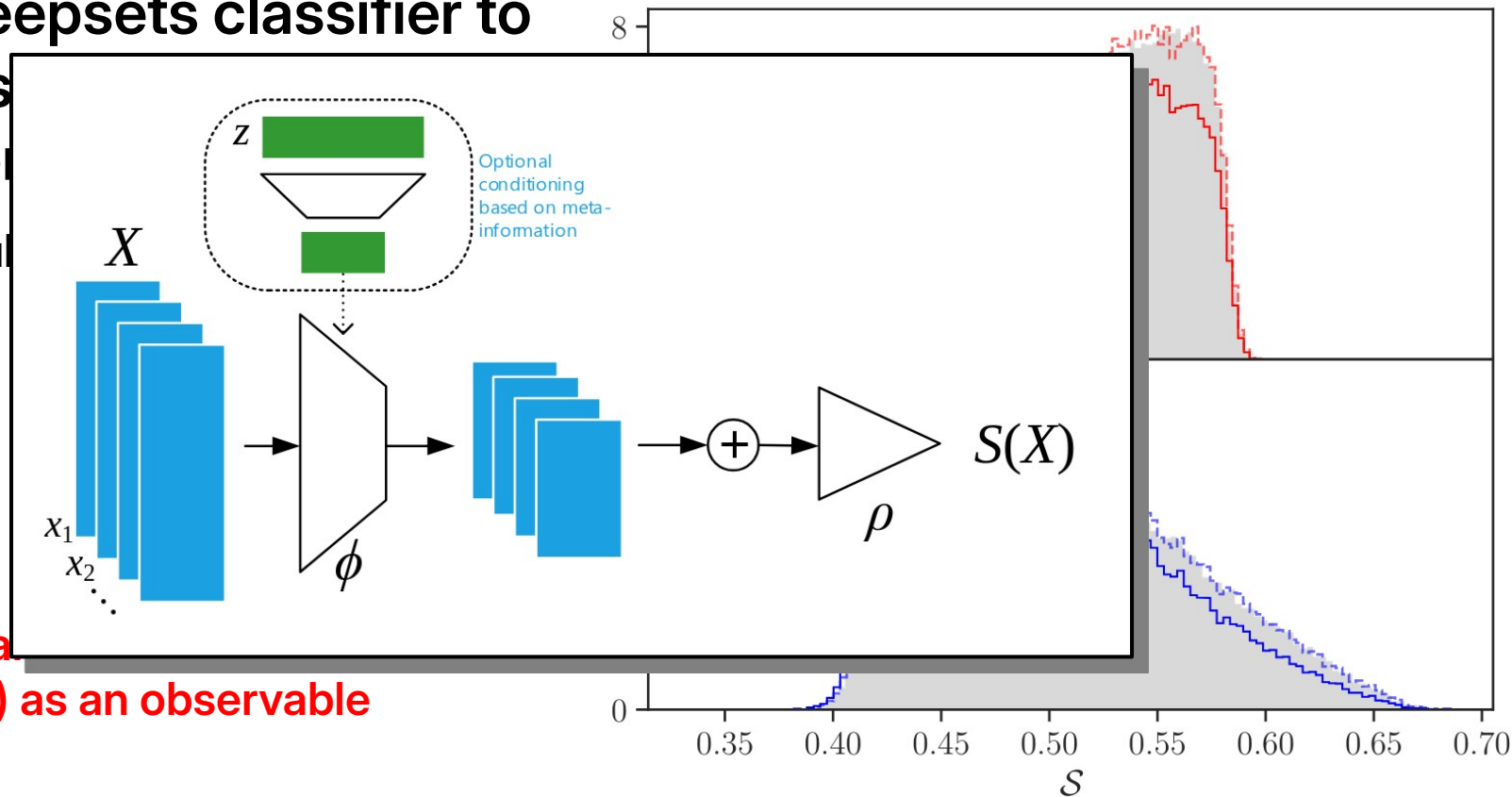


The "score" observable

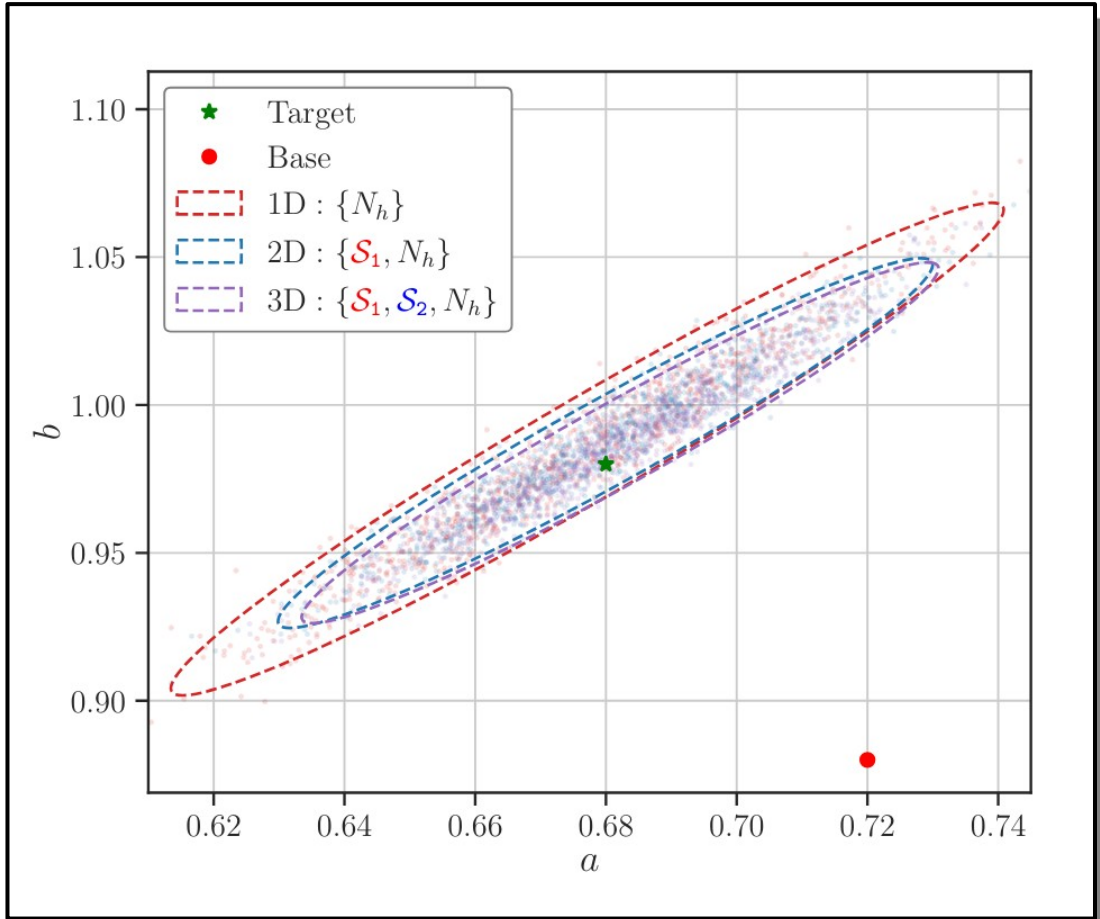
- Train a deepsets classifier to distinguish experimental

– Takes full input

Use training (score) as an observable

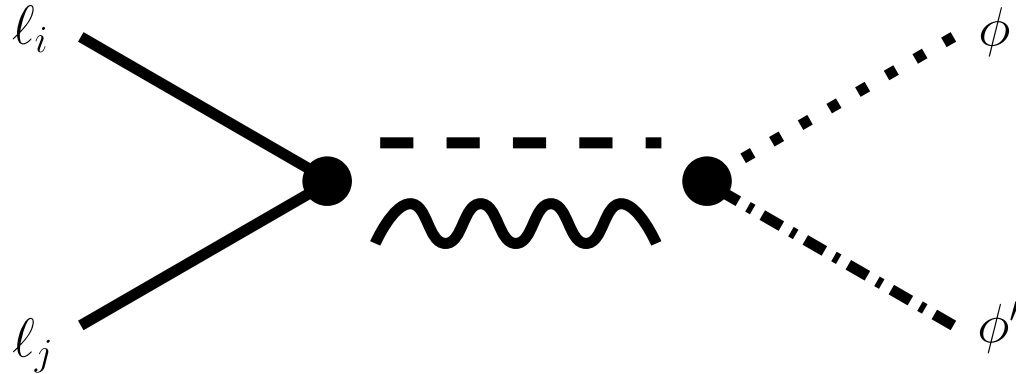


Classifier score with full event
info improves tuning
convergence



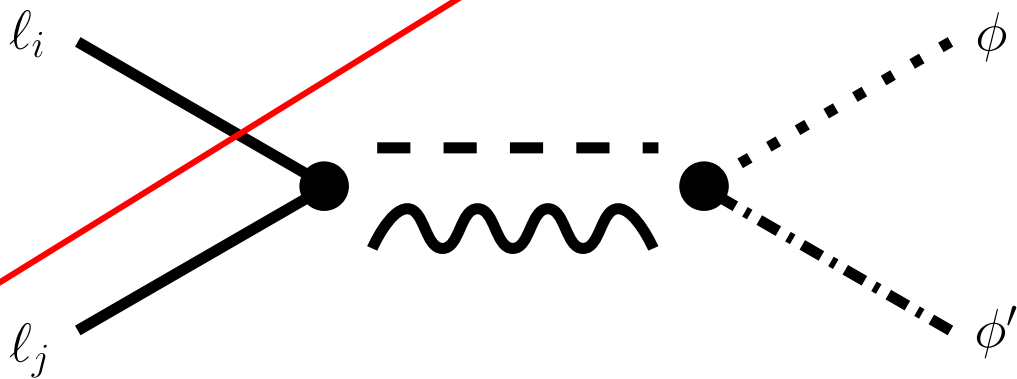
Explicit realization

Consider: $\frac{C_{ij}}{f^2} \phi \partial_\mu \phi' (\bar{l}_i \gamma^\mu l_j)$



Explicit realization

Consider: $\frac{C_{ij}}{f^2} \phi \partial_\mu \phi' (\bar{l}_i \gamma^\mu l_j)$



→ Non-abelian pseudo-NGB + portal

Explicit realization

- Non-abelian pseudo-NGB + $U(1)_D$

$$\mathcal{L}_D \supset \frac{f_{UV}^2}{8} (D_\mu U D^\mu U^\dagger) + \frac{f_{UV}^2}{8} (\chi^\dagger U + U^\dagger \chi) + \dots$$

Explicit realization

- Non-abelian pseudo-NGB + $U(1)_D$ $\rightarrow U \rightarrow e^{i\alpha}U$

$$\mathcal{L}_D \supset \frac{f_{UV}^2}{8} (D_\mu U D^\mu U^\dagger) + \frac{f_{UV}^2}{8} (\chi^\dagger U + U^\dagger \chi) + \dots$$