HADRONIZATION

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1 Lund string model in 1+1 Dimensions

Here we will refer to *hadronization* as the *model* used to convert an initial partonic state such as a quark-antiquark pair ($q\bar{q}$) to a final state consisting of n hadrons h i.e. $q\bar{q} \to h_1 h_2 \cdots h_n$. We reserve the term *fragmentation* to describe one of the many singular hadronization events which collectively make up the full hadronic final state i.e. $q_i\bar{q}_i \to h_i$. In what follows, we analyze a 1+1 (one space, one time) dimensional massless $q_i\bar{q}_i$ system in the center-of-mass frame where the partons, each with flavor index i and initial energy E, travel with equal and opposite momenta. As the separation increases the three-gluon coupling (or vacuum pressure) causes an approximately uniform cylindrical string (flux tube) of color field to form between the quark pair with an approximately uniform energy density $\kappa \approx 1$ GeV/fm ≈ 0.2 GeV^2. As the potential energy of the string becomes larger, it becomes energetically favorable to create $q'\bar{q}'$ pairs out of the vacuum. This production breaks the original string into fragments and produces a composite hadron $h \equiv q_i\bar{q}_j(q_j\bar{q}_i)$ and another $q_j\bar{q}_i(q_i\bar{q}_j)$-string system as depicted in Fig.(1). The hadron is ejected with some energy and momentum $(E_h, \vec{p}_h)$ following the Lund hadronization model implemented in Pythia. These fragmentations continue in a fragmentation *chain* until the energy of the initial two-parton system ($2E$) is entirely converted into hadrons.

In the absence of string breaks our $q\bar{q}$ system will follow the so-called "yo-yo" motion depicted in Fig.(2). At $t = 0$ the massless quarks have equal and opposite momentum along the z-axis each with energy E. The quarks continue to separate at the speed of light until the string contains all of the system's energy at $t = E/\kappa$. At this point the tension in the string reverses the quark motion bringing the quarks back to the origin at $t = 2E/\kappa$ where they cross and continue the same motion with q and $\bar{q}$ interchanged. Assuming that the hadronization process occurs completely within the first quarter of the string motion, the evolution of the energy and momentum of the quark-string system can be described by the following energy-momentum four vectors

$$p_{q/\bar{q}}(t) = (E - \kappa t)(1, 0, 0, \pm 1), \qquad p_{\text{string}}(t) = (2\kappa t, 0, 0, 0), \qquad 0 < t < E/\kappa \quad (1)$$

The system is also conveniently described with light-cone coordinates. We define both energy-momentum coordinates $p^{\pm} \equiv E \pm p_z$ as well as spacetime coordinates $z^{\pm} \equiv t \pm z$ related via the linear energy density of the string κ. We also define $W^{\pm} \equiv p^{\pm}(t = 0) = 2E_{q/\bar{q}} = 2E$ and denote the invariant mass of the system (or area of momentum space) as $W^2 = W^+ W^-$. The first quarter of the motion is then

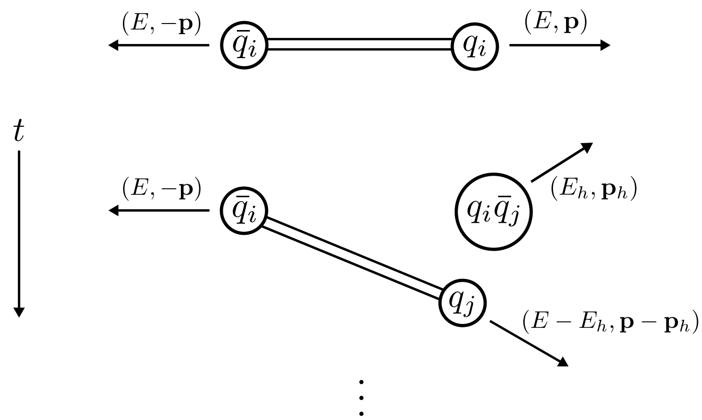

Figure 1: A cartoon depiction of a single fragmentation event in the center of mass frame of the initial string system.

described as

$$p_{q/\bar{q}}(t) = \left(\frac{W^{\pm} - \kappa z^{\pm}}{2}\right)(1,0,0,\pm 1), \quad p_{\mathrm{string}}(t) = (\kappa z^{+},0,0,0) \tag{2}$$

Inclusion of string breaking creates a complex evolution between the produced hadrons, represented as isolated strings following the 'yo-yo' motion described above, and the resulting string system which can further fragment into more hadrons. The space-time locations, or production vertices, where string breaks occur are space-like separated and thus causally disconnected. This makes the space-time ordering of all the individual fragmentation events Lorentz frame dependent. In practice this means that each vertex can be analyzed independently of all other vertices and in any convenient order in time. Equivalently, each fragmentation vertex v_i can be considered as a 'scaled down' version of the previous vertex v_{i-1}.

The fragmentation process implemented in PYTHIA is constructed in momentum space as an iterative walk through production vertices. To describe the fragmentation in momentum space we introduce light-cone coordinates $\hat{x}^{\pm}$ describing the location of production vertices and light-cone fractions $x^{\pm}$ defined by $p^{\pm}_{h_i} = x^{\pm}_{h_i} W^{\pm}$ as shown in Fig.(3). Both coordinates are normalized to the quark turning points such that $0 \leq \hat{x}^{\pm}, x^{\pm} \leq 1$. In $1+1$ dimensions the $q_i\bar{q}_i$ pair produced at a production vertex v_i are created with no energy or momentum and then pulled apart by the color force from the adjacent outgoing quarks. Thus, the phase space of hadron i is fully described by the oscillating string formed between vertices $i-1$ and i

$$E_{h_i} = \kappa(z_{i-1} - z_i), \qquad p_{z,h_i} = \kappa(t_{i-1} - t_i) \tag{3}$$

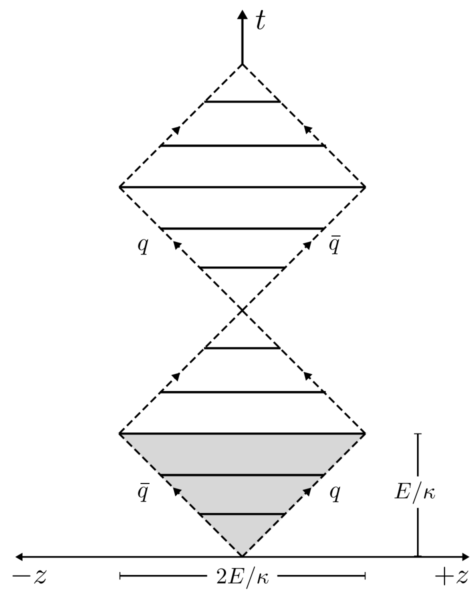

Figure 2: The described 'yo-yo' motion through spacetime of a free string in the absence of string breaks. The shaded region is where the hadronization process occurs.

For a description in momentum space we define the four-vectors $p_0^{\pm} \equiv E(1, 0, 0, \pm 1)$, and fractions $x_{h_i}^{+} = \hat{x}_{i-1}^{+} - \hat{x}_i^{+}$, $x_{h_i}^{-} = \hat{x}_i^{-} - \hat{x}_{i-1}^{-}$. The hadron momentum is then fully described by the system of equations

$$p_{h_i} = x_{h_i}^{+} p_0^{+} + x_{h_i}^{-} p_0^{-} \tag{4}$$

or simply

$$E_{h_i} = E(x_{h_i}^{+} + x_{h_i}^{-}), \qquad p_{z,h_i} = E(x_{h_i}^{+} - x_{h_i}^{-}) \tag{5}$$

constrained by the condition

$$m_{h_i}^2 = p_{h_i}^2 = x_{h_i}^{+} x_{h_i}^{-} W \tag{6}$$

Finally we introduce the longitudinal momentum fractions $z_i^{\pm}$ describing the percentage of longitudinal momentum (light-cone momentum $p^{\pm}$) absorbed by the ith hadron from the remaining longitudinal momentum in the system. The distribution from which z is sampled is called the *Lund left-right symmetric scaling (fragmentation) function* and is proportional to the following

$$f(z)dz \propto \frac{(1-z)^a}{z} \exp\left(-b\frac{m_h^2}{z}\right) dz \tag{7}$$

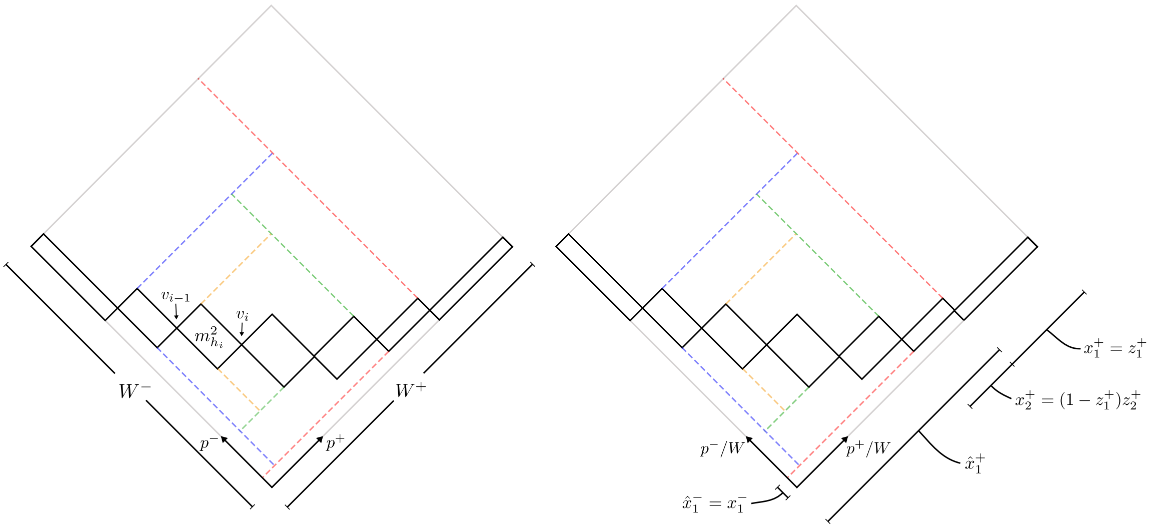

Figure 3: Momentum space representation of a jet fragmentation as implemented in the Lund string model.

Where a and b are phenomenological parameters chosen to match empirical data. Given the mass of the new hadron m_{h_i} and longitudinal momentum fraction z_i the new vertex i can be determined recursively via

$$x_{h_i}^{\pm} = z_i^{\pm} \prod_{j}^{i-1} (1 - z_j^{\pm}) \tag{8}$$

$$x_{h_i}^{\mp} = \frac{m_{h_i}^2}{x_{h_i}^{\pm} W^2} \tag{9}$$

The program iteration for the left-right symmetric Lund model in $1+1$ dimensions proceeds as follows

1. Randomly select from which string end the fragmentation will take place
2. Select new $q'\bar{q}'$ and hadron to be produced
3. Sample z according to the distribution in Eq.(7)
4. Compute production vertices from Eqs.(8)(9)
5. Update all momenta
6. Proceed through steps (1)-(5) until the center of mass energy of the new string system falls below a given cut off threshold E_{cut}.

One of the main drawbacks of the Lund model as a phenomenological model of hadronization is the reliance on the analytic scaling function $f(z)$. The efficacy of which relies entirely on the fit parameters a and b determined from jet observables. In principle, the scaling function could have higher order corrections in z, flavor dependence, or contain other non-trivial evolution in E which may be difficult to discern from current jet data analysis but could be revealed with more sophisticated detector technology or statistical modeling. In this sense, it may prove beneficial to implement an equivalent phenomenological hadronization model with a fragmentation function that is motivated directly from experimental data — this is an active area of research.

2 Lund string model in 3+1 dimensions

References

[1] Sjöstrand, Torbjörn, et al. "An introduction to PYTHIA 8.2." Computer physics communications 191 (2015): 159-177.

[2] Andersson, Bo, et al. "Parton fragmentation and string dynamics." Physics Reports 97.2-3 (1983): 31-145.